ENGI 4430 Advanced Calculus for Engineering
Faculty of Engineering and Applied Science

Problem Set 5 Questions

[Gradient, divergence and curl (Cartesian coordinates)]

---

1. Find the divergence and curl of the vector field $\vec{\mathbf{F}} = xy^2\,\hat{\mathbf{i}} + x^2y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$.

---

2. Find the divergence and curl of the vector field $\vec{\mathbf{F}} = \frac{x}{y}\hat{\mathbf{i}} + \frac{y}{z}\hat{\mathbf{j}} + \frac{z}{x}\hat{\mathbf{k}}$.

---

3. A temperature distribution for a region within 75 metres of the origin is given by

$$T(x, y, z) = \frac{10\,000 - x^2 - y^2}{z + 100}$$

(a) Find the gradient of the temperature function T.
(b) Find the [instantaneous] rate at which the temperature is changing at the point (50, 50, 0) in the same direction as the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$.
(c) Is the field formed by the gradient vector purely radial?
[That is, does the gradient vector point directly towards or directly away from the origin at every point?]

---

4. Find the equations of the tangent plane and the normal line to the surface

$$x^2 + xy + z^3 = 8$$

at the point (0, –3, 2).

---

5. Find the equations of the tangent plane and the normal line to the sphere $x^2 + y^2 + z^2 = 9$ at the point (–2, 1, 2).

---

6. Find the angle between the elliptic paraboloid $z = 3x^2 + 2y^2$ and the parabolic cylinder $7y^2 = 2x + z$ at the point (1, 1, 5), to the nearest 0.01 degree.

---

7. Calculate the directional derivative of $\phi(\vec{\mathbf{r}}) = x \ln y - e^{x/z^3}$ at the point (8, 1, −2) in the direction of the vector $\vec{\mathbf{a}} = 12\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

---

8. Find the family of streamlines associated with the velocity field

$$\vec{\mathbf{v}}(x, y) = y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}$$

and find the streamline through the point (0, –1).

☜ Back to the index of questions On to the solutions to this problem set ☞