

ASSIGNMENT 2

Problem I (25)

You are given an information source having a four symbol alphabet $A = \{\text{start}, -1, +1, \text{stop}\}$.

The source obeys the following rules:

- a) a “start” symbol must always be followed by either a “+1” or a “-1” symbol,
 - b) the probability that A emits a “-1” at any given time is always 3 times the probability that A emits a “+1”,
 - c) the probability a “+1” or a “-1” is followed by a “stop” symbol is 0.2,
 - d) a “stop” symbol is always followed by a “start” symbol.
- 1) **(10)** With the source states defined as S_0 : “start”, S_1 : “-1”, S_2 : “+1” and S_3 : “stop”, draw a state diagram for this source and write the corresponding transition probability matrix.
 - 2) **(6)** Calculate the steady-state probabilities of S_0 , S_1 , S_2 and S_3 .
 - 3) **(3)** Find the entropy rate for this source.
 - 4) **(6)** Calculate the steady-state symbol probabilities and the entropy of a memoryless source having these symbols probabilities. Compare this result with that obtained for 3). Explain.

Problem II (25)

A certain code has an alphabet consisting of the following code words:

0000000	1000111	0101011	0011101
1101100	1011010	0110110	1110001

- a) **(10)** Show that these code words form a vector space
- b) **(10)** Find the minimum Hamming distance of this code and determine its error correction and error detection capabilities.
- c) **(5)** Determine the code rate.

Problem III (10)

An (8,2) repetition code is defined to have code vectors of the form $\bar{\mathbf{C}} = (m_1 m_0 m_1 m_0 m_1 m_0 m_1 m_0)$, where $\bar{\mathbf{m}} = (m_1 m_0)$ is the message.

- a) (5) What is the generator matrix for this code? What are the basis vectors for this code?
- b) (5) What is the minimum Hamming distance of this code? Determine its error correction and detection capabilities.

Problem IV (15)

- a) (5) Using the (7,4) systematic Hamming code given in Examples 4.5.1 and 4.5.2 from the textbook, p. 141, generate the code words for the messages (0100), (0101), (1110) and (1001).
- b) (5) Construct the standard array and syndrome table for the chosen code.
- c) (5) By using the syndrome decoding method, decode the received vectors (1101001), (0010111), (1111100), and (1011001).