1) Observations of the weights w (in Newtons) of two hundred (200) test cables after two weeks of immersion in a corrosive fluid are summarized in this frequency table:

Weight w (N)	Frequency f	$w\,f$	$w^2 f$	Cumulative frequency
$0 \le w < 50$	53	1325	33125	53
$50 \le w < 100$	62	4650	348750	115
$100 \le w < 150$	28	3500	437500	
$150 \le w < 200$	24	4200	735000	
$200 \le w < 300$	21	5250	1312500	
$300 \le w < 400$	7	2450	857500	
$400 \le w < 600$	5	2500	1250000	
Total	200	23875	4974375	

(a) Identify the median class. [2]

(b) Estimate the sample mean weight $\overline{w}$ from this frequency table. *Show your working.* [2]

(c) Estimate the sample standard deviation s_w from this frequency table. *Show your working.* [4]

[Parts (d) and (e) are on the next page.]

(a) The median of 200 ordered data is $\tilde{x} = \dfrac{x_{100} + x_{101}}{2}$.

The 100th and 101st values are both in the second class.
Therefore the median class is

$$\boxed{50 \le w < 100}$$

(b) $$\overline{w} = \frac{\sum w \cdot f}{\sum f} = \frac{23875}{200} \Rightarrow \boxed{\overline{w} = 119.375\text{ N}} = 119\text{ N } (3\text{ s.f.})$$

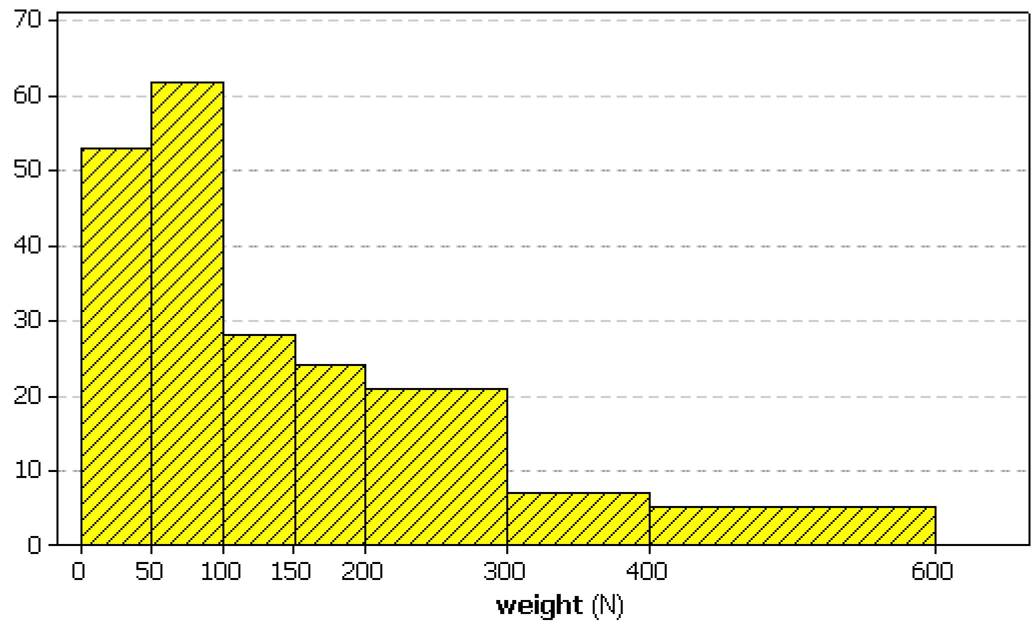
(c) $$s_w^2 = \frac{n\sum w^2 \cdot f - \left(\sum w \cdot f\right)^2}{\sum f\left(\sum f - 1\right)} = \frac{200 \times 4974375 - 23875^2}{200 \times 199} = \frac{424859375}{39800} = 10674.858\ldots$$

$$\Rightarrow s_w = \sqrt{10674.858\ldots} = 103.319\ldots \quad \therefore \boxed{s_w = 103\text{ N } (3\text{ s.f.})}$$

Question 1 continued ☞

1 (d) Do these data provide evidence for positive skew, negative skew or no skew? [2]

(e) Explain briefly why the graph below of the data is ***not*** a histogram. [2]



(d) **Positive skew** - there are many values far above the median and none far below.

