Piping Systems and Flow Analysis
(Chapter 3)
Learning Outcomes (Chapter 3)

• Losses in Piping Systems
  – Major losses
  – Minor losses

• Pipe Networks
  – Pipes in series
  – Pipes in parallel

• Manifolds and Distribution Networks

• Two Phase Flow Models
Mechanical Energy and Flow

- We are interested in flow problems involving pipes, networks, and other systems.
- As we saw earlier, this will involve application of the extended Bernoulli equation or the Mechanical Energy equation when pumps are involved:

\[
\left( \frac{P_1}{\gamma} + \frac{V_1^2}{2g} \right) - \left( \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \right) = \left( z_2 - z_1 \right) + \sum h_{\text{losses}}
\]

- Total Pressure Difference
- Elevation Change
- System Losses
Head Losses or Pressure Drop

• The head loss or pressure drop is due to three contributions:

\[ h_L = \sum \frac{4f_i L_i V_i^2}{D_i} + \sum K_i \frac{V_i^2}{2g} + \sum h_{\text{comp}} \]

\[ \Delta p_L = \sum \frac{4f_i L_i \rho V_i^2}{D_i} + \sum K_i \frac{\rho V_i^2}{2} + \sum \Delta p_{\text{comp}} \]

• Head losses are categorized as either *minor* or *major*.

• Care must be taken to define the “V” through each appropriately. It is better to use mass flow rate:

\[ \dot{m} = \rho \overline{V}_i A_i \]
Minor Losses

• Minor losses are piping losses that result from components such as joints, bends, T’s, valves, fittings, filters, expansions, contractions, etc.

• It does not imply the insignificant!

• On the contrary, minor losses can makeup the majority of pressure drop in small systems dominated by such components.

• Minor losses are modeled two ways:
  – K factors
  – Equivalent pipe length
Minor Losses (cont.)

• The “K” factor method defines the pressure drop according to:

\[ K = \frac{h_f}{V^2/2g} = \frac{\Delta P}{\frac{1}{2} \rho V^2} \]

  – K factors are more widely tabulated.

• The equivalent length method models the loss as an extension of pipe length for each component that yields the same pressure drop.
## Minor Losses (simple)

<table>
<thead>
<tr>
<th>Type of fitting or valve</th>
<th>Loss coefficient (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° elbow, standard</td>
<td>0.35</td>
</tr>
<tr>
<td>45° elbow, long radius</td>
<td>0.2</td>
</tr>
<tr>
<td>90° elbow, standard</td>
<td>0.75</td>
</tr>
<tr>
<td>Long radius</td>
<td>0.45</td>
</tr>
<tr>
<td>Square or miter</td>
<td>1.3</td>
</tr>
<tr>
<td>180° bend, close return</td>
<td>1.5</td>
</tr>
<tr>
<td>Tee, standard, along run, branch blanked off</td>
<td>0.4</td>
</tr>
<tr>
<td>Used as elbow, entering run</td>
<td>1.3</td>
</tr>
<tr>
<td>Used as elbow, entering branch</td>
<td>1.5</td>
</tr>
<tr>
<td>Branching flow</td>
<td>1.0</td>
</tr>
<tr>
<td>Coupling</td>
<td>0.04</td>
</tr>
<tr>
<td>Union</td>
<td>0.04</td>
</tr>
<tr>
<td>Gate valve, open</td>
<td>0.17</td>
</tr>
<tr>
<td>open</td>
<td>0.9</td>
</tr>
<tr>
<td>open</td>
<td>4.5</td>
</tr>
<tr>
<td>open</td>
<td>24.0</td>
</tr>
<tr>
<td>Diaphragm valve, open</td>
<td>2.3</td>
</tr>
<tr>
<td>open</td>
<td>2.6</td>
</tr>
<tr>
<td>open</td>
<td>4.3</td>
</tr>
<tr>
<td>open</td>
<td>21.0</td>
</tr>
<tr>
<td>Globe valve, bevel seat, open</td>
<td>6.4</td>
</tr>
<tr>
<td>open</td>
<td>9.5</td>
</tr>
<tr>
<td>Composition seat, open</td>
<td>6.0</td>
</tr>
<tr>
<td>open</td>
<td>8.5</td>
</tr>
<tr>
<td>Plug disk, open</td>
<td>9.0</td>
</tr>
<tr>
<td>open</td>
<td>13.0</td>
</tr>
<tr>
<td>open</td>
<td>36.0</td>
</tr>
<tr>
<td>open</td>
<td>112.0</td>
</tr>
<tr>
<td>Angle valve, open</td>
<td>3.0</td>
</tr>
<tr>
<td>Y or blowoff valve, open</td>
<td>3.0</td>
</tr>
<tr>
<td>Plug cocks, θ = 5°</td>
<td>0.05</td>
</tr>
<tr>
<td>10°</td>
<td>0.29</td>
</tr>
<tr>
<td>20°</td>
<td>1.56</td>
</tr>
<tr>
<td>40°</td>
<td>17.3</td>
</tr>
<tr>
<td>60°</td>
<td>206.0</td>
</tr>
<tr>
<td>Butterfly valve, θ = 5°</td>
<td>0.24</td>
</tr>
<tr>
<td>10°</td>
<td>0.52</td>
</tr>
<tr>
<td>20°</td>
<td>1.54</td>
</tr>
<tr>
<td>40°</td>
<td>10.8</td>
</tr>
<tr>
<td>60°</td>
<td>118.0</td>
</tr>
<tr>
<td>Check valve, swing</td>
<td>2.0</td>
</tr>
<tr>
<td>Disk</td>
<td>10.0</td>
</tr>
<tr>
<td>Ball</td>
<td>70.0</td>
</tr>
<tr>
<td>Foot valve</td>
<td>15.0</td>
</tr>
<tr>
<td>Water meter, disk</td>
<td>7.0</td>
</tr>
<tr>
<td>Piston</td>
<td>15.0</td>
</tr>
<tr>
<td>Rotary (star-shaped disk)</td>
<td>10.0</td>
</tr>
<tr>
<td>Turbine wheel</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Minor Losses (variable)

