

ENGI 4430
Mid Term Test
2021 June 16

1. A curve in the x - y plane has the equation $y = 3x^2$ in Cartesian coordinates.
- (a) Show that the arc length L along this curve from the point $(0, 0)$ to the point $(2, 12)$ is [3]

$$L = \int_0^2 \sqrt{1+36x^2} dx$$

- (b) Use Simpson's rule with $n = 4$ intervals to estimate L to two decimal places. [8]
(c) Write down the Cartesian equation of the surface of revolution formed when this curve is rotated about the x -axis. [2]
(d) Find the Cartesian symmetric equations of the normal line to this surface at the point $P(1, 0, -3)$. [7]

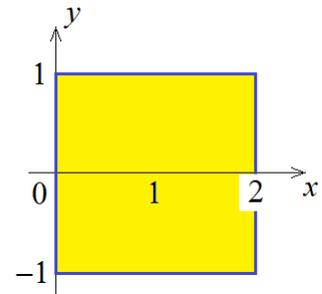
2. Find the equations of the family of lines of force (streamlines) for the vector field $\vec{F} = 2z\hat{j} + (3+4y)\hat{k}$ and find the equation of the line of force that passes through the point $(3, 1, -2)$. [20]

3. A thin sheet of metal has the shape of a square of side 2 m, with its left side along the y axis and a line of symmetry along the x axis, as shown. [20]

Its surface density is

$$\sigma(x, y) = (3x + 5y^2) \text{ kg m}^{-2}$$

Find I_x , its second moment of mass about the x axis.



4. **BONUS QUESTION** [+5]

Without the aid of any graphing software, sketch the curve whose equation in parametric form is

$$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{6t}{1+t^3} \begin{bmatrix} 1 \\ t \end{bmatrix} \quad (t \in \mathbb{R})$$

You must show your working.