Problem Set 6

Engineering 3422, 2005

Due Oct 25th

(In every case: (a) state what the property of the naturals is; (b) state what must be proved in the base step; (c) prove the base step; (d) state what must be proved in the induction step; (e) prove the induction step, being careful to identify the induction hypotheses as such.)

- Exercise 6 from Section 3.3.5 of Gossett
- Exercise 13 from Section 3.3.5 of Gossett
- Exercise 24 from Section 3.3.5 of Gossett
- **Polygons:** It is a well known fact that interior angles of a triangle sum to 180; you may assume this fact. Show that for all $n \geq 3$, any n-sided polygon has interior angles that sum to $180(n-2)$ degrees. (You may assume that no two edges of a polygon intersect, except that two adjacent edges must intersect at their common end-points.)
  - Interpretation note: I found it interesting that the concept of a polygon, when you look at it closely, is actually rather hard to define. For example it is clear that Figure 1 (a) should be a polygon. But is Figure 1 (b)? And if we allow intersections, then is Figure 1 (c)? The problem with Figure 1 (c) is that it is not clear which angles are the interior ones and which the exterior ones. The theorem is only true of Figure 1 (b) if you consider one of the interior angles to be the 270^o angle at the bottom. The theorem is only true of Figure 1 (c) if you consider the angles to be 45, 45, 315, and -45!
  - Let's define the concept of a polygon inductively. A triangle with three distinct vertices (v_0, v_1, v_2) is a 3-sided polygon. I'll use the convention that vertices are numbered "clockwise". So the interior angles are $\measuredangle v_0v_1v_2$, $\measuredangle v_2v_0v_1$, and$\measuredangle v_1v_2v_0$. For any $n \geq 3$, let $(v_0, v_1, ..., v_{n-1})$ be any n-sided polygon and let v be any point not in $\{v_0, v_1, ..., v_{n-1}\}$ such that the segments (v_{n-1}, v) and (v, v_0) do not intersect any of the segments of the n-sided polygon (other than the inevitable intersections at the end points). Then $(v_0, v_1, ..., v_{n-1}, v)$ is an $(n+1)$-sided polygon and its interior angles are those of the smaller polygon,

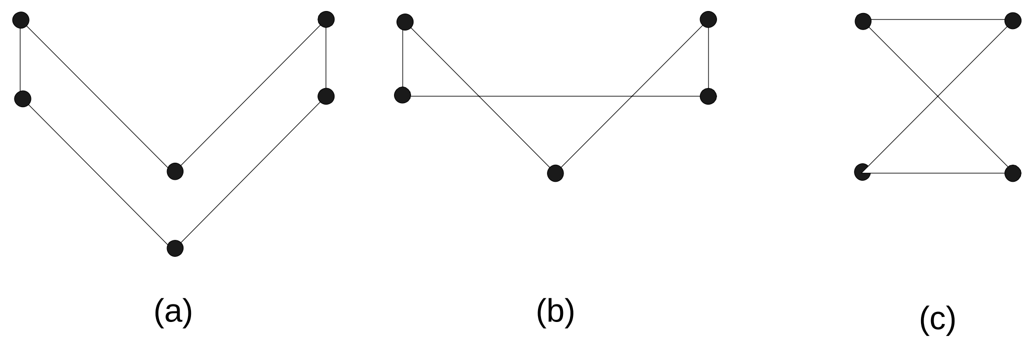

Figure 1:

but with the interior angles $\measuredangle v_{n-2}v_{n-1}v_0$ and $\measuredangle v_{n-1}v_0v_1$ replaced by the three angles $\measuredangle v_{n-2}v_{n-1}v$, $\measuredangle v_{n-1}vv_0$, and $\measuredangle vv_0v$. One consequence of this definition is that the angles of a polygon can not be 0^o, 180^o, or 360^o. Another is that Figures 1 (b) and (c) are ruled out.

- **East Weehawken.** The city of East Weehawken has many one-way streets. Between every pair of (distinct) intersections runs a one-way street. Each one-way street starts at one intersection and ends at another and doesn't pass through any others. (Note that there are a lot of overpasses, so two streets can cross without needing an intersection). Is there a (legal) route that visits (starts at, ends at, or passes through) every intersection exactly once? Prove it.