<table>
<thead>
<tr>
<th>Component</th>
<th>threaded</th>
<th>flanged, welded, glued, bell &amp; spigot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square edged inlet</td>
<td>$K = 0.5$</td>
<td>$K = 1.5$</td>
</tr>
<tr>
<td>Basket strainer</td>
<td>$K = 1.3$</td>
<td>$K = 1.5/(D_1/D_2)^{0.57}$</td>
</tr>
<tr>
<td>Re-entrant inlet or inward projecting pipe</td>
<td>$K = 1.0$</td>
<td>ID from 0.3 to 4 in</td>
</tr>
<tr>
<td>Well rounded inlet or a bell mouth inlet</td>
<td>$K = 0.05$</td>
<td>ID from 1 to 23 in</td>
</tr>
<tr>
<td>Foot valve</td>
<td>$K = 0.8$</td>
<td>long radius $K = 0.2$</td>
</tr>
<tr>
<td>Exit</td>
<td>$K = 1.0$</td>
<td>ID from 1 to 23 in</td>
</tr>
<tr>
<td>Convergent outlet or nozzle</td>
<td>$K = 0.1(1 - D_2/D_1)$</td>
<td>line flow $K = 0.9$ all sizes</td>
</tr>
<tr>
<td>$D_2/D_1$ from 0.5 to 0.9</td>
<td></td>
<td>ID from 0.3 to 4 in</td>
</tr>
<tr>
<td>90° Elbow</td>
<td>$K = 1.4$</td>
<td>branch flow $K = 1.9$</td>
</tr>
<tr>
<td>$K = 1.4/(D_1/D_2)^{0.58}$</td>
<td>$K = 1.9/(D_1/D_2)^{0.58}$</td>
<td>ID from 0.3 to 4 in</td>
</tr>
<tr>
<td>ID from 0.3 to 4 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45° Elbow</td>
<td>$K = 0.35$</td>
<td>$K = 0.083/(D_1/D_2)^{0.69}$</td>
</tr>
<tr>
<td>$K = 0.35/(D_1/D_2)^{0.6}$</td>
<td>ID from 0.4 to 4 in</td>
<td>ID from 0.3 to 23 in</td>
</tr>
<tr>
<td>ID from 0.3 to 4 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45° Elbow</td>
<td>$K = 0.35$</td>
<td>$K = 0.083/(D_1/D_2)^{0.69}$</td>
</tr>
<tr>
<td>$K = 0.35/(D_1/D_2)^{0.6}$</td>
<td>ID from 0.4 to 4 in</td>
<td>ID from 0.3 to 23 in</td>
</tr>
<tr>
<td>ID from 0.3 to 4 in</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Return bend

T joint

Coupling

Reducing bushing

Sudden expansion
## Minor Losses (variable) (cont.)