(e) Relative frequency of a class = area of the bar for that class in the histogram,
so that the height of each bar = relative frequency / class width
But this diagram indicates height of each bar = frequency:
it is a bar chart, not a histogram.

2) Events A, B, C form a partition. A bookmaker offers the following odds:
r_A = 3:1 on, r_B = 7:5 against and r_C = 2:1 against

(a) Show that the corresponding probabilities are not coherent. [4]
(b) If a deposit of \$10 is placed on each of the three outcomes with the quoted odds, then what is the bookmaker's profit (or loss) if event B occurs? [3]
(c) Rescale the three probabilities so that they are coherent. [3]
(d) Convert the coherent probabilities back into odds. [3]

(a) $$r = \frac{p}{1-p} \quad \Rightarrow \quad p = \frac{r}{1+r}$$

$$p_A = \frac{3}{1+3} = \frac{3}{4}, \qquad p_B = \frac{\frac{5}{7}}{1+\frac{5}{7}} = \frac{5}{7+5} = \frac{5}{12}, \qquad p_C = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{2+1} = \frac{1}{3}$$

$$\Rightarrow \quad p_A + p_B + p_C = \frac{3}{4} + \frac{5}{12} + \frac{1}{3} = \frac{9+5+4}{12} = \frac{18}{12} = \frac{3}{2} \neq 1$$

Therefore the three probabilities are not coherent.

(b) $$p_i = \frac{\text{deposit}}{\text{stake}} = \frac{b_i}{s_i} \quad \Rightarrow \quad s_i = \frac{b_i}{p_i} = \frac{10}{p_i} \quad \Rightarrow \quad s_B = \frac{10}{p_B} = 10 \times \frac{12}{5} = 24$$

$$\text{Profit} = \sum b_i - s_B = 3 \times 10 - 24 = \boxed{\$6}$$

(c) The sum of the probabilities is $\frac{3}{2}$. Therefore rescale by $\frac{2}{3}$:

$$p_A = \frac{2}{3} \times \frac{3}{4} = \boxed{\frac{1}{2}}, \qquad p_B = \frac{2}{3} \times \frac{5}{12} = \boxed{\frac{5}{18}}, \qquad p_C = \frac{2}{3} \times \frac{1}{3} = \boxed{\frac{2}{9}}$$

(d) $$r_A = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1:1 \ \boxed{(\text{even odds})}, \qquad r_B = \frac{\frac{5}{18}}{1-\frac{5}{18}} = \frac{5}{18-5} = \frac{5}{13} = \boxed{13:5 \text{ against}},$$

$$r_C = \frac{\frac{2}{9}}{1-\frac{2}{9}} = \frac{2}{9-2} = \frac{2}{7} = \boxed{7:2 \text{ against}}$$

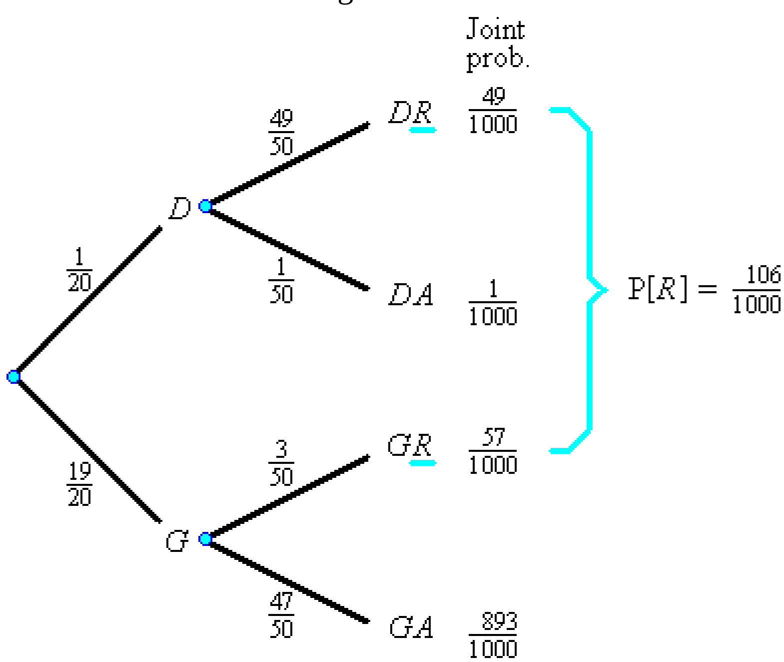
3) A quality control system rejects an item that is defective 98% of the time.
It rejects a good item 6% of the time. It is known that 5% of all items are defective.

