Chapter 5

Heat Exchangers

5.1 Introduction

Heat exchangers are devices used to transfer heat between two or more fluid streams at different temperatures. Heat exchangers find widespread use in power generation, chemical processing, electronics cooling, air-conditioning, refrigeration, and automotive applications. In this chapter we will examine the basic theory of heat exchangers and consider many applications. In addition, we will examine various aspects of heat exchanger design and analysis.

5.2 Heat Exchanger Classification

Due to the large number of heat exchanger configurations, a classification system was devised based upon the basic operation, construction, heat transfer, and flow arrangements. The following classification as outlined by Kakac and Liu (1998) will be discussed:

- Recuperators and regenerators

- Transfer processes: direct contact or indirect contact

- Geometry of construction: tubes, plates, and extended surfaces

- Heat transfer mechanisms: single phase or two phase flow

- Flow Arrangement: parallel flow, counter flow, or cross flow

Figure 1 shows a more comprehensive classification using these basic criteria.

5.3 Heat Exchanger Design Methods

The goal of heat exchanger design is to relate the inlet and outlet temperatures, the overall heat transfer coefficient, and the geometry of the heat exchanger, to the rate of heat transfer between the two fluids. The two most common heat exchanger design problems are those of *rating* and *sizing*. We will limit ourselves to the design of recuperators only. That is, the design of a two fluid heat exchanger used for the purposes of recovering waste heat.

We will begin first, by discussing the basic principles of heat transfer for a heat exchanger. We may write the enthalpy balance on either fluid stream to give:

$$Q_c = \dot{m}_c(h_{c2} - h_{c1}) \tag{5.1}$$

and

$$Q_h = \dot{m}_h(h_{h1} - h_{h2}) \tag{5.2}$$

For constant specific heats with no change of phase, we may also write

$$Q_c = (\dot{m}c_p)_c(T_{c2} - T_{c1}) \tag{5.3}$$

and

$$Q_h = (\dot{m}c_p)_h(T_{h1} - T_{h2}) \tag{5.4}$$

Now from energy conservation we know that $Q_c = Q_h = Q$, and that we may relate the heat transfer rate Q and the overall heat transfer coefficient U, to the some mean temperature difference ΔT_m by means of

$$Q = UA\Delta T_m \tag{5.5}$$

where A is the total surface area for heat exchange that U is based upon. Later we shall show that

$$\Delta T_m = f(T_{h1}, T_{h2}, T_{c1}, T_{c2}) \tag{5.6}$$

It is now clear that the problem of heat exchanger design comes down to obtaining an expression for the mean temperature difference. Expressions for many flow configurations, i.e. parallel flow, counter flow, and cross flow, have been obtained in the heat transfer field. We will examine these basic expressions later. Two approaches to heat exchanger design that will be discussed are the LMTD method and the effectiveness - NTU method. Each of these methods has particular advantages depending upon the nature of the problem specification.

5.3.1 Overall Heat Transfer Coefficient

A heat exchanger analysis always begins with the determination of the overall heat transfer coefficient. The overall heat transfer coefficient may be defined in terms of

individual thermal resistances of the system. Combining each of these resistances in series gives:

$$\frac{1}{UA} = \frac{1}{(\eta_o h A)_i} + \frac{1}{S k_w} + \frac{1}{(\eta_o h A)_o} \qquad (5.7)$$

where η_0 is the surface efficiency of inner and outer surfaces, h is the heat transfer coefficients for the inner and outer surfaces, and S is a shape factor for the wall separating the two fluids.

The surface efficiency accounts for the effects of any extended surface which is present on either side of the parting wall. It is related to the fin efficiency of an extended surface in the following manner:

$$\eta_o = \left(1 - (1 - \eta_f) \frac{A_f}{A} \right) \qquad (5.8)$$

The thermal resistances include: the inner and outer film resistances, inner and outer extended surface efficiencies, and conduction through a dividing wall which keeps the two fluid streams from mixing. The shape factors for a number of useful wall configurations are given below in Table 1. Additional results will be presented for some complex doubly connected regions.

Equation (5.7) is for clean or unfouled heat exchanger surfaces. The effects of fouling on heat exchanger performance is discussed in a later section. Finally, we should note that

$$UA = U_o A_o = U_i A_i \qquad (5.9)$$

however,

$$U_o \neq U_i \qquad (5.10)$$

Finally, the order of magnitude of the thermal resistances in the defintion of the overall heat transfer coefficient can have a significant influence on the calculation of the overall heat transfer coefficient. Depending upon the nature of the fluids, one or more resistances may dominate making additional resistances unimportant. For example, in Table 2 if one of the two fluids is a gas and the other a liquid, then it is easy to see that the controlling resistance will be that of the gas, assuming that the surface area on each side is equal.

