

A Multi-faceted Approach to Teaching Modelling, via Hydrologic Routing

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Abstract

There are many ways to try to model engineering phenomenon. Three well-known modelling categories or methods are (i) the building of physical models, followed by laboratory measurements during system operation, (ii) via deterministic modelling using analytical solutions of, or numerical approximations to, the governing differential equations, (iii) via non-deterministic modelling using 'best-fit' equations to mimic the observed processes (without seeking a deeper understanding of the underlying mechanics). Numerical solutions themselves represent a large number of possible approaches and much is known about the magnitude of the errors that may be expected for a given method. All these approaches are interesting and useful in their own ways, but it is still a challenge to make this material seem interesting to engineering students. Today's engineering professor is also confronted with a large array of ever-changing software that purports to 'skin the cat' of process modelling in new and better ways. This paper describes how the phenomenon of level-pool hydrologic routing was used in a civil engineering course as a vehicle to introduce students to all of these approaches, including a powerful simulation software that writes computer-code based on the user's intuitive understanding of the processes being observed.

Introduction

Dalhousie University course CIVL4720 "Civil Engineering Computations" was originally conceived as one in which various numerical methods would be taught using examples specifically from civil engineering. Except for

this selectivity it was, prior to the fall of 2000, a fairly typical 'number-crunching' course. There was no laboratory component. Our on-going desire is to increase the amount of environmental modeling in the course, thereby increasing both its usefulness and the level of interest experienced by students. This paper describes an experience with teaching modelling using the well-known hydrologic phenomenon of level-pool routing as the 'pedagogic vehicle'. The routing phenomenon was modelled in this course in five ways: (i) physically, using an experimental set-up that was simple and inexpensive to build, (ii) numerically, by executing traditional numerical schemes in Excel®, (iii) analytically, for part of the problem, (iv) statistically, using non-linear transformations and ordinary least squares regression (OLS) curve fitting, and (v) using an intuitive systems simulation software package known as Stella (HPS 2000). It is believed that this last approach represents quite a departure from what civil engineering students normally encounter in their undergraduate programs.

Method

The phrase 'level-pool routing' in hydrology refers either to the manner in which water moves through a pond or reservoir, or to one of a number of algorithms that may be used to simulate this phenomenon, all mathematically based on the simple principle of the conservation of volume. A series of such reservoirs is sometimes referred to as a cascade. A typical outflow sequence for such a cascade is shown in Figure 1.

