Chapter 4

Turbomachinery

4.1 Introduction

In this chapter we will examine the performance characteristics of turbomachinery. The word *turbo* implies a spinning action is involved. In turbomachinery a blade or row of blades rotates and imparts or extracts energy to or from the fluid. Work is generated or extracted by means of enthalpy changes in the working fluid.

In general, two kinds of turbomachines are encountered in practice. These are open and closed turbomachines. Open machines such as propellers, windmills, and unshrouded fans act on an infinite extent of fluid, whereas, closed machines operate on a finite quantity of fluid as it passes through a housing or casing. We will examine only turbomachines of the closed type.

We also categorize turbomachines according to the type of flow. When the flow is parallel to the axis of rotation, we denote this type of machine as axial flow, and when flow is perpendicular to the axis of rotation, we denote this machine as radial flow. Finally, when both radial and axial flow velocity components are present, the machine is denoted as mixed flow.

Turbomachines may be further classified into two additional categories: those that absorb energy to increase the fluid pressure, i.e. pumps, fans, and compressors, and those that produce energy such as turbines by expanding to lower pressures. Of particular interest are applications which contain pumps, fans, compressors, and turbines. These components are essential in almost all mechanical equipment systems such as power and refrigeration cycles. We will examine each of these components in detail, and address a number of operating issues in systems when more than one component exists.

4.2 Pumps and Fans

We begin by considering pumps and fans first as their performance and operational characteristics are similar, with the exception that pumps are used with liquids and

fans are used for gases usually air.

Pumps are fluid machines which increase the pressure of a liquid, to enable the fluid to be moved from one location to another. Pumps are typically used to overcome losses due to friction in pipes over long distances, losses due to fittings, losses due to components, and elevation differences. Pumps usually fall into the following groups:

- positive displacement

- centrifugal

- axial flow

- special design or application.

Fans on the other hand are used to move gases from one region to another without an appreciable change in density. Fans must overcome losses in a system due to friction in ducting, minor losses due to fittings, and flow through components such as filters, cleaners, etc. Fans usually fall into the following classes:

- centrifugal

- axial flow

- special design or application.

4.2.1   Ideal Centrifugal Flow Machines

Simple analysis of centrifugal pump or fan impeller dynamics leads to the following theoretical head relationship (Potter and Wiggert, 1997):

$$H_t = \frac{\omega^2 r_2^2}{g} - \frac{\omega \cot \beta_2}{2\pi l_2 g} Q \tag{4.1}$$

where ω is the angular velocity of the impeller, β_2 is the exit blade angle, r_2 is the exit radius, l_2 is the exit width of the impeller, and Q is the volumetric flow rate.

This theoretical head increase in the fluid is a linear relationship and may be written as:

$$H_t = A - BQ \tag{4.2}$$

In reality, losses due to irreversibilities in the impeller housing result in actual pump/fan performance of the form:

$$H_p = A - BQ^2 \tag{4.3}$$

Pump Performance

Actual Head

$$H_p = H_t - h_L$$

Actual Fluid Power

$$\dot{W}_f = \rho g H_p Q$$

Brake (Impeller) Power

$$\dot{W}_p = \omega T$$

Efficiency

$$\eta_p = \frac{\dot{W}_f}{\dot{W}_p} = \frac{\rho g H_p Q}{\omega T} < 1$$

Pump impellers and fan blades are designed to give maximum efficiency at a given flow and head. For all other off design flow conditions, efficiency deteriorates. A typical pump/fan performance curve provides the following information: head-flow characteristic, net positive suction head (NPSH) required for pumps, brake horspower, and efficiency, see handout.

4.3 Sizing, Selection, and Performance of Turbomachines

We now consider some of the basic operational aspects of pumps and fans in mechanical equipment systems. This includes determining the operating point or finding the resultant mass flow rate in the system, and how to analyze systems with prime movers in series and parallel arrangements.