<table>
<thead>
<tr>
<th>Valve Type</th>
<th>threaded</th>
<th>flanged, welded, glued, bell &amp; spigot</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Globe Valve</strong></td>
<td><em>K = 10</em></td>
<td><em>K = 10</em></td>
</tr>
<tr>
<td></td>
<td>$K = \exp[2.158 - 0.459 \ln(ID)]$</td>
<td>$K = \exp[2.565 - 0.916 \ln(ID)]$</td>
</tr>
<tr>
<td></td>
<td>+ 0.259[ln(ID)]$^2$</td>
<td>+ 0.399[ln(ID)]$^2$</td>
</tr>
<tr>
<td></td>
<td>$- 0.123[ln(ID)]^3$</td>
<td>$- 0.01416[ln(ID)]^3$</td>
</tr>
<tr>
<td></td>
<td><em>ID</em> from 0.3 to 4 in</td>
<td><em>ID</em> from 0.3 to 4 in</td>
</tr>
<tr>
<td><strong>Gate Valve</strong></td>
<td><em>K = 0.15</em></td>
<td><em>K = 0.15</em></td>
</tr>
<tr>
<td></td>
<td>$K = 0.24(ID)^{0.75}$</td>
<td>$K = 0.78(ID)^{1.14}$</td>
</tr>
<tr>
<td></td>
<td><em>ID</em> from 0.3 to 4 in</td>
<td><em>ID</em> from 1 to 20 in</td>
</tr>
<tr>
<td><strong>Angle Valve</strong></td>
<td><em>K = 2.0</em></td>
<td><em>K = 2.0</em></td>
</tr>
<tr>
<td></td>
<td>$K = 4.5(ID)^{1.08}$</td>
<td>$K = \exp[1.569 - 1.43 \ln(ID)]$</td>
</tr>
<tr>
<td></td>
<td><em>ID</em> from 0.6 to 4 in</td>
<td>+ 0.8[ln(ID)]$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$- 0.137[ln(ID)]^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><em>ID</em> from 1 to 20 in</td>
</tr>
<tr>
<td><strong>Ball Valve</strong></td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0$</td>
</tr>
<tr>
<td></td>
<td>$K = 0.05$</td>
<td>$K = 0.05$</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
<td>2.56</td>
</tr>
<tr>
<td></td>
<td>1.65</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>1.73</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>1.81</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>2.06</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>2.56</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>3.05</td>
<td>5.50</td>
</tr>
<tr>
<td></td>
<td>3.50</td>
<td>6.00</td>
</tr>
<tr>
<td></td>
<td>3.95</td>
<td>6.50</td>
</tr>
<tr>
<td></td>
<td>4.50</td>
<td>7.00</td>
</tr>
<tr>
<td></td>
<td>5.00</td>
<td>7.50</td>
</tr>
<tr>
<td></td>
<td>5.50</td>
<td>8.00</td>
</tr>
<tr>
<td><strong>Check Valves</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Swing Type</td>
<td><em>K = 2.5</em></td>
<td><em>K = 2.5</em></td>
</tr>
<tr>
<td>Ball Type</td>
<td><em>K = 70.0</em></td>
<td><em>K = 70.0</em></td>
</tr>
<tr>
<td>Lift Type</td>
<td><em>K = 12.0</em></td>
<td><em>K = 12.0</em></td>
</tr>
</tbody>
</table>

### Expansion Losses

![Expansion Loss Coefficient](image-url)

\[ h_a = \frac{V_1^2}{2a} \]

\[ h_a = \frac{V_2^2}{2} \]

\[ K_{SC} \approx 0.42(1 - \sigma^2)^2 \]

\[ K_{SE} \approx (1 - \sigma)^2 \]

\[ 0 < \sigma = \frac{A_1}{A_2} < 1 \]
Major Losses

• Major losses are due to piping and due to major components such as heat exchangers or other device for which the flow passes through.

• Piping losses are dealt with using friction factor models, while the major component losses are dealt with using performance data for the component or first principles, i.e. you develop a model for it!
Friction Factors

- Friction factors depend on whether the flow is *laminar* or *turbulent*.
- Pipe geometry also affects the value of the friction factor: *Circular* or *Non-Circular*.
- Surface roughness is also important in turbulent flows.
- Finally in laminar flows, entrance effects (boundary layer development) can be significant if the pipe is short.
- There are many models for pipe friction.
Friction Factors (cont.)

- There are also two definitions of the friction factor.
- The Fanning friction factor is defined according to:
  \[ f_F = \frac{\tau}{\frac{1}{2} \rho V^2} \quad \tau = -\frac{A}{P} \frac{dp}{dx} = -\frac{D_h}{4} \frac{dp}{dx} \]
- The Darcy friction factor is defined according to:
  \[ f_D = \frac{-\frac{dp}{dx}}{\frac{1}{2} \rho V^2} \]
- They are related through:
  \[ f_D = 4 f_F \]
Friction Factors (cont.)

