1) A continuous random quantity X is known to be normally distributed with a population mean $\mu = 20.4$ and a population variance $\sigma^2 = 25.1$.

(a) Evaluate $\mathrm{P}\left[X \le 15.0\right]$. [3]

(b) A random sample of size 4 is taken from this population. $\overline{X}$ is the sample mean. Evaluate $\mathrm{P}\left[\overline{X} \le 15.0\right]$. [5]

Note: You do ***not*** need to use linear interpolation in this question.
Quote your answers correct to only two significant figures.

[Also provided with this question paper were tables of the standard normal c.d.f. (the z tables)]

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2) The joint probability mass function $p(x, y)$ for random quantities X, Y is defined by the table:

	$p(x, y)$	Y = –1	0	1	
X	–1	.20	.15	.15	
	0	.15	.14	.11	
	1	.05	.01	.04	

(a) Find the covariance $\mathrm{Cov}(X, Y)$. [8]

(b) Are the random quantities X, Y independent? Why or why not? [4]

3) A box contains twelve (12) gear wheels, of which three (3) are protected with a rust-proofing treatment and the other nine (9) are not protected. A random sample of two (2) gear wheels is drawn, both at once, from the box. Let the random quantity X represent the number of gear wheels in the random sample that are protected.

(a) Show why the probability mass function (p.m.f.) for X is ***not*** binomial. [2]

(b) Find $\text{P}[X = 3]$. [2]

(c) Find the exact probability mass function $p(x)$ for X. [10]

(d) If the sample were drawn with replacement, then would the p.m.f. for X be binomial? Why or why not? [2]

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4) A function $f(x)$ of a continuous variable x is defined by

$$f(x) = \begin{cases} 105\left(x^4 - 2x^5 + x^6\right) & (0 < x < 1) \\ 0 & (\text{otherwise}) \end{cases}$$

(a) Show that $f(x)$ is a well-defined probability density function (p.d.f.). [2]

(b) Find the cumulative distribution function (c.d.f.) $F(x)$ for this p.d.f. [8]

(c) Hence evaluate $\text{P}\left[X > \frac{1}{2}\right]$ exactly. Leave your answer as a fraction. [4]

BONUS QUESTION:

(d) Find the population mean μ as a fraction reduced to its lowest terms. [+3]

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