4.3.1 Matching System Characteristics to Pumps and Fans

Pumps and fans are chosen to meet the special requirements of a particular system. In any given system, the operating point of a pump or fan is determined by comparing the system performance curve or pressure loss characteristic to the system performance curve of the prime mover. Given a pump or fan characteristic which is usually of a polynomial form:

$$\Delta p_p = a_o + a_1 \dot{m} + a_2 \dot{m}^2 + \cdots + a_n \dot{m}^n \tag{4.4}$$

and the following system characteristic which may be fitted to a polynomial or may be in the form of an actual mathematical model (in which case it will not be a polynomial):

$$\Delta p_s = b_o + b_1 \dot{m} + b_2 \dot{m}^2 + \cdots + b_n \dot{m}^n \tag{4.5}$$
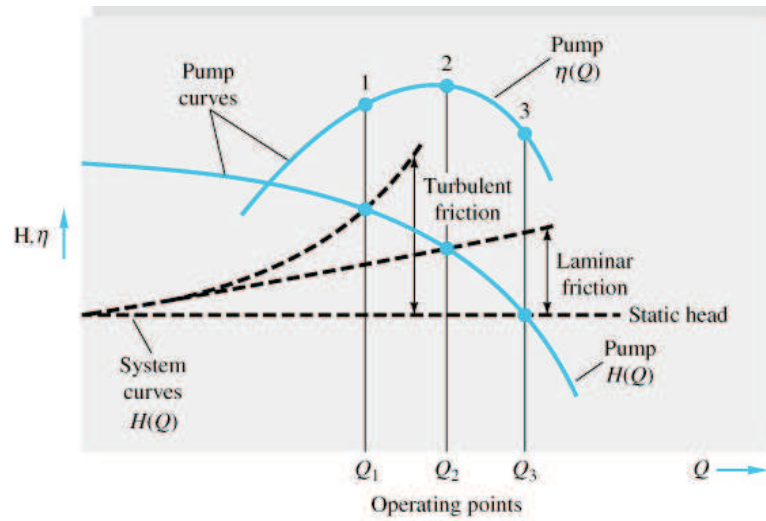
**Fig. 4.1 - Matching System Characteristics to Pump Characteristics
From Fluid Mechanics, F.M. White (2000)**

The operating point occurs when the pump or fan pressure drop equals the system pressure drop:

$$\Delta p_p = \Delta p_s \tag{4.6}$$

which yields

$$a_o + a_1\dot{m} + a_2\dot{m}^2 + \cdots + a_n\dot{m}^n = b_o + b_1\dot{m} + b_2\dot{m}^2 + \cdots + b_n\dot{m}^n \tag{4.7}$$

or

$$(a_o - b_o) + (a_1 - b_1)\dot{m} + (a_2 - b_2)\dot{m}^2 + \cdots + (a_n - b_n)\dot{m}^n = f(\dot{m}) = 0 \tag{4.8}$$

This equation is a non-linear expression which requires the root, $\dot{m}$, to be determined. This type of equation is easily solved using a Newton-Raphson root finding procedure, or easily dealt with using most computer software. For a Newton-Raphson procedure one applies the following equation repeatedly until convergence is acheived:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{4.9}$$

where x_i the value of the solution variable for the ith iteration, and x_0 denotes an initial guess. Provided a reasonable initial guess is made, Newton-Raphson iteration converges after two to three iterations. Thus if $x_i = \dot{m}$, we may write

$$\dot{m}_{i+1} = \dot{m}_i - \frac{(a_o - b_o) + (a_1 - b_1)\dot{m}_i + (a_2 - b_2)\dot{m}_i^2 + \cdots + (a_n - b_n)\dot{m}_i^n}{(a_1 - b_1) + 2(a_2 - b_2)\dot{m}_i + \cdots + n(a_n - b_n)\dot{m}_i^{n-1}} \quad (4.10)$$

where $\dot{m}_i$ is an initial guess. The procedure may then be repeated with $\dot{m}_i = \dot{m}_{i+1}$ until desired convergence is achieved.