• We will use the Fanning friction factor. The pressure drop is defined according to:

\[ \Delta P = 4f \frac{L}{D_h} \left( \frac{1}{2} \rho V^2 \right) \]

• For non-circular ducts and channels we use the hydraulic diameter rather than “D”, but \( D = D_h \) for a tube:

\[ D_h = \frac{4A_c}{P} \]
Pipe Friction

Friction factor $f$

Reynolds number $Re = \frac{UD}{\nu}$

Relative roughness $k/d$

Materials:
- Riveted steel
- Concrete
- Wood stave
- Cast iron
- Galvanized steel
- Asphalted cast iron
- Commercial steel
- or wrought iron
- Drawn tubing

Laminar flow $Re = \frac{16}{\nu}$

Critical flow $Re_{crit}$

Complete turbulence, rough pipes

Smooth pipes
Friction Factor Models

• Laminar flow ($Re<2300$)

\[ fRe_Dh = 16 \]

\[ f = \frac{16}{Re_Dh} \]

\[ Dh = \frac{4A_c}{P} \]

\[ f = \frac{C}{Re_Dh} \]

– For non-circular ducts we can use:

\[ fRe_Dh = \frac{12\beta^{3/2}}{(1 + \beta) \left[ 1 - \frac{192}{\pi^5\beta} \tanh \left( \frac{\pi}{2\beta} \right) \right]} \left( \frac{4\sqrt{A_c}}{P} \right) \]
Friction Factor Models (cont.)

• Developing Flows:

\[ L_e \approx 0.05D_h Re_{D_h} \]

\[ L << L_e \quad \text{Short Duct} \]
\[ L >> L_e \quad \text{Long Duct} \]

- Entrance effects are negligible if \( L > 10L_e \)

\[ f_{app} Re_{D_h} = \left[ \left( \frac{3.44}{\sqrt{L^*}} \right)^2 + (f Re_{D_h})^2 \right]^{1/2} \]

\[ L^* = \frac{L}{D_h Re_{D_h}} \quad Re_{D_h} = \frac{\rho V D_h}{\mu} \]

\[ \Delta p = \frac{2(f_{app} Re_{D_h}) \dot{m} \nu L}{D_h^2 A_c} \]
Friction Factor Models (cont.)

• Turbulent flow (Re > 4000)
  – Blasius Model (smooth pipes)

\[ f = \frac{0.0791}{Re_{D_h}^{1/4}} \quad 4000 < Re_{D_h} < 100,000 \]

– Swamee and Jain Model (rough pipes)

\[ f = \frac{1}{16 \left[ \log \left( \frac{k/D_h}{3.7} + \frac{5.74}{Re_{D_h}^{9/10}} \right) \right]^2} \quad 4000 < Re_{D_h} \]
Friction Factor Models (cont.)

- Turbulent flow \((Re>4000)\) (cont.)
  - Churchill Model of the Moody Diagram

\[
f = 2 \left[ \left( \frac{8}{Re_{D_h}} \right)^{12} + \left( \frac{1}{(A_1 + A_2)^{3/2}} \right) \right]^{1/12}
\]

\[
A_1 = \left\{ 2.457 \ln \left[ \frac{1}{(7/Re_{D_h})^{0.9} + (0.27k/D_h)} \right] \right\}^{16}
\]

\[
A_2 = \left( \frac{37530}{Re_{D_h}} \right)^{16}
\]
# Pipe Roughness

<table>
<thead>
<tr>
<th>Pipe Material</th>
<th>ε, ft</th>
<th>ε, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steel</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial</td>
<td>0.00015</td>
<td>0.0046</td>
</tr>
<tr>
<td>Corrugated</td>
<td>0.003-0.03</td>
<td>0.09-0.9</td>
</tr>
<tr>
<td>Riveted</td>
<td>0.003-0.03</td>
<td>0.09-0.9</td>
</tr>
<tr>
<td>Galvanized</td>
<td>0.0002-0.0008</td>
<td>0.006-0.025</td>
</tr>
<tr>
<td><strong>Mineral</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brick sewer, Cement-asbestos</td>
<td>0.001-0.01</td>
<td>0.03-0.3</td>
</tr>
<tr>
<td>Clays</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wood stave</td>
<td>0.0006-0.003</td>
<td>0.018-0.09</td>
</tr>
<tr>
<td><strong>Cast iron</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphault coated</td>
<td>0.0004</td>
<td>0.012</td>
</tr>
<tr>
<td>Bituminous lined</td>
<td>0.000008</td>
<td>0.00025</td>
</tr>
<tr>
<td>Cement lined</td>
<td>0.000008</td>
<td>0.00025</td>
</tr>
<tr>
<td>Centrifugally spun</td>
<td>0.00001</td>
<td>0.00031</td>
</tr>
<tr>
<td><strong>Drawn tubing</strong></td>
<td>0.000005</td>
<td>0.00015</td>
</tr>
<tr>
<td><strong>Miscellaneous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brass, Copper, Glass, Lead,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plastic, Tin, Galvanized</td>
<td>0.0002-0.0008</td>
<td>0.006-0.025</td>
</tr>
<tr>
<td>Wrought iron</td>
<td>0.00015</td>
<td>0.0046</td>
</tr>
<tr>
<td>PVC</td>
<td>Smooth</td>
<td>Smooth</td>
</tr>
</tbody>
</table>
Pipe Flow Problems