Table 1 - Shape Factors

Geometry	S
Plane Wall	$\dfrac{A}{t}$
Cylindrical Wall	$\dfrac{2\pi L}{\ln\left(\dfrac{r_o}{r_i}\right)}$
Spherical Wall	$\dfrac{4\pi r_i r_o}{r_o - r_i}$

Table 2 - Order of Magnitude of h
Kakac (1991)

Fluid	$h \ [W/m^2 K]$
Gases (natural convection)	5-25
Gases (forced convection)	10-250
Liquids (non-metal)	100-10,000
Liquid Metals	5000-250,000
Boiling	1000-250,000
Condensation	1000-25,000

5.3.2   LMTD Method

The log mean temperature difference (LMTD) is derived in all basic heat transfer texts. It may be written for a parallel flow or counterflow arrangement. The LMTD has the form:

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}} \tag{5.11}$$

where ΔT_1 and ΔT_2 represent the temperature difference at each end of the heat exchanger, whether parallel flow or counterflow. The LMTD expression assumes that

the overall heat transfer coefficient is constant along the entire flow length of the heat exchanger. If it is not, then an incremental analysis of the heat exchanger is required. The LMTD method is also applicable to crossflow arrangements when used with the crossflow correction factor. The heat transfer rate for a crossflow heat exchanger may be written as:

$$Q = FUA\Delta T_{LMTD} \tag{5.12}$$

where the factor F is a correction factor, and the log mean temperature difference is based upon the counterflow heat exchanger arrangement.

The LMTD method assumes that both inlet and outlet temperatures are known. When this is not the case, the solution to a heat exchanger problem becomes somewhat tedious. An alternate method based upon heat exchanger effectiveness is more appropriate for this type of analysis. If $\Delta T_1 = \Delta T_2 = \Delta T$, then the expression for the LMTD reduces simply to ΔT.

5.3.3 $\epsilon - $ NTU Method

The effectiveness / number of transfer units (NTU) method was developed to simplify a number of heat exchanger design problems. The heat exchanger effectiveness is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate if there were infinite surface area. The heat exchanger effectiveness depends upon whether the hot fluid or cold fluid is a minimum fluid. That is the fluid which has the smaller capacity coefficient $C = \dot{m}C_p$. If the cold fluid is the minimum fluid then the effectiveness is defined as:

$$\epsilon = \frac{C_{max}(T_{H,in} - T_{H,out})}{C_{min}(T_{H,in} - T_{C,in})} \tag{5.13}$$

otherwise, if the hot fluid is the minimum fluid, then the effectiveness is defined as:

$$\epsilon = \frac{C_{max}(T_{C,out} - T_{C,in})}{C_{min}(T_{H,in} - T_{C,in})} \tag{5.14}$$

We may now define the heat tansfer rate as:

$$Q = \epsilon C_{min}(T_{H,in} - T_{C,in}) \tag{5.15}$$

It is now possible to develop expressions which relate the heat exchanger efectiveness to another parameter referred to as the nmber of transfer units (NTU). The value of NTU is defined as:

$$NTU = \frac{UA}{C_{min}} \tag{5.16}$$

It is now a simple matter to solve a heat exchanger problem when

$$\epsilon = f(NTU, C_r) \tag{5.17}$$

where $C_r = C_{min}/C_{max}$.

Numerous expressions have been obtained which relate the heat exchanger effectiveness to the number of transfer units. The handout summarizes a number of these solutions and the special cases which may be derived from them.

For convenience the $\epsilon - NTU$ relationships are given for a simple double pipe heat exchanger for parallel flow and counter flow:

Parallel Flow

$$\epsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r} \tag{5.18}$$

or

$$NTU = \frac{-\ln[1 - \epsilon(1 + C_r)]}{1 + C_r} \tag{5.19}$$

Counter Flow

$$\epsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 + C_r \exp[-NTU(1 - C_r)]}, \quad C_r < 1 \tag{5.20}$$

and

$$\epsilon = \frac{NTU}{1 + NTU}, \quad C_r = 1 \tag{5.21}$$

or

$$NTU = \frac{1}{C_r - 1} \ln\left(\frac{\epsilon - 1}{\epsilon C_r - 1}\right), \quad C_r < 1 \tag{5.22}$$

and

$$NTU = \frac{\epsilon}{1 - \epsilon}, \quad C_r = 1 \tag{5.23}$$

For other configurations, the student is referred to the Heat Transfer course text, or the handout. Often manufacturer's choose to present heat exchanger performance in terms of the inlet temperature difference $ITD = (T_{h,i} - T_{c,i})$. This is usually achieved by plotting the normalized parameter $Q/ITD = Q/(T_{h,i} - T_{c,i})$. This is a direct consequence of the $\epsilon - NTU$ method.

