Problem Set 5

Engineering 3422, 2005

Due Oct 18

Q0 Prove the following facts
(a) $\forall a \in \mathbb{Z}, (2|a \leftrightarrow 4|a^2)$
(b) $\forall a \in \mathbb{Z}, (4|2a \leftrightarrow 2|a)$
(c) $\forall a \in \mathbb{Z}, (2|a^2 \leftrightarrow 2|a)$

Q1 Show that $\sqrt{2}$ is not a rational number. [Hint. Use proof by contradiction. Consider the root of 2 expressed as a fraction in lowest terms. Make use of the facts shown in Q0 to show that the square of the numerator both is and is not divisible by 4.]

Q2. Let P be the property of natural numbers (i.e. the predicate) defined by

$$P(n) \triangleq \left(\sum_{j=1}^{n} j = \frac{n(n+1)}{2} \right)$$

Show that $\forall n \in \mathbb{N}, (P(n) \rightarrow P(n+1))$.

Q3. Use prime decomposition to calculate the gcd and the lcm of $a = 2^6 \cdot 3^4 \cdot 5^3 \cdot 7^2 = 31\,752\,000$ and $b = 2^3 \cdot 3^6 \cdot 7^3 = 2000\,376$.

Q4. $n!$ is $1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$. At the end of $10! = 3628\,800$ there are 2 zeros. How many zeros are at the end of 100! ? [Hint. Consider the prime decompositions of the numbers from 1 to 100.