• There are three types of pipe flow problems:
  – Type I: $\Delta P$ (unknown), $Q$, $L$, and $D$ (known)
  – Type II: $Q$ (unknown), $\Delta P$, $L$, and $D$ (known)
  – Type III: $L$ or $D$ (unknown), $\Delta P, Q$, $L$ or $D$ (known)

• Type I and II problems are “Analysis” problems since the system dimensions are known, and the pressure/flow characteristics is to be solved.

• Type III problems are “Design” or sizing problems, since the flow characteristics are known, and the system dimensions are solved.

• Type II and III problems are “Iterative” as the Reynolds number is unknown when “Q” or “D” are solution variables.
Example 3-1 (Problem 3-7)

- Examine the system given below. The water distribution system is to be designed to give equal mass flow rate to each of the two locations, which are not of equal distance from the source. In order to achieve this, two pipes of different diameter are used. Determine the size of the longer pipe which yields the same mass flow rate. You may assume that all of the kinetic energy is lost at the terminations of the pipeline and that the pressure is atmospheric. The density of water at 20°C is 1000 (kg/m³) and the viscosity is 0.001 (Pa.s). Assume $K=1.5$ for the junction connection.

a) Develop the basic equation for each branch of the system.

b) Determine the required diameter of the longer pipe.

c) Find $P_p$.
Example 3-2 (Problem 3-3)

Examine the electronics packaging enclosure described below. Nine circuit boards are placed in an enclosure with dimensions of $W=50$ (cm), $H=25$ (cm), and $L=45$ (cm) in the flow direction. If the airflow required to adequately cool the circuit board array is $3$ (m/s) over each board, determine the fan pressure required to overcome the losses within the system. Assume each board has an effective thickness of $5$ (mm), which accounts for the effects of the circuit board and components. You may further assume that the roughness of the board is $2.5$ (mm). The air exhausts to atmosphere pressure. In your analysis include the effect of entrance and exit effects due to the reduction in area. The density of air at $20$ (C) is $1.2$ (kg/m$^3$) and the viscosity is $1.81 \times 10^{-5}$ (Pa.s).
Example 3-3 (Problem 3-9)

You are to analyze the flow through a flat plate solar collector system as shown below. The system consists of a series of pipes connected to distribution and collection manifolds. Make any necessary assumptions. Predict the inlet manifold, core, and exit manifold losses for the mechanical component shown below which is to be used in a solar water pre-heater. The design mass flow rate through the system is to be 5 kg/s of water. The inner diameter of the pipes is 12.5 mm and there are 10 in total, each having a length of 80 cm. Assume a pipe roughness for copper tubing. You may neglect friction in the manifolds. The density of water at 20 C is 998.1 kg/m$^3$, and the viscosity is 0.001 Pa.s.
Example 3-4

- Examine the system sketched below. Water is to be pumped from a lake at a rate of 5000 (L/hr) through a pipeline of 30 (cm) diameter, to an elevated reservoir whose free surface is 30 (m) above the lake surface. The pipe intake is submerged 5 (m) below the surface of the lake and the total length of pipe is 250 (m). The pipeline contains four 90 degree flanged large radius elbows.
  - Develop the mechanical energy balance which gives the pressure rise (or head) required by a pump to overcome all losses and changes in elevation to get the water from the lake to the reservoir.
  - If the density of water is 1000 (kg/m$^3$) and the viscosity of water is 0.001 (Pa.s) at 15 (C), calculate the required pump pressure rise at the given flow rate. Assume that the pipe is a commercial grade steel.
  - If a pump capable of delivering a pressure rise of 350 kPa at a desired flow of 1.5 kg/s (5400 L/hr) is chosen, what diameter pipe should be used to achieve this goal. You may neglect roughness for this part only.
Pipe Networks