5.4   Heat Exchanger Pressure Drop

Pressure drop in heat exchangers is an important consideration during the design stage. Since fluid circulation requires some form of pump or fan, additional costs are incurred as a result of poor design. Pressure drop calculations are required for both fluid streams, and in most cases flow consists of either two internal streams or an internal and external stream. Pressure drop is affected by a number of factors, namely the type of flow (laminar or turbulent) and the passage geometry.

First, a fluid experiences an entrance loss as it enters the heat exchanger core due to a sudden reduction in flow area, then the core itself contributes a loss due to friction and other internal losses, and finally as the fluid exits the core it experiences a loss due to a sudden expansion. In addition, if the density changes through the core as a result of heating or cooling an accaleration or decelaration in flow is experienced. This also contributes to the overall pressure drop (or gain). All of these effects are discussed below.

Entrance Loss

The entrance loss for an abrupt contraction may be obtained by considering Bernoulli's equation with a loss coefficient combined with mass conservation to obtain:

$$\Delta p_i = \left(1 - \sigma_i^2 + K_c\right) \frac{1}{2} \frac{G^2}{\rho_i} \tag{5.24}$$

where σ is the passage contraction ratio and $G = \dot{m}/A$, the mass flux of fluid. In general,

$$\sigma = \frac{\text{minimum flow area}}{\text{frontal area}} \tag{5.25}$$

Core Loss

In the core, we may write the pressure drop in terms of the *Fanning* friction factor:

$$\Delta p_c = \frac{4fL}{D_h} \frac{1}{2} \frac{G^2}{\rho_m} \tag{5.26}$$

Since the fluid density may change appreciably in gas flows, acceleration or deceleration may occur. We consider a momentum balance across the core

$$\Delta p_a A_c = \dot{m}(V_e - V_i) \tag{5.27}$$

which may be written as

$$\Delta p_a = G^2 \left(\frac{1}{\rho_e} - \frac{1}{\rho_i}\right) \tag{5.28}$$

after writing $V = G/\rho$, since $G_i = G_e$.

Exit Loss

Finally, as the flow exits the core, the fluid may pass through a sudden expansion. Application of Bernoulli's equation with mass conservation results in

$$\Delta p_e = -\left(1 - \sigma_e^2 - K_e\right) \frac{1}{2} \frac{G^2}{\rho_e} \tag{5.29}$$

where we have assumed that pressure drop (rise) is from left to right. Once again σ is the area contraction ratio and G is the mass flux of fluid.

Total Pressure Drop

The total pressure drop across the heat exchanger core is obtained by taking the sum of all of these contributions. That is

$$\Delta p = \Delta p_i + \Delta p_c + \Delta p_a + \Delta p_e \tag{5.30}$$

Combining all of the effects and rearranging, yields the following general expression for predicting pressure drop in a heat exchanger core:

$$\Delta p = \frac{G^2}{2\rho_i} \left[\left(1 - \sigma_i^2 + K_c\right) + f\frac{4L}{D_h}\left(\frac{\rho_i}{\rho_m}\right) + 2\left(\frac{\rho_i}{\rho_e} - 1\right) - \left(1 - \sigma_e^2 - K_e\right)\left(\frac{\rho_i}{\rho_e}\right) \right] \tag{5.31}$$

Now the fluid pumping power is related to the overall pressure drop through application of conservation of energy

$$\dot{W}_p = \frac{1}{\eta_p}\frac{\dot{m}}{\rho}\Delta P \tag{5.32}$$

where η_p is the pump efficiency. The efficiency accounts for the irreversibilities in the pump, i.e. friction losses.

Entrance and exit loss coefficients have been discussed earlier in Chapter 3. The handout provides some additional information useful in the design of heat exchangers. It is clear that a Reynolds number dependency exists for the expansion and contraction loss coefficients. However, this dependency is small. For design purposes we may approximate the behaviour of these losses by merely considering the $Re = \infty$ curves. These curves have the following approximate equations:

$$K_e = (1 - \sigma)^2 \tag{5.33}$$

and

$$K_c \approx 0.42(1 - \sigma^2)^2 \tag{5.34}$$

5.5   Analysis of Extended Surfaces

Extended surfaces also known as fins, are widely used as a means of decreasing the thermal resistance of a system. The addition of fins as a means of increasing the overall heat transfer rate is widely employed in compact heat exchanger and heat sink design. The aim of this chapter is to develop and present the theory of extended surfaces. A large number of analytic solutions for various types of fins will be presented in detail in addition to the development of numerical methods for complex fin geometries whose solution are not possible by analytic means.

5.5.1 One Dimensional Conduction with Convection

We begin first, by deriving the equation for one dimensional conduction with convection from first principles. The governing equation will be derived in general terms such that the results may be applied to both axial flow systems such as longitudinal and pin fins, as well as radial flow systems such as the family of circular annular fins.

