ENGI 4430 Advanced Calculus for Engineering
Faculty of Engineering and Applied Science

Problem Set 8 Questions

[Surface integration]

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1. Show that no potential function exists for the vector field $\vec{\mathbf{F}} = \begin{bmatrix} x & y & \cos xy \end{bmatrix}^{\mathrm{T}}$.

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2. For the vector field $\vec{\mathbf{F}} = e^{-kr}\vec{\mathbf{r}}$, where $\vec{\mathbf{r}} = \begin{bmatrix} x & y & z \end{bmatrix}^{\mathrm{T}}$ and $k > 0$ is a constant,
   (a) find the divergence of $\vec{\mathbf{F}}$.
   (b) find the curl of $\vec{\mathbf{F}}$.
   (c) find where div $\vec{\mathbf{F}} = 0$ and classify this surface.
   (d) find where the magnitude F of the vector field $\vec{\mathbf{F}}$ attains its maximum value.
   (e) show that $V(r) = \dfrac{-(1+kr)}{k^2} e^{-kr}$ is a potential function for the vector field $\vec{\mathbf{F}}$.
   (f) find the work done to move a particle from the origin to a place where div $\vec{\mathbf{F}} = 0$.

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3. Find the centroid (centre of mass when the surface density is constant) of the frustum of the circular cone $z^2 = x^2 + y^2$ between the origin and the plane $z = a$, (where a is a positive constant).

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4. Find the total flux that passes through the hemisphere $x^2 + y^2 + z^2 = a^2$, $z > 0$, due to the electric flux density $\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} = \dfrac{Q}{4\pi r^3} \vec{\mathbf{r}}$, (where $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$), using:
   (a) the surface projection method and
   (b) the parametric surface net method.
   (c) What can you say about the relative orientation of the vector field **D** and the normal **N** to the hemisphere?

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5. Find the total flux of the vector field $\vec{\mathbf{F}} = \left(\dfrac{x}{2}\right)\hat{\mathbf{i}}$ through that part of the circular paraboloid $z = 4 - x^2 - y^2$ that lies in the first octant.

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6. A thin sheet, in the shape of the paraboloid $z = 16 - \left(x^2 + y^2\right)$,

where $x \geq 0$ and $y \geq 0$, has a surface density of $\rho(\vec{\mathbf{r}}) = \dfrac{xy}{\sqrt{1+4\left(x^2+y^2\right)}}$. It lies within the coaxial region between the circular cylinders $C_1: \left(x^2+y^2=1,\ z \in \mathbb{R}\right)$ and $C_2: \left(x^2+y^2=9,\ z \in \mathbb{R}\right)$.

(a) Determine the total mass of the sheet, and
(b) Locate the centre of mass for the sheet.

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7. Fluid is flowing along a cylindrical pipe. The circular cross section inside the pipe has a constant radius of *a* (m). The Cartesian coordinate system is aligned with the *x* axis along the line of symmetry (centre line) of the pipe. The other two axes are in the plane of one of the circular cross sections. The velocity of fluid at any point (*x*, *y*, *z*) inside the pipe is

$$\vec{\mathbf{v}} = \frac{v_0}{a}\sqrt{a^2 - y^2 - z^2}\,\hat{\mathbf{i}} \quad \left(\mathrm{ms}^{-1}\right)$$

Find the flux Q (in m^3/s) (the rate at which fluid is flowing through the pipe).

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8. Find the mass and the location of the centre of mass of the upper half of the ellipsoid, whose equation in Cartesian coordinates is

$$x^2 + y^2 + 4z^2 = 4, \quad (z \geq 0)$$

and whose surface density is

$$\rho(x,y,z) = \frac{1}{\sqrt{1+3z^2}}$$

(a) using a parametric net method. and
(b) using a projection method.

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9. Find the mass of a shell that is in the shape of the upper half ($z > 0$) of the sphere of radius 1 m, centre the origin, and whose surface density is

$$\sigma(r,\theta,\phi) = r\cos\theta \quad \left(\mathrm{kg/m^2}\right)$$

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10. Find the moment of inertia I of a spherical shell of radius a and constant surface density σ about any axis that passes through the centre of the sphere, in terms of the radius a and the mass m of the spherical shell.

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