1. Find the derivative ***and*** the integral with respect to x of $f(x) = \dfrac{x}{1-x^2}$.

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2. Find $\dfrac{d}{dx}\coth x$.

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3. Show that $\tanh^{-1} x \equiv \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ for $|x| < 1$ and hence find the value of $\tanh^{-1}\dfrac{1}{2}$.

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4. Find the exact value of $I = \int_0^{\pi/3} x\sec^2 x \, dx$.

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5. Find $I(x) = \int e^{ax}\cos bx \, dx$, where a and b are constants, not both zero.

   Check your solution by showing that $\dfrac{dI}{dx} = e^{ax}\cos bx$.

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6. Find $\displaystyle\int x^3 e^{\left(-x^2\right)} dx$ and check your solution by differentiating it.

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7. An arch is in the shape of that arc of the downward-opening parabola $y = 6x - x^2$ for which $y \geq 0$. Find the coordinates (x, y) of the vertex (the highest point on the arch)

   (a) by a method that does not use calculus; and
   (b) by a calculus-based method.
   (c) Sketch the parabola.

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8. For the curve in $\mathbb{R}^2$ that is given in parametric form by

$$\vec{\mathbf{r}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t^2 \\ t^3 \end{bmatrix}$$

   Sketch the curve.

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9. For the curve whose Cartesian equation is

$$\left(x^2 + y^2\right)^{3/2} = 2x^2$$

(a) Find and simplify the equation in polar coordinates.
(b) Sketch the curve.

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10 (a) Sketch the graph of the curve whose equation in Cartesian form is

$$y = \cos(3x)$$

Indicate on your sketch the values of any two of the x-axis intercepts.

(b) Hence sketch the graph of the curve whose equation in polar form is

$$r = \cos(3\theta)$$

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11. Complex numbers z can be represented in three completely equivalent ways: the Cartesian form $(x + jy)$, the polar form $(r\angle\theta = r\cos\theta + jr\sin\theta)$ or the exponential form $r\,e^{j\theta}$, where $j = \sqrt{-1}$. Any non-zero number z has exactly n distinct nth roots, best found using the polar or exponential forms.

Find the exact values of the three cube roots of $z = 4 + 4j\sqrt{3}$.
Sketch z and its cube roots on an Argand diagram.

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12. For the curve whose equation in polar form is $r = 2\sec\theta\tan\theta$,
(a) Find the Cartesian form of the equation of the curve.
(b) Hence classify the curve [what type of curve is it?].
(c) Sketch the curve, labelling the points where $\theta = -\pi/4$, 0, $\pi/4$ and $3\pi/4$.

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13. Sketch the curve whose equation in polar form is $r^2 = 4\cos 3\theta$.
Include the following features:
(a) Sketch guide circle(s) for the maximum and minimum values of r.
(b) Sketch guide lines for the distinct tangents to the curve at the pole.
(c) Indicate the range of values of θ for which r is not real.
(d) Sketch the regions of the curve where $r < 0$ in a different colour from the distinct regions of the curve where $r > 0$.
(e) Label all distinct points on the curve where r attains its maximum and minimum values and specify a pair of polar coordinates (r, θ) for each such point.

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