The governing equation for one dimensional flow with convection may be derived in general terms for longitudinal, radial, or pin fin configurations. Beginning with an arbitrary control volume, the conduction into the left face is given by the Fourier rate equation

$$Q_{cond,u} = -kA\frac{dT}{du} \tag{5.35}$$

where k is the thermal conductivity, $A = A(u)$ is the cross-sectional area, $T = T(u)$ is the temperature, and u is the inward directed normal with respect to a particular coordinate system, i.e., for longitudinal or pin fins $u = x$ and for radial fins $u = r$.

The conduction leaving the right face is

$$Q_{cond,u+du} = -kA\frac{dT}{du} + \frac{d}{du}\left(-kA\frac{dT}{du}\right)du \tag{5.36}$$

where du is the width of the control volume in the direction of conduction.

The convection loss at the surface of the control volume is obtained from Newton's Law of cooling

$$Q_{conv} = hPds(T - T_f) \tag{5.37}$$

where h is the convection heat transfer coefficient, $P = P(u)$ is the perimeter, ds is the arc length of the lateral surface, and T_f is the ambient fluid temperature.

Taking an energy balance over the control volume requires that

$$Q_{cond,u} - Q_{cond,u+du} - Q_{conv} = 0 \tag{5.38}$$

which results in the following differential equation

$$\frac{d}{du}\left(kA\frac{dT}{du}\right) - hP\frac{ds}{du}(T - T_f) = 0 \tag{5.39}$$

Now, introducing the temperature excess $\theta = T(u) - T_f$

$$\frac{d}{du}\left(A\frac{d\theta}{du}\right) - \frac{hP}{k}\frac{ds}{du}\theta = 0 \tag{5.40}$$

gives the governing equation for one dimensional heat conduction with convection. Expansion of the differential yields

$$A\frac{d^2\theta}{du^2} + \frac{dA}{du}\frac{d\theta}{du} - \frac{hP}{k}\frac{ds}{du}\theta = 0 \tag{5.41}$$

The governing equation is valid for both axial and radial systems having varying cross-sectional area and profile. The term ds/du is the ratio of lateral surface area to the projected area. It is related to the profile function $y(u)$ through

$$\frac{ds}{du} = \sqrt{1 + \left(\frac{dy}{du}\right)^2} \tag{5.42}$$

The above equation may be taken as unity, i.e. $(dy/du)^2 \approx 0$ for slender fin profiles, without incurring large errors. Thus, for slender fins having varying cross-sectional area and profile the governing equation becomes

$$A\frac{d^2\theta}{du^2} + \frac{dA}{du}\frac{d\theta}{du} - \frac{hP}{k}\theta = 0 \tag{5.43}$$

The governing equation for one dimensional conduction with convection is applicable to systems in which the lateral conduction resistance is small relative to the convection reistance. Under these conditions the temperature profile is one dimensional. The conditions for which Eq. (5.37) is valid are determined from the following criterion:

$$Bi = \frac{hb}{k} < 0.1 \tag{5.44}$$

where Bi is the Biot number based upon the maximum half thickness of the fin profile. The fin Biot number is simply the ratio of the lateral conduction to lateral convection resistance

$$Bi = \frac{R_{conduction}}{R_{convection}} = \frac{\dfrac{b}{kA}}{\dfrac{1}{hA}} \tag{5.45}$$

5.5.2   Boundary Conditions

The general fin equation is subject to the following boundary conditions at the fin tip $(u = u_e)$

$$\frac{d\theta(u_e)}{du} + \frac{h_e}{k}\theta(u_e) = 0 \tag{5.46}$$

for truncated fins where h_e is the convection heat transfer coefficient for the edge surfaces, or

$$\frac{d\theta(u_e)}{du} = 0 \tag{5.47}$$

the adiabatic tip condiction. At the fin base $(u = u_o)$

$$\theta(u_o) = \theta_o \tag{5.48}$$

is generally prescribed.

For axial fins it will be convenieint to take $u_e = 0$ and $u_o = L$, while for radial fins $u_e = r_o$ and $u_o = r_i$. In subsequent sections, analytic results will be obtained for each class of fin for various profile shapes. Once the solution for the temperature excess for a particular case has been found, the solution for the heat flow at the fin base may be obtained from the Fourier rate equation

$$Q_b = -kA\frac{d\theta}{du} \tag{5.49}$$

applied to the base of the fin.

5.5.3 Fin Performance

Fin performance has traditionally been measured by means of the fin efficiency or fin effectiveness. Fin efficiency may be defined as

$$\eta_f = \frac{Q_b}{Q_{max}} = \frac{Q_b}{hA_s\theta_b} \tag{5.50}$$

where Q_{max} is the maximum heat transfer rate if the temperature at every point within the fin were at the base temperature θ_b. The fin effectiveness may be defined as

$$\epsilon = \frac{Q_{b,fin}}{Q_{b,bare}} = \frac{Q_b}{hA_b\theta_b} \tag{5.51}$$

where $Q_{b,bare}$ is the heat transfer from the base of the fin when the fin is not present, i.e. $L \to 0$.

