

9. Introduction to Ordinary Differential Equations

Equations involving only one independent variable and one or more dependent variables, together with their derivatives with respect to the independent variable, are ordinary differential equations (**ODEs**, to be studied in ENGI 4425).

Similar equations involving derivatives and more than one independent variable are partial differential equations (**PDEs**, to be studied in ENGI 5434).

The **order** of a differential equation is determined by the highest order derivative present.

A **linear** ordinary differential equation is an equation in which the dependent variable and its derivatives each appear only to the power 1 (or not at all) and do not multiply each other. The most general linear ODEs of first, second and fourth order can each be written in the forms

$$\frac{dy}{dx} + p(x)y = r(x),$$

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = r(x)$$

and

$$\frac{d^4y}{dx^4} + a_3(x)\frac{d^3y}{dx^3} + a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = r(x)$$

respectively.

Most ODEs of practical interest in engineering and science are of first or second order. Some problems involving deflections of beams, elasticity, etc. can be of fourth order.

Example 9.01

Which of the following ODEs are linear?

$$\frac{dy}{dx} + 2y = e^{-2x}$$

$$\frac{d^2y}{dx^2} + y \frac{dy}{dx} + 4y = x$$

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 3x^2y = e^{-x}$$

$$e^{-x} \frac{dy}{dx} = (y+1) \cos x$$

$$\frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + y = 0$$

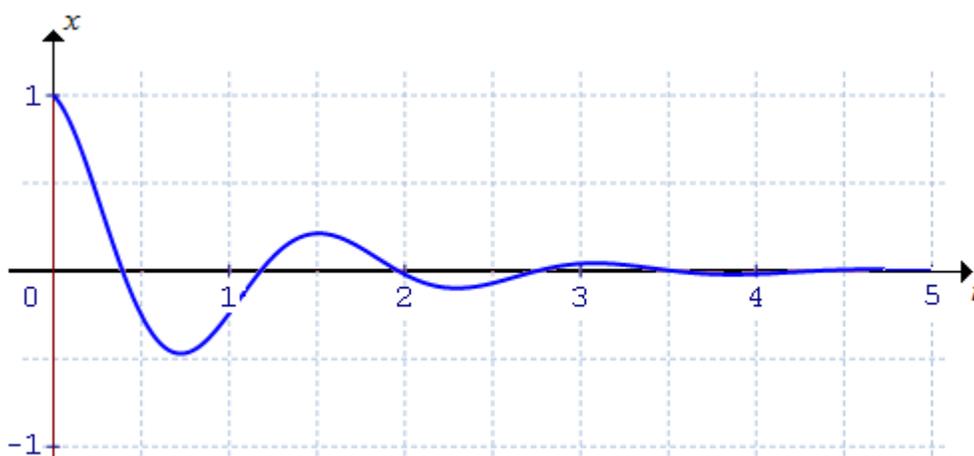
$$\frac{d^2x}{dt^2} + 8 \frac{dx}{dt} + 17x = e^{-t}$$

An **initial value problem** consists of a differential equation together with one or more initial conditions.

Example 9.02

Verify that $x(t) = e^{-t} \cos 4t$ is a solution to the initial value problem

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 17x = 0, \quad x(0) = 1, \quad \left. \frac{dx}{dt} \right|_{t=0} = -1$$



Only first order and linear second order ODEs will be studied in detail in ENGI 4425.

[Space for additional notes]

[End of Chapter 9]
[End of ENGI 3425!]
