1) Find the intersection of the planes whose equations in Cartesian coordinates are [10]

$$\begin{aligned} x + y + z &= 6 \\ 2y - z &= 1 \\ 3x + y + 4z &= 17 \end{aligned}$$

Show your working.

What type of geometric object is the intersection? (point? line? etc.)

BONUS QUESTION

Explain, in geometrical terms, why the linear system has no solution if the equation of the third plane is changed to $3x + y + 4z = 18$. [+4]

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2) By any valid method, find the function $x(t)$ whose Laplace transform is [10]

$$X(s) = \frac{5\,e^{-2s}}{s^2 + 7s + 12}.$$

Hence (or otherwise) solve the initial value problem for a mass-spring system

$$\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 12x = 5\delta(t-2), \qquad x(0) = x'(0) = 0,$$

where $\delta(t-2)$ is the Dirac delta function, (modelling a sudden hammer blow to the system at time $t = 2$ seconds). Sketch a graph of the solution.

Is this system under-damped, critically damped or over-damped?

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3) Determine the nature (node, saddle point, centre or focus) and stability of the critical point for the linear system of differential equations [10]

$$\begin{aligned} \frac{dx}{dt} &= 2x - y \\ \frac{dy}{dt} &= 7x - 6y \end{aligned}$$

Sketch the orbits near the critical point and label any asymptotes.

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4) Use a Frobenius series method to find the general solution of the ordinary differential equation [10]

$$x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + 4y = 18x^2$$

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