ENGI 3425 Mathematics for Civil Engineering 1

Problem Set 7 Questions

(Sections 7.5 - 7.8 – Gradient Vector, Maxima & Minima)

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1. A scalar field $V(x, y, z)$ in $\mathbb{R}^3$ is defined by

$$V(x, y, z) = \frac{x^2 + y^2}{1 + z^2}$$

(a) Evaluate the gradient vector $\vec{\nabla} V$ at the point P (3, 4, 0)

(b) Hence find the instantaneous rate at which V is changing when one moves through the point P in the direction of the vector $\vec{\mathbf{a}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

(c) In what direction must one travel at point P in order to experience the greatest possible instantaneous increase in V?

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2. The temperature T inside a material is modelled by

$$T = 10\, r\, e^{-r/10}$$

where r is the distance of the point (x, y, z) from the origin.

(a) Find the rate at which the temperature is [instantaneously] increasing at the point (4, 4, 7) when one is moving in the direction of $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.

(b) At what location(s) is the temperature not changing in any direction?

(c) At all other locations, in what direction is the temperature increasing most rapidly?

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3. Find the Cartesian equations of the normal line and the tangent plane to the ellipsoid

$$\frac{x^2}{12} + \frac{y^2}{27} + \frac{z^2}{75} = 1$$

at the point P (2, –3, 5).

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4. Find the angle between the surfaces

$$x^2 + y^2 + z^2 = 4 \quad \text{and} \quad z = 1$$

at the point $P\left(-1, \sqrt{2}, 1\right)$.

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5. Determine the location and nature of all critical points of the function

$$f(x, y) = xy\left(1 - xy^2\right)$$

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6. Determine the location and nature of all critical points of the function $z = f(x, y)$, where
$$x^2 + 2x + y^2 + 4y + 2z + 5 = 0$$

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7. Determine the location (x, y, z) and nature (relative minimum, relative maximum or saddle point) of the critical point(s) of the function
$$f(x, y) = x^2 - y e^{-y}$$

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8. Determine the location (x, y, z) and nature (relative minimum, relative maximum or saddle point) of the critical point(s) of the function
$$f(x, y) = x^3 - 3xy + y^3$$

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9. Find the location (x, y, z) and nature (local minimum, local maximum or saddle point) of all critical points of the function
$$z = f(x, y) = x^2 e^{\left(1-x^2\right)} + y^2$$

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10. Find the maximum and minimum values of $f(x, y) = 4x^2 + 9y^2$ on the circle $x^2 + y^2 = 1$.

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11. A window is to be constructed in the shape of a rectangle surmounted by an isosceles triangle. In order to meet building code requirements for the room in which it is to be placed, the window must have a fixed glass area of 3 square metres. Determine the dimensions of the window such that the cost of the materials is minimized.

[Hint: Since the glass area is fixed, the only way to accomplish the task is to minimize the amount of the material needed for the perimeter of the window.]

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12. Determine the location and nature of all critical points for the function $h(x, y, z) = y$ on the ellipse of intersection of the cone $f(x, y, z) = x^2 - y^2 + z^2 = 0$ and the plane $g(x, y, z) = x + 3y + 2z = 3$.

[Hint: This is a Lagrange Multiplier problem with two constraints.]

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13. Find the point on the sphere, radius 1, centre the origin, the sum of whose Cartesian coordinates is the greatest.

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14. Determine the location and nature of all critical points of the function
$$z = x^3 + y^3 - 3x - 3y$$

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