When dealing with polynomial forms, one must be careful as multiple roots often exist. These may be real and/or complex and the correct solution is the only real positive root. However, there are some situations when dealing with systems of multiple fans, in which multiple positive real roots are obtained, i.e. multiple operating points. This can lead to some operational instabilities. This is a result of the peculier fan curves that often result.
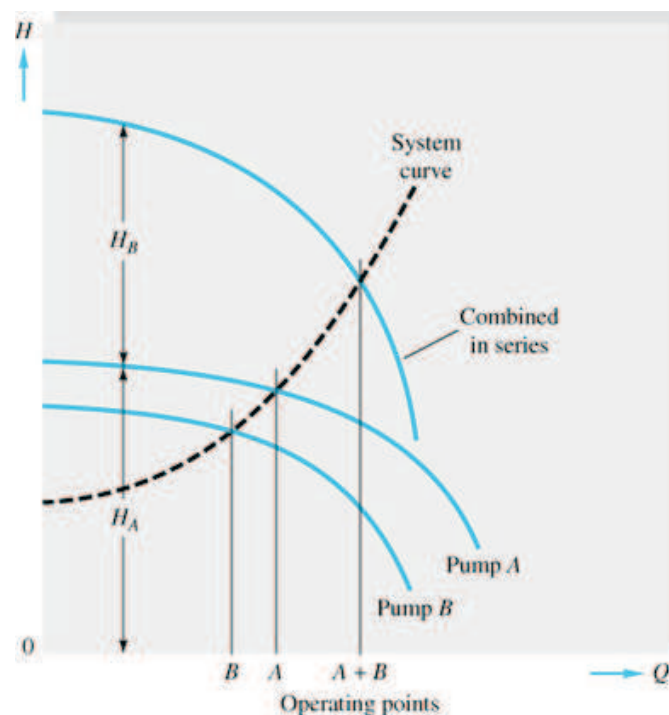

Fig. 4.2 - Combining Pumps or Fans in Series
From Fluid Mechanics, F.M. White (2000)

4.3.2 Components in Series

Pumps and fans are often arranged in series for a high impedance system, i.e. a system which has a high head characteristic. Pumps or fans may be combined in series to provide the necessary head required in a high impedance system when the

discharge characteristics are satisfactory but the head characteristics are not. In a series configuration, the discharge from one pump or fan is fed into the intake of the second pump or fan. The rule for combining components in series is that the two heads are summed at constant discharge rate. This leads to a new operating curve for the combined system. In general, given two or more pump or fan curves, the equivalent series system curve is given by:

$$H_s = H_1 + H_2 + \cdots H_n \tag{4.11}$$

4.3.3 Components in Parallel

Pumps and fans may also be combined in parallel. A parallel arrangement is used in a low impedance system, that is, a system which has a low head characteristic. Pumps or fans are used in parallel when the head characteristics are adequate, but the discharge characteristics are not. The rule for combining pumps or fans in parallel is that the discharge is summed at constant head. Pumps combined in parallel must be run with care if the operating characteristics are very different. When dissimilar pumps are combined in parallel, the pump with the lower head characteristic cannot be operated until the head in the larger pump has decreased to that of the smaller pump. In general, given two or more pump or fan curves, the equivalent parallel system curve is given by:
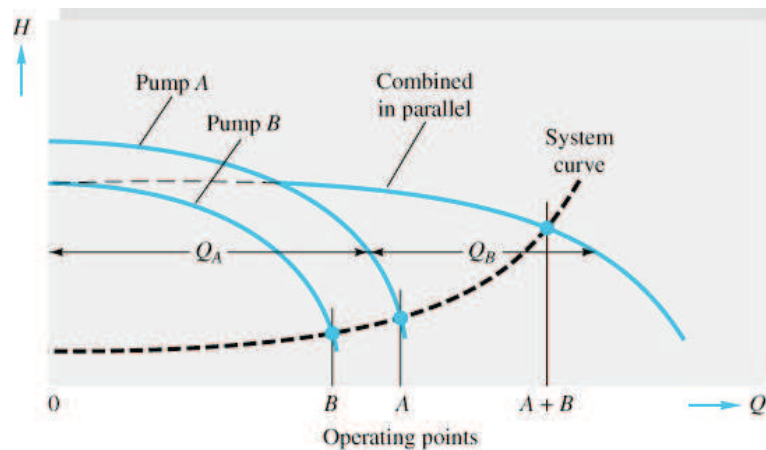
$$Q_p = Q_1(H) + Q_2(H) + \cdots Q_n(H) \tag{4.12}$$

