Problem Set 9

Engineering 3422, 2004

To do for Nov 16

Q0 Which of the following equations describes a function from $\mathbb{R}$ to $\mathbb{R}$. Explain why.
(a) $f(x) = 1/x$
(b) $f(x) = \sqrt{x}$
(c) $f(x) = \pm\sqrt{x^2+1}$

Q1 For each of the following relations, which of the following adjectives apply (it could be more than one).

- Partial Function
- Total Relation
- Function
- One-one
- Onto

In each case, the domain is $\{0,1,2,3\}$ and the range is $\{0,1,2,3\}$ and the graph is given below.

1. $\{(x,y) \mid y = x \bmod 4\}$
2. $\{(x,y) \mid x = y \bmod 4\}$
3. $\{(x,y) \mid x = y \bmod 2\}$
4. $\{(x,y) \mid y = x \bmod 2\}$
5. $\{(x,y) \mid x < 2\}$
6. $\{(x,y) \mid y < 2\}$

Q2.
(a) Show that R is a partial function iff R^{-1} is one-one.
(b) Show that R is a total relation iff R^{-1} is onto.

Q3.
The *composition of S following R*, written $S \circ R$ is a relation such that

- $\mathrm{dom}(S \circ R) = \mathrm{dom}(R)$
- $\mathrm{rng}(S \circ R) = \mathrm{rng}(S)$
- $\mathrm{graph}(S \circ R)$ is such that

$$(x\,(S \circ R)\,y \;\leftrightarrow \exists z, xRz \wedge zSy)\text{, for all } x \in \mathrm{dom}(R), y \in \mathrm{rng}(S)$$

(a) Show that this operation is not commutative.
(b) Show that this operation is associative.
(c) Show that if R and S are both partial functions, then $S \circ R$ is a partial function
(d) Show that if $\mathrm{rng}(R) \subseteq \mathrm{dom}(S)$ and R and S are both total relations, then $S \circ R$ is a total relation
(e) Show that $S \circ R = (R^{-1} \circ S^{-1})^{-1}$