(a) Given that an item has been rejected, find the probability that it is defective. [8]
Express your answer as a fraction reduced to its lowest terms ***and*** as a decimal correct to two significant figures.

BONUS QUESTION

(b) Given that the quality control system has tested the item twice in independent tests and has rejected it both times, find the probability that the item is defective. [+3]
Express your answer as a decimal correct to two significant figures.

(a) Let D = "item is defective", R = "item is rejected",
$G = \tilde{D}$ = "item is good" and $A = \tilde{R}$ = "item is accepted", then



$$\Rightarrow \quad P[\, D \mid R \,] = \frac{P[\, DR \,]}{P[\, R \,]} = \frac{49}{1000} \times \frac{1000}{106} =$$

$$\Rightarrow \quad P[\, D \mid R \,] = \boxed{\frac{49}{106}} \approx \boxed{46\%}$$

(b) $P[\, R_2 \mid DR_1 \,] = P[\, R \mid D \,] = \dfrac{49}{50} \quad \Rightarrow \quad P[\, DRR \,] = \dfrac{1}{20} \times \dfrac{49}{50} \times \dfrac{49}{50} = \dfrac{2401}{50\,000}$

Similarly, $P[\, GRR \,] = \dfrac{19}{20} \times \dfrac{3}{50} \times \dfrac{3}{50} = \dfrac{171}{50\,000}$

$$\Rightarrow \quad P[\, RR \,] = P[\, DRR \,] + P[\, GRR \,] = \frac{2\,401 + 171}{50\,000} = \frac{2\,572}{50\,000} = \frac{643}{12\,500}$$

$$\Rightarrow \quad P[\, D \mid RR \,] = \frac{P[\, DRR \,]}{P[\, RR \,]} = \frac{2\,401}{50\,000} \times \frac{50\,000}{2\,572} = \frac{2\,401}{2\,572} \approx \boxed{93\%}$$

Note how the second test has increased the reliability of the quality control dramatically.

To the next question ☞

4) It is known that [7]

$\text{P}[A] = .60,\quad \text{P}[B] = .55,\quad \text{P}[C] = .50,$

$\text{P}[AB] = .40,\quad \text{P}[BC] = .30,\quad \text{P}[CA] = .25\quad \text{and}\quad \text{P}[ABC] = .20.$

Find the probability that ***none*** of events A, B, C occur.

Let E = "none of events A, B, C occur" = $\tilde{A} \cap \tilde{B} \cap \tilde{C}$

The general addition law of probability for three events is

$$\text{P}[A \cup B \cup C] = \text{P}[A] + \text{P}[B] + \text{P}[C] - \text{P}[AB] - \text{P}[BC] - \text{P}[CA] + \text{P}[ABC]$$

$$\Rightarrow\quad \text{P}[A \cup B \cup C] = .60 + .55 + .50 - .40 - .30 - .25 + .20 = 1.85 - 0.95 = .90$$

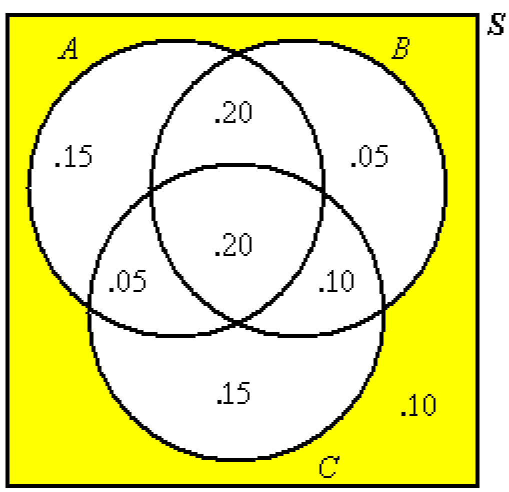
By the total probability law and one of deMorgan's laws,

$$\text{P}[E] = \text{P}[\tilde{A} \cap \tilde{B} \cap \tilde{C}] = 1 - \text{P}\left[\sim\left(\tilde{A} \cap \tilde{B} \cap \tilde{C}\right)\right] = 1 - \text{P}[A \cup B \cup C] = 1 - .90$$

Therefore

$$\boxed{\text{P}[E] = .10}$$

The Venn diagram for this situation is



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