ENGI 3425 Mathematics for Civil Engineering I

Practice Questions for the Mid Term Test

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1. Evaluate the integral

$$I(x) = \int x^3 \ln\left(x^4\right) dx$$

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2. A curve in the *x*-*y* plane is defined by the parametric equations

$$x = (t-1)^2, \quad y = t(t-2)^2$$

(a) Find all values of *t* and the corresponding values of *y* at which $x = 0$.
(b) Find all values of *t* and the corresponding values of *x* at which $y = 0$.
(c) Find all values of t at which the tangent line to the curve is horizontal.
(d) Show that the curve is concave up if and only if $t > 1$.
(e) Sketch the curve. Label the coordinates of all axis intercepts on your sketch.

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3. A thin cable is suspended by its ends from the points $(x, y) = (-1, \cosh 1)$ and $(x, y) = (+1, \cosh 1)$. It hangs under its own weight along the curve $y = \cosh x$.

(a) Find the length of the cable. Assume SI units.
(b) Find the area between the cable, the ground (at $y = 0$), $x = -1$ and $x = 1$.

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4. Find the exact value of the sum S of the series

$$S = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

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5. The equation of a curve, in polar coordinates, is given by

$$r = 1 + 2\sin\theta, \qquad (-\pi < \theta \le \pi)$$

(a) Show that $r < 0$ for $-\frac{5\pi}{6} < \theta < -\frac{\pi}{6}$.
(b) Sketch the curve. Show your work (either a preliminary Cartesian sketch or a table of values).
(c) Show that the total area enclosed by the outer loop $(r > 0)$ of the curve is

$$A = \int_{-\pi/6}^{+\pi/2} \left(3 + 4\sin\theta - 2\cos 2\theta\right) d\theta$$

and find the exact value of this integral.

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6. Find the exact value of the sum S of the series

$$S = 8 + 4 + 2 + 1 + \frac{1}{2} + \ldots$$

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7. Evaluate the integral

$$I = \int_0^1 x^3 \sinh 2x \, dx$$

You may leave your answer in terms of $\sinh 2$ and $\cosh 2$.

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8. A curve C in $\mathbb{R}^3$ is defined parametrically by

$$\vec{\mathbf{r}} = 72t^2\,\hat{\mathbf{i}} - 21t^2\,\hat{\mathbf{j}} + 50t^3\,\hat{\mathbf{k}}$$

(where the parameter t is any real number).

(a) Find the arc length L along the curve from the origin to the point $(72, -21, 50)$.

(b) Show that the unit tangent vector is

$$\hat{\mathbf{T}} = \frac{+1}{25\sqrt{1+t^2}} \begin{bmatrix} 24 \\ -7 \\ 25t \end{bmatrix} \quad \text{when } t > 0\text{, but is } \hat{\mathbf{T}} = \frac{-1}{25\sqrt{1+t^2}} \begin{bmatrix} 24 \\ -7 \\ 25t \end{bmatrix} \quad \text{when } t < 0.$$

(c) What happens to the unit tangent $\hat{\mathbf{T}}$ as the curve passes through the origin?

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9. A curve is defined by the polar equation

$$r = 2 - \cos\theta$$

(a) Find the values of θ in the interval $0 \le \theta < 2\pi$ at which the tangent line is vertical.

(b) Sketch this curve.

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10. Find the Cartesian equation of the ellipse of eccentricity $e = \frac{12}{13}$ whose foci are at the points $(\pm 12, 0)$. Sketch the ellipse, showing the locations of the foci, centre, vertices, directrices and the ends of the minor axis.

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11. Express the recurring decimal $1.\dot{6}\dot{3}$ as an exact fraction, reduced to its lowest terms.

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12. A conic section has the Cartesian equation $4x + y^2 = 8$

(a) Classify this conic section.

(b) Sketch the conic section, identifying the location of its focus, vertex and directrix.

(c) Find the equation of the surface of revolution formed when the conic section is rotated around the *x*-axis.

(d) Classify the quadric surface formed by this surface of revolution.

(e) Find the volume enclosed by this surface of revolution and the *y*-*z* plane.

(f) Find the curved surface area of this surface of revolution to the right of the *y*-*z* plane.

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13. Classify the following quadric surfaces:

(a) $x + 2y^2 + 3z^2 = 1$

(b) $x - 2y^2 + 3z^2 = 1$

(c) $x + 2y^2 = 1$

(d) $2y^2 - 3z^2 = 1$

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14. Find the sum of the series $S = \frac{4}{3\times1} + \frac{4}{4\times2} + \frac{4}{5\times3} + \frac{4}{6\times4} + \ldots$

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