- Pipes in series and parallel or series/parallel
Pipes in Series

\[ Q_1 = Q_2 = Q_3 = \cdots = Q_N = \text{constant} \]

\[ \Delta p_{A \to B} = \Delta p_1 + \Delta p_2 + \Delta p_3 + \cdots + \Delta p_N \]
Pipes in Parallel

\[ \Delta p_1 = \Delta p_2 = \Delta p_3 = \cdots = \Delta p_N = \text{constant} \]

\[ Q_{A \rightarrow B} = Q_1 + Q_2 + Q_3 + \cdots + Q_N \]
Piping Networks

At any junction:

\[ \sum Q_i = 0 \]

Around any loop:

\[ \sum \Delta p_i = 0 \]
Manifolds and Distribution Networks

- **Manifolds** are used to distribute a fluid within a mechanical system, usually on a small scale.
- On a larger scale, a *distribution network* must be used.
Example 3-5 (Problem 3-8)

- You are to design an air distribution system having the following layout: main line diameter \( D = 50 \) (cm) and four equally spaced branch lines having diameter \( d = 30 \) (cm). Each branch line is to have the same air flow. To achieve this, you propose using a damper having a well defined variable loss coefficient, to control the flow in each branch. Determine the value of the loss coefficient for each damper, such that the system is balanced. Each section of duct work is 5 (m) in length. A total flow of 10 \((m^3/s)\) is to be delivered by a fan. In your solution consider the minor losses at the junctions \( K_B = 0.8 \), \( K_L=0.14 \), and exits \( K = 1.0 \). What fan pressure is required? Assume air properties to be \( \rho = 1.1 \) (kg/m\(^3\)), and \( \mu = 2 \times 10^{-5} \) (Pa·s). Also assume the main line has a fixed damper with a \( K = 25 \).
Example 3-6 (Problem 3-4)

- Examine the series piping system consisting of three pipes each having a length L=1 (m). The diameter of the first pipe is $D_1=0.02664$ (m), the diameter of the second pipe is $D_2=0.07792$ (m), and the diameter of the third pipe is $D_3=0.05252$ (m). Assume the roughness $\epsilon=0.0005$ (m) for each pipe. The fluid properties may be taken to be $\rho=1000$ (kg/m$^3$) and $\mu=0.001$ (Pa.s). Develop the pressure drop versus mass flow rate characteristic for the system assuming at first no minor losses and then include minor losses. What is the pressure drop when the mass flow rate is $m=20$ (kg/s)? How significant are the minor losses relative to the piping losses?
Example 3-7 (Problem 3-5)

Examine the parallel piping system consisting of three pipes each having a length \( L = 1 \) (m). The diameter of the first pipe is \( D_1 = 0.02664 \) (m), the diameter of the second pipe is \( D_2 = 0.07792 \) (m), and the diameter of the third pipe is \( D_3 = 0.05252 \) (m). Assume the roughness \( \varepsilon = 0.0005 \) (m) for each pipe. The fluid properties may be taken to be \( \rho = 1000 \) (kg/m\(^3\)) and \( \mu = 0.001 \) (Pa.s). Develop the pressure drop versus mass flow rate characteristic for each of pipes in the system and the pressure drop versus total mass flow rate characteristic, assuming no minor losses. What is the pressure drop when the total mass flow rate is \( m = 50 \) (kg/s)? At this flow rate what fraction of flow occurs in each branch?
**Example 3-8**

- Consider the parallel piping system shown below. The system contains two heat exchangers with different pressure loss characteristics. If the system as a whole is limited to a 1 (Mpa) pressure drop, determine the flow that occurs through each branch (and hence each heat exchanger). Consider minor losses for all piping elements and pipe friction. The working fluid is water at standard temperature conditions. Also discuss, how the “equivalent” system curve can be developed for this system i.e. a the pressure drop versus total flow rate.

\[
\begin{align*}
L_1 &= L_4 = L_5 = L_6 = 20 \text{ (m)} \\
L_2 &= L_3 = L_6 = L_7 = 50 \text{ (m)} \\
L_i &= L_o = 10 \text{ (m)} \\
D &= 0.1 \text{ (m) (all pipes)} \\
\varepsilon/D &= 0.001 \text{ (all pipes)} \\
\Delta P_1 &= 35.58m_1 + 122.45m_1^2 \\
\Delta P_2 &= 106.74m_2 + 367.35m_2^2
\end{align*}
\]
Example 3-9

