ENGI 4430 Advanced Calculus for Engineering
Faculty of Engineering and Applied Science

Problem Set 3 Questions

[Multiple Integration; Lines of Force]

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1. Evaluate

$$\iint_D x^3 y^2 \, dA$$

over the triangular region D that is bounded by the lines $y = x$, $y = -x$ and $x = 2$.

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2. Evaluate

$$\iint_R y \, dA$$

over the region R that is bounded by the lines $y = 1 + x$, $y = 1 - x$ and $y = 0$.

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3. Evaluate

$$\iint_R (x - 3y) \, dA$$

over the region R that is bounded by the triangle whose vertices are the points (0, 0), (2, 1) and (1, 2):

(a) directly

(b) using the transformation of variables $x = 2u + v$, $y = u + 2v$.

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4. Find the mass and the location of the centre of mass of the lamina D defined by $\{ 0 \le x \le 2,\ -1 \le y \le 1 \}$ and whose surface density is $\sigma = xy^2$.

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5. Find the location of the centre of mass of the lamina D defined by the part of $x^2 + y^2 \le 1$ that lies in the first quadrant and whose surface density is directly proportional to the distance from the x-axis.

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6. Evaluate

$$\iiint_R z \, dV$$

where R is the region in the first octant that is between 1 and 2 units away from the origin.

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7. Use the transformation of variables $x = u / v$, $y = v$ to evaluate

$$\iint_R x y \, dA$$

over the region R (in the first quadrant) that is bounded by the lines $y = x/3$, $y = 3x$ and the hyperbolae $xy = 1$ and $xy = 3$.

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8. Find the centre of mass for a plate of surface density $\sigma = \dfrac{k}{x^2 + y^2}$, whose boundary is the portion of the annulus $a^2 < x^2 + y^2 < b^2$ that is inside the first quadrant. k, a and b are positive constants.

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9. Find the centre of mass for a cylinder, centre the z-axis, radius 2 m, height 3 m, with its base on the x-y plane, with volume density $\rho = \dfrac{kz}{\sqrt{x^2 + y^2}}$.

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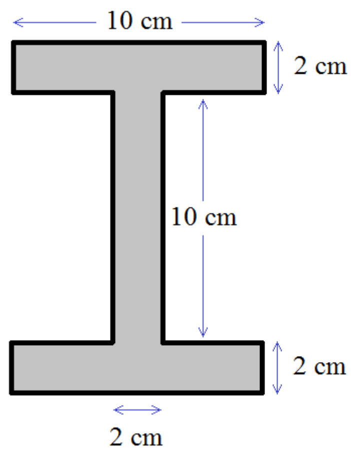
10. Find the moment of inertia of this cross section of a uniform guide rail of uniform thickness and uniform surface density $\sigma = 0.5 \text{ kg cm}^{-2}$ about its centroid.



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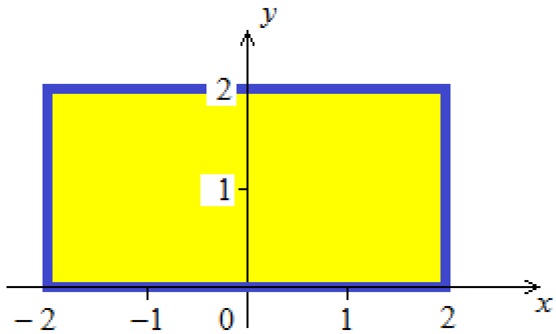
11. When the density is not constant, the centre of mass (the balance point) can be in a different location from the centroid (the geometric centre).

The rectangle shown below has a surface density $\sigma = 3y \text{ kg m}^{-2}$.



(a) Find the location of its centroid.
(b) Find the second moments of area around the centroid.
(c) Find the second moments of area around the origin of the coordinate system shown.
(d) Find the location of its centre of mass.
(e) Find the moments of inertia around the centre of mass.
(f) Find the moments of inertia around the origin of the coordinate system shown.

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12. For the vector field $\vec{\mathbf{F}} = \begin{bmatrix} \frac{1}{x} & e^y & -1 \end{bmatrix}^T$,

(a) find the equations of the lines of force and
(b) find the equations of the particular line of force passing through the point (2, 0, 4).

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13. For the vector field $\vec{\mathbf{F}} = 2e^z \hat{\mathbf{j}} - \cos y \,\hat{\mathbf{k}}$,

(a) find the equations of the lines of force and
(b) find the equations of the particular line of force passing through the point (3, π, 0).

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14. Find the family of vector fields in $\mathbb{R}^3$ whose lines of force are straight lines.

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15. In ENGI 3424, example 4.8.4, we solved the problem of finding the direction of steepest ascent at the point $P(500, 300, 3390)$ on the hill modelled by

$$h(x, y) = 4000 - \frac{x^2}{1000} - \frac{y^2}{250}$$

Find the equation on the *x*-*y* plane of the projection of the path that must be taken from the point P to reach the summit at $S(0, 0, 4000)$, while following the path of steepest ascent at all points on the hill. Sketch this path on the *x*-*y* plane.

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