In many heat sink design applications, it is often more convenient to consider the fin resistance defined as

$$R_{fin} = \frac{\theta_b}{Q_b} \tag{5.52}$$

The use of the fin resistance is more appropriate for modelling heat sink systems, since additional resistive paths may be considered.

5.5.4 Analytical Solutions

In this section we examine the many analytical solutions which have been obtained various fin configurations. In most cases, the solutions for the temperature distribution involve special functions such as the modified Bessel functions. A thorough review of analytical methods pertaining to extended surfaces may be found in the classic text by Kern and Kraus (1972). While more brief reviews are found in most advanced texts on heat conduction such as those by Arpaci (1966) and Schnieder (1955), along with the more general advanced heat transfer texts such as those by

Jakob (1949) and Eckert and Drake (1972). Analytical methods have been success-fully applied to a number of applications of extended surfaces such as longitudinal fins, pin fins, and circular annular fins. A table of widely used solutions is provided in the class notes.

5.6 Typical Heat Exchanger Designs

We will now examine several common heat exchanger designs and discuss examples which highlight the differences in each of the configurations. We shall consider: Double Pipe Heat Exchangers, Shell and Tube Heat Exchnagers, Compact Heat Exchangers, Plate and Frame Heat Exchangers, and Boilers, Condensers, and Evaporators.

5.6.1 Douple Pipe Exchangers

The double pipe heat exchanger is probably one of the simplest configurations found in applications. It consists of two concentric circular tubes with one fluid flowing inside the inner tube and the other fluid flowing inside the annular space between the tubes. Its primary uses are in cooling process fluids where small heat transfer areas are required. It may be designed in a number of arrangements such as parallel flow and counterflow, and combined in series or parallel arrangements with other heat exchangers to form a system.

For this configuration the overall heat transfer coefficient is given by:

$$\frac{1}{UA} = \frac{1}{h_i(2\pi r_i L)} + \frac{\ln(r_o/r_i)}{2\pi k_w L} + \frac{1}{h_o(2\pi r_o L)} \tag{5.53}$$

where r_i and r_o denote the radii of the inner pipe. The heat transfer coefficient h_i is computed for a pipe while the heat transfer coefficient h_o is computed for the annulus. If both fluids are in turbulent flow, the heat transfer coefficients may be computed using the same correlation with $D = D_h$, otherwise, special attention must be given to the annular region.

The pressure drop for each fluid may be determined from:

$$\Delta p = \left[\Sigma \frac{4fL}{D_h} + \Sigma K \right] \frac{1}{2}\rho V^2 \tag{5.54}$$

However, care must be taken to understand the nature of the flow, i.e. series, parallel, or series-parallel.

Example 5.1

Examine the following double pipe heat exchanger. Water flowing at 5000 kg/hr is to be heated from 20 C to 35 C by using hot water from another source at 100 C. If the temperature drop of the hot water is not to exceed 15 C, how much tube length is needed in a parallel flow arrangement if a nominal 3 inch outer pipe and nominal 2 inch inner pipe are used. Assume teh inner pipe wall thickness is 1/8 inch

carbon steel with thermal conductivity $k = 54 \ W/mK$. If the heat exchanger is to be composed of 1.25 m U-tubes connected in series, how many will be required. Also, determine the pressure drop for each fluid stream. The heat exchanger is insulated to prevent losses, and the hot water flows in the in the annulus while the cold water which is to be heated flows in the in inner pipe.

Example 5.2

You are to consider three flow arrangements for a double pipe heat exchanger: series, parallel, series-parallel. Each configuration consists of eight pipes of equal length ($L = 1 \ m$). The inner pipe has a diameter $D = 1 \ cm$. Each configuration of heat exchanger is to have the same overall heat transfer rate $Q = 15 \ kW$ and have the same inlet $T_i = 100 \ C$ and exit $T_e = 85 \ C$ temperatures. Assuming that these criteria may be met by the coolant side (annulus), calculate the expected pressure drop for each case. In your analysis, consider minor losses and additional flow length due to the pipe bends. The radius of the pipe bend is $R = 12.5 \ cm$, and each bend has a loss coefficient for a long radius flanged connection $K = 0.2$. Assume that the working fluid in the pipe is ethylene glycol. Comment on the advantages or disadvantages of each configuration.

5.6.2 Shell and Tube Exchangers

Shell and tube heat exchangers are widely used as power condensers, oil coolers, preheaters, and steam generators. They consist of many tubes mounted parallel to each other in a cylindrical shell. Flow may be parallel, counter, or cross flow and in some cases combinations of these flow arrangements as a result of baffling. Shell and tube designs are relatively simple and most often designed according to the Tubular Exchanger Manufacturer's Association (TEMA) standards.