**Fig. 4.3 - Combining Pumps or Fans in Parallel
From Fluid Mechanics, F.M. White (2000)**

Example 4.1

Consider a pump with the following performance characteristic:

$$\Delta p = 100,000 - 25\dot{m}^2$$

Find the equivalent characteristic for two pumps in series (2PS) and two pumps in parallel (2PP).

Example 4.2

Consider the closed loop pumping system sketched in class. If the total length of piping is 60 [m], with a diameter of 5 [cm], and a roughness $\epsilon = 0.0001$ [m], what is the resultant flow in the system if the pump has the following characteristic:

$$H_p = 250 - 0.25\dot{m}^2$$

and the filter has the following pressure loss:

$$H_f = 50 + 0.5\dot{m}^2$$

If the desired discharge were $\dot{m} = 25$ [kg/s] and the pump was normally run at 1750 RPM, can the desired discharge be achieved with two pumps in series or two pumps in parallel.

4.4 Flow Control

Often, the resultant flow in a system may not meet design requirements, or depending upon system demand, a variable flow may be desired. Flow control in a system can be achieved in a number of ways. These include:

- valve or damper control, i.e. change the system curve

- speed control, i.e. change the pump or fan curve

- speed and damper/valve control, i.e. manage both curves

The use of valve or damper control allows the system curve to shifted to the left or right by means of a variable resistance or K factor. This leads to a decrease or an increase in flow while the prime mover rotates at a fixed speed.

The use of variable speed allows the prime mover curve to be shifted up or down and hence increasing or decreasing the flow in the system with a fixed system curve.

Finally, the use of both speed and valve/damper control allows more flexability in the operation of a system. This mode of operation is the most efficient from a cost point of view.

4.5   Similarity Laws for Turbomachinery

The use of dimensionless performance variables for pumps and fans is desirable in a systems analysis. This allows for greater flexibility in the use of pump or fan performance data. Pump or fan performance in general, is a function of several variables:

$$\phi\left(\dot{W}, \omega, D, Q, H, \rho, \mu, \frac{l_1}{D}, \frac{l_2}{D}, \dots\right) = 0 \tag{4.13}$$

where l_i represent other dimensions of the turbomachine. For *geometrically* similar machines where all dimensions are scaled proportionally, only the diameter of the impeller becomes critical, and we may write:

$$\phi(\dot{W}, \omega, D, Q, H, \rho, \mu) = 0 \tag{4.14}$$

Using dimensional analysis it can be shown that the following dimensionless relationships may be obtained to characterize turbomachine performance for incompressible fluids:

$$C_w = \frac{\dot{W}}{\rho(\omega D)^3 D^2} \tag{4.15}$$

$$C_p = \frac{\Delta p}{\rho(\omega D)^2} \tag{4.16}$$

$$C_Q = \frac{Q}{(\omega D)D^2} \tag{4.17}$$

where C_w, C_p, C_Q are the power coefficient, pressure coefficient, and discharge coefficient, respectively, for the turbomachine.