- Consider the three reservoir pumping problem sketched below. Develop the necessary equations and solve using a direct solver and Newton-Raphson method.

\[ L_1 = 4000 \text{ (ft)}, \quad L_2 = 1000 \text{ (ft)}, \quad L_3 = 3000 \text{ (ft)} \]
\[ D_1 = 4 \text{ (in)}, \quad D_2 = 2 \text{ (in)}, \quad D_3 = 4 \text{ (in)} \]
\[ Z_1 = 20 \text{ (ft)}, \quad Z_2 = 10 \text{ (ft)}, \quad Z_3 = 30 \text{ (ft)} \]
\[ W_s = 150 - 50Q^2 \]

The system which must be solved is:

\[
\begin{align*}
  f_1: 160 - 50Q_1^2 - 326.2Q_1^{7/4} - 2194.4Q_2^{7/4} &= 0 \\
  f_2: 140 - 50Q_1^2 - 326.2Q_1^{7/4} - 244.6Q_2^{7/4} &= 0 \\
  f_3: Q_1 - Q_2 - Q_3 &= 0
\end{align*}
\]
Example 3-10 (Problem 3-6)

- Water flows in a pipe network shown below. The pipes forming the network have the following dimensions: \( L_1=1777.7 \) (m), \( D_1=0.2023 \) (m), \( L_2=1524.4 \) (m), \( D_2=0.254 \) (m), \( L_3=1777.7 \) (m), \( D_3=0.3048 \) (m), \( L_4=914.6 \) (m), \( D_4=0.254 \) (m), \( L_5=914.6 \) (m), and \( D_5=0.254 \) (m). If the mass flow rate entering the system is \( m_A=50 \) (kg/s) and \( m_B=25 \) (kg/s) and \( m_C=25 \) (kg/s) are drawn off the system at points B and C, compute the pressure drops and flow in each section of pipe. Ignore minor losses and assume that each junction is at the same elevation.
Two Phase Flows

- Two phase flows occur when gas/liquid, liquid/liquid, or solid/liquid flow together.
- Most calculations are done with simple models, but more accurate predictions use phenomenological models (models for special types of flow).
- For gas/liquid two phase flows, we frequently use flow maps to determine the types of flow.
- For pressure drop calculations we will use simple models.
Two Phase Flows (Vertical)

GAS FLOW INCREASING
Two Phase Flows (Vertical) (cont.)

Fig. 3.14 - Two Phase Flow Map for Vertical Up Pipe Flow
Two Phase Flows (Horizontal)
Fig. 3.16 - Two Phase Flow Map for Horizontal Pipe Flow
Two Phase Flows: Models

- Two phase flow pressure drop is composed of three contributions:
  - Friction (due to mixture shear stress at walls and interface motion)
  - Acceleration (due to changes in density)
  - Gravitational (due to elevation changes)

\[-\frac{dp}{dz} = \frac{4\tau_w}{D_h} + \bar{\rho}g \sin(\theta) + G^2 \frac{d}{dz} \left( \frac{1}{\bar{\rho}} \right)\]
Two Phase Flows: Models (cont.)

• We must consider the basic rules of mixtures for undertaking calculations:
  
  – Mixture density:
    \[ \bar{\rho} = \alpha_g \rho_g + \alpha_l \rho_l = \left[ \frac{x}{\rho_g} + \frac{1-x}{\rho_l} \right]^{-1} \]

  – Void and liquid fractions:
    \[ \alpha_g = \frac{V_g}{V} = \frac{A_g}{A} \quad \alpha_l = \frac{V_l}{V} = \frac{A_l}{A} \quad \alpha_g + \alpha_l = 1 \]

  – Mass flux:
    \[ G = \rho U = \frac{\dot{m}}{A} \quad G_g = \rho_g U_g \quad G_l = \rho_l U_l \]

  – Mixture quality:
    \[ x = \frac{G_g}{G_g + G_l} \]

  – Phase (actual) velocity:
    \[ u_g = \frac{U_g}{\alpha_g} = \frac{G_g}{\rho_g \alpha_g} \quad u_l = \frac{U_l}{\alpha_l} = \frac{G_l}{\rho_l \alpha_l} \]
Two Phase Flows: Models (cont.)

- Two phase flow models utilize the concept of a “multiplier” to correct a reference pressure drop or pressure gradient.
  - Based on component (phase) mass flux:
    \[
    \phi_g^2 = \frac{(dp/dz)}{(dp/dz)_g} \\
    \phi_i^2 = \frac{(dp/dz)}{(dp/dz)_l}
    \]
  - Based on an individual phase but with total mass flow:
    \[
    \phi_{g_0}^2 = \frac{(dp/dz)}{(dp/dz)_{g_0}} \\
    \phi_{i_0}^2 = \frac{(dp/dz)}{(dp/dz)_{i_0}}
    \]
Two Phase Flows: Models (cont.)