For this configuration the overall heat transfer coefficient is given by (ignoring fouling):

$$\frac{1}{UA} = \frac{1}{h_i A_i} + R_w + \frac{1}{h_o A_o} \tag{5.55}$$

where A_i and A_o denote the inner and outer areas of the tubes. The heat transfer coefficient h_i is computed for a tube while the heat transfer coefficient h_o is computed for tube bundles in either parallel or cross flow depending on whether baffling is used. Special attention must be given to the internal tube arrangement, i.e. baffled, single pass, multi-pass, tube pitch and arrangement, etc., to properly predict the heat transfer coefficient. Often, unless drastic changes occur in the tube count, the shell side heat transfer coefficient will not vary much from an initial prediction. Often a value of $h_o = 5000 \ W/m^2 K$ is used for preliminary sizing.

The heat transfer surface area is calculated from:

$$A_o = \pi d_o N_t L \tag{5.56}$$

where d_o is the outer diameter of the tubes, N_t is the number of tubes, and L is the length of the tubes.

The number of tubes that can fit in a cylindrical shell is calculated from:

$$N_t = CTP \frac{\pi D_s^2}{4CLP_t^2} \tag{5.57}$$

The factor CTP is a constant that accounts for the incomplete covereage of circular tubes in a cylindrical shell, i.e. one tube pass $CTP = 0.93$, two tube passes $CTP = 0.9$, and three tube passes $CTP = 0.85$. The factor CL is the tube layout constant given by $CL = 1$ for 45 and 90 degree layouts, and $CL = 0.87$ for 30 and 60 degree layouts. Finally, P_t is the tube pitch and D_s is the shell diameter.

The shell diameter may be solved for using the above two equations, to give:

$$D_s = 0.637 \sqrt{\frac{CL}{CTP}} \left[\frac{A_o P_t^2}{d_o L} \right]^{1/2} \tag{5.58}$$

The shell side heat transfer coefficient is most often computed from the following experimental correlation:

$$Nu_{D_e} = 0.36 Re_{D_e}^{0.55} Pr^{1/3} \tag{5.59}$$

for $2 \times 10^3 < Re_{D_e} < 1 \times 10^6$. The effective diameter D_e is obtained from

$$D_e = \frac{4(P_t^2 - \pi d_o^2/4)}{\pi d_o} \tag{5.60}$$

for a square tube arrangement, and

$$D_e = \frac{8(\sqrt{3}P_t^2/4 - \pi d_o^2/8)}{\pi d_o} \tag{5.61}$$

for a triangular tube arrangement. The tube side heat heat transfer coefficient is computed from an appropriate tube model depending on the type of flow, i.e. laminar or turbulent.

The pressure drop for the tube side is often predicted using the following formula (Kakac and Liu, 1998):

$$\Delta p_t = \left[\frac{4fLN_p}{d_i} + 4N_p \right] \frac{1}{2} \rho V^2 \tag{5.62}$$

where N_p is the number of tube passes.

This accounts for tube friction and internal losses due to the return bends. The friction factor f is computed from an appropriate tube model depending on teh type of flow, i.e. laminar or turbulent. The shell side pressure drop may be predicted from

$$\Delta p_s = \frac{fG_s^2(N_b + 1)D_s}{2\rho D_e} \tag{5.63}$$

and

$$G_s = \frac{\dot{m}P_t}{D_s C B} \tag{5.64}$$

where N_b is the number of baffles, C is the clearance between adjacent tubes, B is the baffle spacing, while f is determined from

$$f = \exp[0.576 - 0.19\ln(G_s D_e/\mu)] \tag{5.65}$$

which is valid for $400 < G_s D_e/\mu < 1 \times 10^6$. These correlations have been tested agianst many shell and tube designs and have been found to provide very good results. However, care must be taken to understand the nature of the flow, i.e. parallel, cross flow, or cross flow with baffles. In general when designing shell and tube heat exchangers, the TEMA standards should be followed.

Example 5.3

Design a single pass shell and tube type heat exchanger which requires $\dot{m}_h = 25$ kg/s with $T_{h,i} = 30\ C$ and $\dot{m}_c = 75\ kg/s$ with $T_{c,i} = 20\ C$. What heat transfer rate is obtained using 150 2 m tubes ($ID = 16\ mm$ and $OD = 19\ mm$ with $k_w = 400$ W/mK). The hot fluid is water and it flows in the tubes, while the cold fluid is a 50/50 mix of ethylene glycol and water which flows in the shell. Assume $h_c = 5000$ $W/m^2 K$. Also, determine the minimum shell diameter assuming that the tubes are arranged in a square pattern with a pitch of 50 mm.