Further, we may write the pressure coefficient as:

$$C_H = \frac{Hg}{(\omega D)^2} \tag{4.18}$$

Finally, the Reynolds number for the turbomachine may be defined as:

$$Re_D = \frac{\rho(\omega D)D}{\mu} \tag{4.19}$$

The turbomachine efficiency may also be defined in terms of these dimensionless groups. It is not difficult to show that:

$$\eta = \frac{C_Q C_H}{C_W} \tag{4.20}$$

We use these affinity or similarity laws to scale pump or fan performance curves for families of similar machines. However, caution must be excerised when applying

these relationships. It is not a simple matter to equate the dimensionless coefficients for similar machines. We see from the relationships above that ωD appears as a group. This is the characteristic rotor or blade tip speed. The remaining parameter appears as a product D^2, which is proportional to area. Thus, if the pump housing is not scaled proportionally, that is only the impeller is scaled, this group may be interpreted as being proportional to a characteristic flow area, Shepherd (1964). This leads to very different scaling laws. For example, for *geometrically* similar machines we may write:

$$\frac{\dot{W}_2}{\dot{W}_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 \tag{4.21}$$

$$\frac{H_2}{H_1} = \left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 \tag{4.22}$$

$$\frac{Q_2}{Q_1} = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2}{D_1}\right)^3 \tag{4.23}$$

These are the scaling laws reported in virtually all fluids texts and most introductory turbomachine texts. However, in the case of only an impeller change, the turbomachines are no longer *geometrically* similar machines. This type of scaling occurs quite frequently and needs to be properly addressed. Using the above laws, we may still write:

$$\frac{\dot{W}_2}{\dot{W}_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^3\left(\frac{D_2}{D_1}\right)^3_i\left(\frac{D_2}{D_1}\right)^2_h = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^3\left(\frac{D_2}{D_1}\right)^3 \tag{4.24}$$

$$\frac{H_2}{H_1} = \left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 \tag{4.25}$$

$$\frac{Q_2}{Q_1} = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2}{D_1}\right)_i\left(\frac{D_2}{D_1}\right)^2_h = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2}{D_1}\right) \tag{4.26}$$

Here, i denotes the impeller and h denotes the housing. We then see that for the same housing, the ratio $(D_2/D_1)_h$ is unity. This leads to very different proportionality or scaling laws. In a number of cases it is more practical to change the characteristic of a pump by changing only the impeller size over a moderate range of operation variables for a fixed pump speed. In this way a manufacturer does not have to design a new housing for each different pump of different capacity over a nominal range of operating conditions. It is not always clear whether a manufacturer's pump performance curves for a family of pumps, are for *geometrically* similar pumps or for *partially* similar machines. This will be left as an excerise in the problem sets to verify for a give set of curves.

We generally apply these scaling laws within families of similar machines or to examine the effect of a change in one or more operating variables, such as the effect

of pump speed or working fluid. The latter is quite important, as many pumps are tested using water, but many applications use other fluids with very different properties.

Unfortunately, pump efficiency, does not scale well according the relationships given above. However, it has been observed that

$$\frac{1 - \eta_2}{1 - \eta_1} \approx \left(\frac{D_1}{D_2}\right)^n \tag{4.27}$$

where n is found to vary somewhat, but is close to a single value of $n = 1/4$. This is sufficient to obtain a reasonable estimate.

Example 4.3

Consider two pumps having the same housing but different impellers. The following data were obtained for the two pumps:

13 inch Impeller

$$H = 188, 184, 181, 175, 165, 152, 132, 111$$

$$Q = 0, 125, 175, 225, 275, 325, 375, 425$$

9 inch Impeller

$$H = 87, 86, 84, 81, 77, 72, 67, 62$$

$$Q = 0, 100, 125, 150, 175, 200, 225, 250$$

Show using the pump affinity laws that either the 13" or 9" data can be scaled such that a single curve is obtained for both impellers.

Example 4.4

Reconsider Example 4.2. Do one or two pumps in series or parallel operating at twice the speed provide the desired demand of $\dot{m} = 25$ [kg/s].