• Three common models used in practice:
  – Lockhart-Martinelli (simplest)
  – Chisholm (more complex)
  – Freidel (even more complex)

• Other more complex models exist, but should only be used when you know the type of flow that you have, i.e. slug, stratified, annular, etc.

• There is great uncertainty in all models due to complex nature of the flow. Expect (+/-) 20% error for a good model and (+/-) 50% or more for a simple model.
Lockhart-Martinelli Model

- The simplest and first model:

\[
\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2}
\]

\[
\phi_g^2 = 1 + CX + X^2
\]

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Gas</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Turbulent</td>
<td>20</td>
</tr>
<tr>
<td>Laminar</td>
<td>Turbulent</td>
<td>12</td>
</tr>
<tr>
<td>Turbulent</td>
<td>Laminar</td>
<td>10</td>
</tr>
<tr>
<td>Laminar</td>
<td>Laminar</td>
<td>5</td>
</tr>
</tbody>
</table>
Lockhart-Martinelli Model (cont.)

![Graph showing Lockhart-Martinelli Model results](image-url)
Chisholm Model

- The Chisholm model is more complex (and accurate to some extent):

\[
\phi_{lo}^2 = 1 + (Y^2 - 1) \left[ B x^{(2-n)/2} (1 - x)^{2-n}/2 + x^{2-n} \right]
\]

\[
B = \begin{cases} 
55/G^{1/2} & 0 < Y < 9.5 \\
520/(Y \cdot G^{1/2}) & 9.5 < Y < 28 \\
15000/(Y^2 \cdot G^{1/2}) & Y > 28 
\end{cases}
\]

\[
Y^2 = \frac{(dp/dz)_{go}}{(dp/dz)_{lo}}
\]
Freidel Model

- The Freidel model is yet more accurate (based on more than 25000 datapoints):

\[
\phi_{lo}^2 = E + \frac{3.24F \cdot H}{Fr^{0.045} We^{0.035}}
\]

\[
E = (1 - x)^2 + x^2 \left( \frac{\rho_l f_{go}}{\rho_g f_{lo}} \right)
\]

\[
F = x^{0.78} (1 - x)^{0.24}
\]

\[
H = \left( \frac{\rho_l}{\rho_g} \right)^{0.91} \left( \frac{\mu_g}{\mu_l} \right)^{0.19} \left( 1 - \frac{\mu_g}{\mu_l} \right)^{0.7}
\]

\[
Fr = \frac{G^2}{gD_h \bar{D}^2}
\]

\[
We = \frac{G^2 D_h}{\bar{D} \sigma}
\]
Model selection criteria

• Choose models based on the following for greater accuracy:
  • For $\mu_l/\mu_g < 1000$, the Friedel model should be used.
  • For $\mu_l/\mu_g > 1000$ and $\hat{m} > 100$, the Chisholm model should be used.
  • $\mu_l/\mu_g > 1000$ and $\hat{m} < 100$, the Lockhart-Martinelli should be used.

• Otherwise can use other models to estimate limits or bounds on parameters. (see notes)
Example 3-11

- Air and water flow in a three inch diameter pipe. The mass flux is $G=500 \text{ (kg/s/m}^2\text{)}$ and the quality is $x=0.1$. Determine the frictional pressure gradient required to move the flow using the Lockhart-Martinelli, Chisolm, and Friedel models. Assume $T=30 \text{ (C)}$. 
Example 3-12

- Oil and gas flow in a vertical oil well approximately 1000 (m) deep and roughly 4 inches in diameter. Once at the surface the mixture is separated and it is determined that the oil flow rate is 150,000 (L/hr) (roughly 1300 (barrels/hr)) and the gas flow rate is 285,000 (L/hr) (roughly 10,000 (ft³/hour)). The density and viscosity of the oil phase are approximately 920 (kg/m³) and 0.12 (Pa.s) while the density and viscosity of the gas phase at surface conditions are approximately 0.68 (kg/m³) and 0.0001027 (Pa.s). Determine:

  - the phase quality of the mixture, i.e. “x”
  - the mixture density
  - the frictional pressure gradient of the two phase mixture in the well
  - the pressure at the bottom of the well assuming that the quality remains constant throughout the flow to the surface and the pressure in the separator is 300 (kPa)