Example 5.4

Design a return pass shell and tube type heat exchanger using the TEMA E shell with shell side fluid mixed, see handout. The heat exchanger is required to cool hot water, $\dot{m}_h = 20\ kg/s$ with $T_{h,i} = 100\ C$ using cold water with $\dot{m}_c = 25\ kg/s$ with $T_{c,i} = 20\ C$. What heat transfer rate is obtained using 200 5 m tubes ($ID = 16\ mm$ and $OD = 19\ mm$ with $k_w = 400\ W/mK$). The hot fluid flows in the tubes, while the cold fluid flows in the shell. Also, determine the minimum shell diameter assuming that the tubes are arranged in a triangular pattern with a pitch of 38 mm. What is the tube side and shell side pressure drop?

5.6.3 Compact Heat Exchangers

Compact heat exchangers offer a high surface area to volume ratio typically greater than 700 m^2/m^3 for gas-gas applications, and greater than 400 m^2/m^3 for liquid-gas applications. They are often used in applications where space is usually a premium such as in aircraft and automotive applications. They rely heavily on the use of extended surfaces to increase the overall surface area while keeping size to a minimum. As a result, pressure drops can be high. Typical applications include gas-to-gas and gas-to-liquid heat exchangers. They are widely used as oil coolers, automotive radiators, intercoolers, cryogenics, and electronics cooling applications.

For this configuration the overall heat transfer coefficient is given by:

$$\frac{1}{UA} = \frac{1}{(\eta_o h A)_i} + \frac{t}{k_w A_w} + \frac{1}{(\eta_o h A)_o} \tag{5.66}$$

where η_o is the overall surface efficiency. In most compact heat exchanger design problems, the heat transfer and friction coefficients are determined from experimental performance charts or models for enhanced heat transfer surfaces. The pressure drop is also computed using the general method discussed section 5.4.

Example 5.5

Air enters a heat exchanger at 300 C at a rate of $\dot{m} = 0.5$ kg/s. and exits with a temperature of 100 C. If a surface similar to $1/9 - 22.68$ (see handout) is used, calculate the pressure drop in the core, heat transfer coefficient for the core, and the surface efficiency for the core. The heat exchanger core has the following dimensions: $W = 10$ cm, $L = 30$ cm, $k_f = 200$ W/mK, $t_w = 1.5$ mm.

Example 5.6

You wish to cool an electronic package having dimensions of 5 cm by 5 cm which produces 100 W. The package and heat sink are to be mounted on a circuit board which forms a channel with another circuit board. The spacing between the two circuit boards is 25 mm and each board is approximately 40 cm x 40 cm. The air speed in the channel is 3 m/s and has a temperature of 25 C. An aluminum, ($k = 200$ W/mK, $\epsilon = 0.8$), finned surface similar to that used in compact heat exchangers is to be considered, i.e $3/32 - 12.22$ (see handout). It is to be attached using a conductive adhesive tape having a thermal conductivity ($k_a = 50$ W/mK) and thickness of 1 mm. As the chief packaging engineer you must determine:

- the package temperature with the heat sink attached?

- the pressure loss coefficient for the heat sink, i.e. K_s?

- the pressure drop in the channel including the effect of the heat sink?

Assume air properties to be $\rho = 1.1$ kg/m^3, $k = 0.03$ W/mK, $\mu = 2 \times 10^{-5}$ Pas, $C_p = 1007$ J/kgK, $Pr = 0.70$.

Example 5.7

A counter flow gas turbine cooler is to be designed to recover heat from exhaust gases. The mass flow rate for each gas stream is 2.5 kg/s, i.e. nearly balanced counter flow. The incoming air of the cold stream is 20 C and the incoming air of the high temperature gas stream is 500 C. Due to space limitations, the heat exchanger is composed of 25 channels for each gas stream and utilizes the internal fin structure with performance characteristics shown in the figure below, see handout. The dimensions of each channel are $L = 30$ cm, $W = 30$ cm, and $H = 6.35$ mm. Determine the heat transfer rate Q and the outlet temperatures of each gas stream. You may neglect the wall conduction resistance. What pressure drop results for each gas stream? Are they the same, and if not explain. Note: the properties of air change drastically with

temperature, however you do not need to re-iterate the solution for the heat transfer, merely comment on the effect of the mean bulk temperature change. In the case of the pressure drop calculations, use the properties given below to interpolate the outlet air densities for each stream.

Assume properties for air at 500 C are: $\rho = 0.456\ kg/m^3$, $\nu = 78.5 \times 10^{-6}\ m^2/s$, $k = 0.056\ W/mK$, $C_p = 1.093\ KJ/kgK$, $Pr = 0.70$, and at 20 C are: $\rho = 1.205\ kg/m^3$, $\nu = 15.0 \times 10^{-6}\ m^2/s$, $k = 0.025\ W/mK$, $C_p = 1.006\ KJ/kgK$, $Pr = 0.72$.

