ENGI 3425 Mathematics for Civil Engineering 1

Problem Set 8 Questions

(Sections 8.1 – 8.3 – Multiple Integration)

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1. Evaluate

$$\iint_D x^3 y^2 \, dA$$

over the triangular region D that is bounded by the lines $y = x$, $y = -x$ and $x = 2$.

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2. Evaluate

$$\iint_R y \, dA$$

over the region R that is bounded by the lines $y = 1 + x$, $y = 1 - x$ and $y = 0$.

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3. Evaluate

$$\iint_R (x - 3y) \, dA$$

over the region R that is bounded by the triangle whose vertices are the points (0, 0), (2, 1) and (1, 2):

(a) directly

(b) using the transformation of variables $x = 2u + v$, $y = u + 2v$.

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4. Find the mass and the location of the centre of mass of the lamina D defined by $\{ 0 \le x \le 2, \ -1 \le y \le 1 \}$ and whose surface density is $\sigma = xy^2$.

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5. Find the location of the centre of mass of the lamina D defined by the part of $x^2 + y^2 \le 1$ that lies in the first quadrant and whose surface density is directly proportional to the distance from the x-axis.

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6. Evaluate

$$\iiint_R z \, dV$$

where R is the region in the first octant that is between 1 and 2 units away from the origin.

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7. Use the transformation of variables $x = u/v$, $y = v$ to evaluate

$$\iint_R x\,y\,dA$$

over the region R (in the first quadrant) that is bounded by the lines $y = x/3$, $y = 3x$ and the hyperbolae $xy = 1$ and $xy = 3$.

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8. Find the centre of mass for a plate of surface density $\sigma = \dfrac{k}{x^2 + y^2}$, whose boundary is the portion of the annulus $a^2 < x^2 + y^2 < b^2$ that is inside the first quadrant. k, a and b are positive constants.

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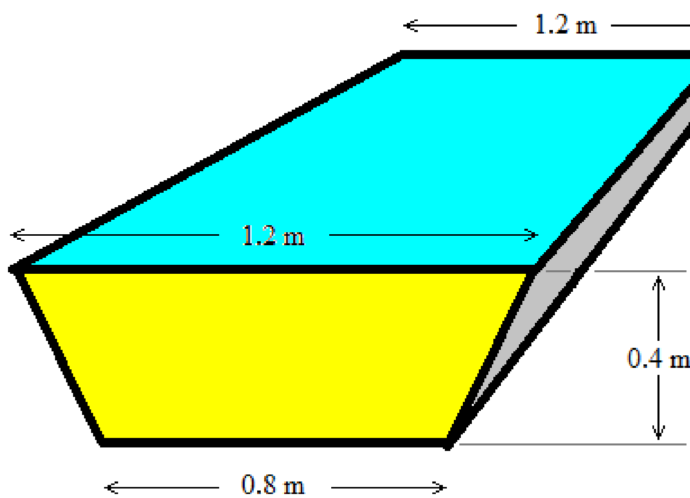
9. Find the net hydrostatic force F on each end wall of the trough shown, due to the weight of the water that it contains. The end walls have the shape of a regular trapezium.



Note that the pressure p at depth h below the surface of the water is $p = \rho g h$, where $\rho \approx 1000\,\text{kg m}^{-3}$ is the density of water and $g \approx 9.81\,\text{m s}^{-2}$ is the [approximately constant] acceleration due to gravity. The element of force ΔF on an element of area ΔA due to the pressure p is $\Delta F = p \cdot \Delta A$.

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