Problem Set 3

Engineering 3422, 2005

To do for Oct 4th.

Part 0. From Section 2.6.2 of Gossett:
(Be sure to include hints in all algebraic proofs and derivations.)

- **Q0** Exercise 2 parts (e), (f), and (g) Carefully define all names of sets and predicates you use.
- **Q1** Exercise 4 parts (b), (c), (d), and (e) and also $\exists y, \forall x, (x + y = x - y)$
- **Q2** Exercise 8 parts (b), (c), and (d)
- **Q3** Exercise 21

Part 1
Q4 Which are the free occurrences of variables in the following expressions.

- (a) $\sum_{i=0}^{n} k \times i^3$
- (b) $\forall i \in \{m, m+1, ..., n\}, A[i] = 10$

Q5
Suppose that we have the following C++ program variables where N is a constant.

```
int i, j ;
double A[N], B[N], x ;
```

Write boolean expressions, using quantifiers as required, for the following assertions. Use *Int* for the set of all numbers representable by int variables in this particular implementation of C++ and *Double* for the set of rational numbers representable by double variables. Be sure that the free variables of your expression match the free variables of my English statement. I'll do the first one for you

- (a) No item of A is equal to x.
  - Solutions: $\neg\exists k \in \{0, 1, ..., N-1\}, A[k] = x$. Equivalently, $\forall k \in \{0, 1, ..., N-1\}, A[k] \neq x$.

- (b) i is the smallest index such that $A[i] = x$. Hint: think " $A[i] = x$ and there is no smaller index k such that $A[k] = x$"
- (c) All items of A from index i up to but excluding index j are equal to x.
- (d) All items of A from index i up to but excluding index j are equal to each other.
- (e) Every item of A is bigger than every item of B.
- (f) For every item of A there is an equal item of B.
- (g) There is an item of A that is equal to every item of B.

Q6

Show (with an algebraic proof) that $(\forall x \in S, f(x) \rightarrow g(x))$ is equivalent to $\neg(\exists x \in S, f(x) \wedge \neg g(x))$

Q7. Consider time to be a nonnegative real number measured in seconds from a common 0. (The set of nonnegative real numbers is usually written as $\mathbb{R}^+$.) Let the following be predicates of time.

- *gearDown*(t) means the landing gear is down at time t
- *gearButtonDepressed*(t) means the button labelled "landing gear down" is depressed at time t
- *gearDownLight*(t) means the light labelled "landing gear" is turned on.

Translate the following English sentences into statements. I'll do the first.

- Whenever the landing gear is has been continuously down for 1 second, the landing gear light should be illuminated.
  - A solution

    $\forall t \in \mathbb{R}^+, \big(\forall u \in \mathbb{R}^+, t \leq u \leq t + 1.0 \rightarrow \mathit{gearDown}(u)\big) \rightarrow \mathit{gearDownLight}(t+1.0)$
- Any time the landing gear has not been down for the previous 0.5 seconds, the landing gear light should be off.
- After the landing gear down button is pressed, the landing gear should be down within 10 seconds.

Q8. Consider as universe, U, the set that contains all sets. Some sets will contain themselves, for example U; some will not, for example $\emptyset$. Let R be a set defined as the set of all sets that not contain themselves $R \triangleq \{x \mid x \notin x\}$.

- (a) Prove that $R \in R$.
- (b) Prove that $R \notin R$.
- (c) Resolve this paradox.