4.6   Cavitation in Turbomachinery

Cavitation in pumps and turbines occurs when the pressure at any point inside the pump falls below the vapour pressure of the liquid. At this point the liquid vaporizes and the bubbles are carried through the pump. If at any point the pressure increases above the vapour pressure, the bubbles collapse. This collapsing of bubbles can lead to mechanical wear in the form of pitting. This phenomena is known as caviation.

Cavitation in pumps usually occurrs on the inlet side of pumps where suction or low (negative gage) pressures are found.

In order to prevent cavitation from occuring, the pressure anywhere inside the pump must be higher than the vapour pressure of the working liquid. To prevent cavitation from ocurring, a new parameter referred to as the Net Positive Suction Head (NPSH) is defined. It is defined as the difference between the head at the pump inlet and the vapour pressure head:

$$NPSH = \frac{p_i}{\rho g} + \frac{V_i^2}{2g} - \frac{p_v}{\rho g} \qquad (4.28)$$

We may rewrite the expression for NPSH in terms of the location Z_i above a fluid reservoir whose pressure is at p_a, using Bernoulli's equation to obtain:

$$NPSH = \frac{p_a}{\rho g} - Z_i - h_{fi} - \frac{p_v}{\rho g} \qquad (4.29)$$

where the h_{fi} term represents the headloss incurred between the surface of the reservoir and the pump inlet. The parameter Z_i as we shall see, may be positive or negative.

All pump performance curves contain a plot of the NPSH that is required to avoid cavitation. When matching a pump to a system, the NPSH available, i.e. the right hand side of Eq. (4.29), must be greater than the NPSH required on the pump performance plot. We usually denote this requirement as:

$$NPSH_A > NPSH_R \qquad (4.30)$$

By comparing $NPSH_R$ with the expression for $NPSH_A$, we may solve for pump elevation or inlet headloss h_{fi} which ensures cavitation will not occur. In general, we may encounter three typical problems involving cavitation prevention:

- vertical placement, i.e. find Z_i

- horizontal placement, i.e. find L_i from h_{fi}

- size a valve, i.e. find K_v from h_{fi}

Example 4.5

A particular pump is required to pump 24,000 [gpm] of water whose free surface is at 1 [atm]. If the losses in the system up to the inlet of the pump are 6 [ft] of water, where should the pump be placed to avoid cavitation if $NPSH_R = 40$ [ft].

Example 4.6

In Example 4.2, if the required NPSH is given by:

$$NPSH_R = 2 + 0.0016\dot{m}^2$$

determine where the pump should be placed in the system if there is 20 [m] of piping leading to the pump inlet. Include minor losses in the analysis. If the system were designed with the valve at 50 percent open, is cavitation more likely to occur when the valve is opened further or closed further. Justify your answer by performing a calculation with $K_v = 3$ and $K_v = 9$

4.7   Operational Aspects of Fans

Fan performance is reported in a similar fashion as pump performance. However, on many fan curves used in HVAC systems, the following information is provided:

- Total Pressure P_t vs Flow

- Static Pressure P_s vs Flow

- Total Efficiency η_t vs Flow

- Static Efficiency η_s vs Flow

- Power Requirement

The total and static fan efficiencies are defined as follows:

$$\eta_t = \frac{\dot{m}(P_{t1} - P_{t2})}{\rho \dot{W}_{sh}} \tag{4.31}$$

and

$$\eta_s = \frac{\dot{m}(P_{s1} - P_{s2})}{\rho \dot{W}_{sh}} \tag{4.32}$$

When matching fan/system characteristics care must be take to work with the appropriate fan curve. If the inlet and exit areas of the fan are not equal then the total pressure curve must be used. If it is not given, then it must be generated using knowledge of the static fan curve, the discharge, and the exit area. This is done using:

$$P_t = P_s + \frac{\rho V^2}{2} \tag{4.33}$$

For equal inlet and exit areas the static and total pressure curves are equal. This is most often the case with pumps but not with fans. In general when working with axial flow fans, this is not an issue as the inlet and exit areas are nearly identical, and hence only static pressure is required. Since fan pressures are much lower than those of liquids, it is usually reported in terms of inches of water or mm of water. Thus, when working with fan curves care must be taken to convert the fan curve to match the pressure losses which assume air as the working fluid, or to convert the losses into inches or mm of water. To make this conversion in SI units the following is required:

$$\Delta p \ [mmH_2O] = \frac{\Delta p \ [Pa]}{g} \tag{4.34}$$

Fans may be operated in series or parallel as are pumps for the same reasons. In a number of applications such as electronics cooling it may be easier to work with the system on a per fan basis. Thus if a heat sink requires more than one axial flow fan to direct air flow between fins, it is often easier to consider only a single fan and a

system control volume for the portion of the heat sink or electronic enclosure covered by the fan. On the other hand, we may combine the fan characteristics for all fans and consider the entire heat sink or enclosure as the system control volume.

Air flow in HVAC systems is most often controlled using a variable flow method based on damper and speed control. This allows for precise management of air distribution in these systems. In many HVAC systems an important consideration is what is often referred to as the "system effect". The system effect accounts for the fact that the system curves are based on the assumption of uniform velocity profiles in ducting and at the exits of fans. However, as a result of the complex nature of the flow through a fan into a duct, there may be a required minimum length to allow for flow development. This additional effect influences both the fan curve and the system curve by lowering the actual fan curve and raising the system curve, resulting in a reduced discharge. This is evident in the Laboratory setup used in this course.

To minimize this problem, one should consider providing a reasonable length for flow development as recommended by the HVAC guidlines most often found in the ASHRAE guide, and to match the fan exit area and duct inlet area such that the ratio of these areas is between $85\% < A_f/A_d < 110\%$. Also, the transitioning for these sections should not exceed 15 degrees for converging sections and 7 degrees for diverging sections.

Example 4.7

Consider a CPU cooler consists of a fan (299 DS from handout) and a heat sink. The sink and fan are configured such that impinging air flow is directed at the base of the sink and ducted outwards into two parallel flow paths. If the sink is 9 [cm] by 9 [cm] square and contains 11 fins of thickness 2 [mm] and height 2 [cm], determine the approximate channel air velocity available for cooling the fins and hence the CPU. In the solution, approximate the fan performance curve as a linear.

4.8 Compressors

Compressors move gases in systems where an appreciable change in gas density occurs. Compressors are found in many applications, such as refrigeration systems, gas processing and storage, and pneumatic systems to name a few. Compressors are classified according to:

- centrifugal

- positive displacement (reciprocating)

- axial flow

Most compressor problems involve determining the power required to compress a certain volume of gas at one particular to another state. Compression can be achieved in a number of ways. Assuming a polytropic process, that is

$$pv^n = C \tag{4.35}$$

we may compress a gas isothermally $n = 1$, isentropically $n = k = C_p/C_v$ and polytropically $n \neq k$. We will consider this in more detail and also consider compressor performance.

The most common type of compressor is the reciprocating piston type, while axial flow compressors are found in gas turbine systems. Ideal compressor theory may be applied with the following assumptions:

- polytropic constant n is the same for compression/expansion

- heat transfer is negligible

The ideal compression cycle gives:

$$h_2 - h_1 = \int_1^2 v dp = \int_1^2 \left(\frac{C}{p}\right)^n dp \tag{4.36}$$

Evaluating gives:

$$h_2 - h_1 = \frac{nC^{1/n}}{n-1} \left[p_2^{(n-1)/n} - p_1^{(n-1)/n} \right], \quad n \neq 1$$

$$h_2 - h_1 = C \ln\left(\frac{p_2}{p_1}\right), \qquad\qquad n = 1 \tag{4.37}$$

Combining with the ideal gas law, i.e.