5.6.4   Plate and Frame Exchangers

Plate heat exchangers consist of a series of thin corrugated formed metal plates. Each pair of plates forms a complex passage in which the fluid flows. Each pair of plates are then stacked together to form a sandwich type construction in which the second fluid flows in the spaces formed between successive pairs of plates. These types of heat exchnagers provide for a compact and lightweight heat transfer surface. As a result of the small plate spacing and corrugated design, high heat transfer coefficients result along with strong eddy formation which helps minimize fouling. Because of the simple construction, they are easily cleaned and find wide use in food processing applications.

5.6.5   Boilers, Condensers, and Evaporators

A condenser and evaporator are heat exchangers in which a change of phase results. In a condenser, a vapour is converted into a liquid, while in an evaporator (and a boiler) liquid is converted into a vapour. Due to the two phase nature of these devices, design is not as straight forward. Two phase fluid flows are much more complex than their single phase counterparts. Additional understanding of the phase make up and distribution is required to perform the necessary design calculations. In addition, design correlations for two phase flows can be somewhat complicated.

5.7   Fouling of Heat Exchangers

Fouling in heat exchangers represents a major source of performance degradation. Fouling not only contributes to a decrease in thermal efficiency, but also hydraulic efficiency. The build up of scale or other deposit increases the overall thermal resistance of the heat exchanger core which directly reduces the overall thermal efficiency. If build up of a fouling deposit is significant, it can also increase pressure drop due to the reduced flow area in the heat exchanger core. The two effects combined can lead to serious performance degradation. In some cases the degradation in hydraulic performance is greater than than the degradation in thermal performance which necessitates cleaning of the heat exchanger on a regular basis. Fouling of heat exchangers results in a number of ways. The two most common are corrosion and scale build

up. However, depending upon the nature of the fluid other factors may contribute to fouling.

Table 3 - TEMA Design Fouling Resistances R_f
for a Number of Industrial Fluids

Fluid	$R_f'' = R_f A \ [m^2 K/kW]$
Engine Oil	0.176
Fuel Oil no.2	0.352
Fuel Oil no.6	0.881
Quench Oil	0.705
Refrigerants	0.176
Hydraulic FLuids	0.176
Ammonia Liquids	0.176
Ethylene Glycol Solutions	0.352
Exhaust Gases	1.761
Natural Gas Flue Gases	0.881
Coal Flue Gases	1.761

Fouling in heat exchangers is traditionally treated usng the concept of a fouling resistance. This resistance is added in series to either side of the wall resistance in the definition of the overall heat transfer coefficient.

$$\frac{1}{UA} = \frac{1}{\eta_i h_i A_i} + R_{f,i} + \frac{1}{S k_w} + R_{f,o} + \frac{1}{\eta_o h_o A_o} \tag{5.67}$$

The fouling resistance may be computed from

$$R_f = \frac{t_f}{k_f A_w} \tag{5.68}$$

for a plane wall, and

$$R_f = \frac{\ln(d_f/d_c)}{2\pi k_f L} \tag{5.69}$$

for a tube.

Unfortunately, fouling in heat exchangers has not been modelled adequately for predictive purposes. Some typical values of fouling resistances are given in Table 3 for a number of fluids.

Another method of designing for fouling, is through the specification of *percentage over surface*, that is increaseing the surface area to initially provide for a heat exchanger which exceeds the design heat transfer rate. This is often done for heat exchangers which cannot be cleaned easily such as shell and tube heat exchangers. Often as a rule 25 percent over surface is prescribed. The percentage over surface is defined as

$$\%OS = \left(\frac{A_f}{A_c} - 1 \right) \times 100 \tag{5.70}$$

The effect that fouling has on a heat exchanger's performance can be seen by examining the the change in pressure drop:

$$\frac{\Delta p_f}{\Delta p_c} = \frac{f_f}{f_c} \frac{d_c}{d_f} \left(\frac{V_f}{V_c} \right)^2 \tag{5.71}$$

If the mass flow rate is constant, then $\dot{m} = \rho V_c A_c = \rho V_f A_f$, and we obtain

$$\frac{\Delta p_f}{\Delta p_c} = \frac{f_f}{f_c} \left(\frac{d_c}{d_f} \right)^5 \tag{5.72}$$

If the pressure drop is maintained constant, then we obtain, after substituting for mass flow rate:

$$\frac{\dot{m}_f}{\dot{m}_c} = \frac{f_c}{f_f} \left(\frac{d_f}{d_c} \right)^3 \tag{5.73}$$

For any other condition, such as pump driven flow, we may solve for the new operating point. Fouling will effectively increase the system curve, which shifts the operating point to the left, i.e. lowering the actual flow. If the flowrate decreases, then the heat transfer coefficient also decreases. This decrease combined with the increased resistance due to the fouling layer, leads to an overall decrease in thermal performance, i.e. a lowering of the $Q/(T_{h,i} - T_{c,i})$ curve.

5.8　References

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