$$pv = RT \tag{4.38}$$

and using $k = C_p/C_v$ and $R = C_p - C_v$, we obtain the temperature ratio across the compressor:

$$\frac{T_2}{T_1} = \left(\frac{k-1}{k}\right)\left(\frac{n}{n-1}\right)\left[r^{(n-1)/n} - 1\right] + 1, \quad n \neq 1$$

$$\frac{T_2}{T_1} = \left(\frac{k-1}{k}\right)\ln r + 1, \qquad\qquad n = 1 \tag{4.39}$$

where $r = p_2/p_1$ is the compression ratio. The value of n has two special cases. For an isentropic process $s_2 = s_1$, $n = k$, while for an isothermal process $T_2 = T_1$, $n = 1$.

Compressor Work and Efficiency

The work done by a compressor is obtained from the enthalpy balance on the gas flowing through it,

$$\dot{W}_c = \dot{m}C_p(T_2 - T_1) \tag{4.40}$$

Using the appropriate relationships we may obtain the following results for the compressor work:

$$\dot{W}_c = \dot{m}C_pT_1\left(\frac{k-1}{k}\right)\left(\frac{n}{n-1}\right)\left[r^{(n-1)/n}-1\right]-1, \quad n \neq 1$$

$$\dot{W}_c = \dot{m}C_pT_1\left(\frac{k-1}{k}\right)\ln r, \qquad\qquad n = 1$$

(4.41)

The isentropic efficiency is defined as:

$$\eta_s = \frac{\dot{m}C_p(T_{2,s}-T_1)}{\dot{m}C_p(T_2-T_1)} = \frac{r^{(k-1)/k}-1}{T_2/T_1-1}$$

(4.42)

The overall or mechanical compressor effciency is defined as:

$$\eta_m = \frac{\dot{m}C_p(T_2-T_1)}{\dot{W}_{shaft}}$$

(4.43)

We may now examine two possible routes to compression, isothermal compression and isentropic compression. For the two processes, it can be shown that the ratio of compression work for isothermal compression to compression work for isentropic compression is:

$$\frac{\dot{W}_{c,t}}{\dot{W}_{c,s}} = \frac{k-1}{k}\frac{\ln r}{r^{(k-1)/k}-1}$$

(4.44)

Examination of this result reveals that less work is required to compress a gas isothermally. In practice this may be difficult to achieve, however through multistage compression with intercooling, it is possible to produce a more efficient compression process. After each stage of compression, heat is removed from the gas, ideally returning it to its original inlet temperature T_1. Intercoolers are usually water cooled heat exchangers. Intercooling is usually incorporated if the the discharge temperature reach 300 F.

In a multistaged compression process the mechanical efficiency may be written as:

$$\eta_m = \frac{\displaystyle\sum_{i=1}^{N}\dot{W}_{cycle}}{\dot{W}_{shaft}}$$

(4.45)

We conclude with a discussion of two important compressor performance issues: *surge* and *choking*. On a given compressor performance plot which usually gives the pressure versus mass or volumetric flow, there are two lines which connect performance curves at low flows and high flows. These are the surge line and the choke line.

The point of minimum flow/maximum pressure where the developed pressure is insuffucient to overcome the system resistance, is referred to as surge. When surge

occurs, gas in discharge piping flows back into the compressor. With no discharge flow, discharge pressure drops and the cycle repeats itself. Surging can cause compressors to overheat.

The point of maximum flow/minumum pressure is referred to as the choke line. Choking occurs when the gas reaches the sonic velocity at any point in the compressor. When this is achieved, flow through the compressor can not increase unless the compressor is modified.

Finally, compressor discharge on performance plots may be given in either actual cubic feet per minute (ACFM), i.e. at process conditions, or in standard cubic feet per minute (SCFM), i.e. at 60 F and 14.7 psi. Care must be taken to understand which reference is used. If flow is given in SCFM, then conversion to actual conditions must be made.

4.9   References

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