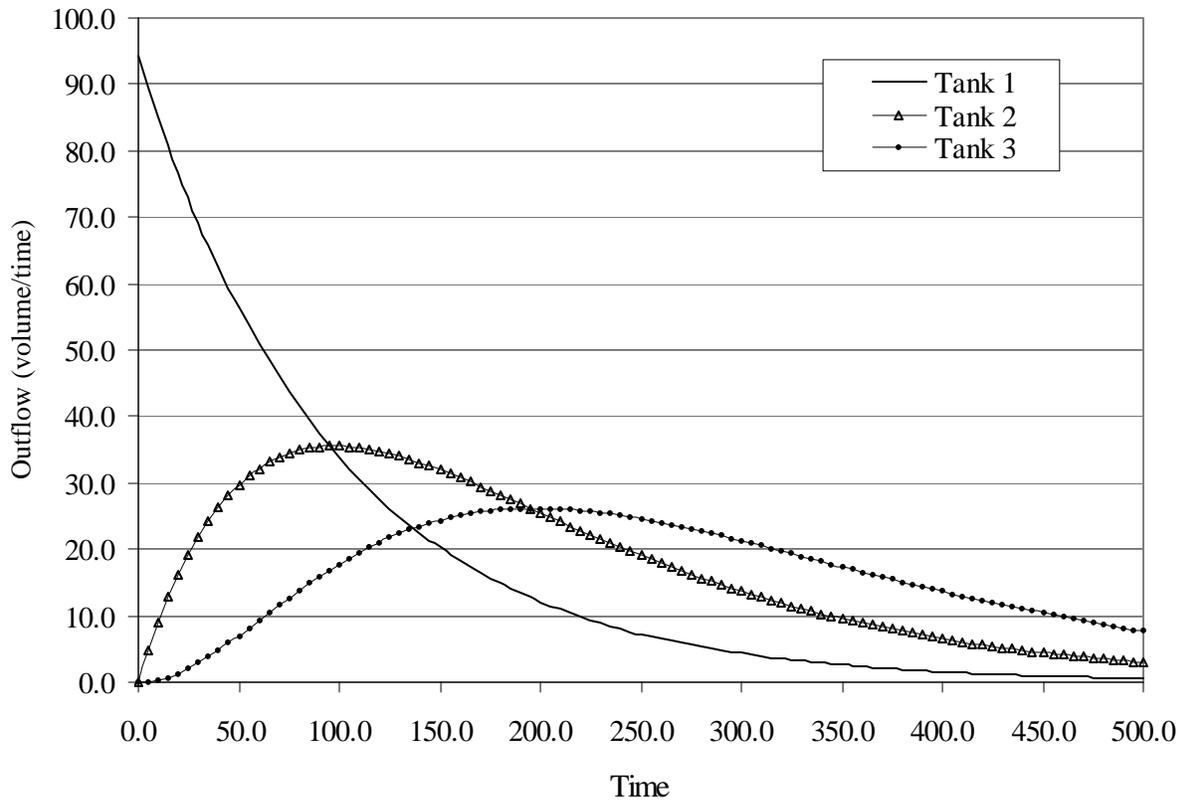


Figure 1. Outflow hydrographs for a cascade of three reservoirs.
 (The first reservoir had no inflow; in this case it drained an imposed initial volume).

It was desired to make the students' modelling experience more than just successfully generating graph(s) like Figure 1. It was hoped that by making the modelling effort more experiential ('hands-on') and by including an element of competition, that the level of student interest would be increased. Therefore, in preparation for the fall 2000 offering of this course a simple cascade of three small reservoirs was designed and assembled (see Figure A1 in Appendix A). The outflow from each of these 30 cm diameter clear plastic reservoirs, placed in series, was controlled by interchangeable sharp-edged circular orifices a few mm in diameter. For the set-up that was used for CIVL4720 the top tank (having the largest orifice) drained the most quickly and the bottom one (having the smallest orifice) the most slowly.

Students used a vernier caliper and a measuring tape to obtain "all relevant physical dimensions" of the apparatus,

such as tank heights, tank diameters, and orifice diameters. Naturally, some students returned to obtain measurements that they later realized were needed in order to do the modelling. The procedure was as follows: water coloured with fluorescent green dye drained through a series of tanks and the students took pictures at regular intervals of the state of the system while the water is making its way through it. This was initiated by having one student pull the plug in the top tank and start a clock at the same moment. Another student took a sequence of colour photographs with a digital camera, capturing a number of water levels in each tank. Because there was a clock in the view of the camera, the students were able to obtain the time that had elapsed for any given set of water levels. These images were emailed to the students and they used the standard orifice equation to convert the water levels, as measured from the images, into outflows. In this way they compiled all the data associated with the complete passage of the water that was initially

only in the top tank. This system for physically modelling hydrologic routing was much less expensive and complex than a data acquisition system with three water level pressure transducers connected to a Lab-View equipped PC.

Basic Theory

The phenomenon associated with how the water runs in, and out of, any given tank is governed by the following equation, founded on the conservation of volume:

$$Q_{in} - Q_{out} = \frac{ds}{dt} \quad [1]$$

where 's' is the volume in any given reservoir. Q_{in} is the hydrograph 'supplied' by the next-most upstream reservoir. For orifice outlets Q_{out} is controlled by:

$$Q = C_D A_0 \sqrt{2gh} \quad [2]$$

Due to a sabbatical taken by the regular hydrology professor, it turned out that the students at this point in their program had no working understanding of eqn [1] nor did they have any computational ability to perform routing. Basic knowledge about routing was therefore taught in this course, instead of in the hydrology course. However, the students were deliberately not given equation [2] on the grounds that it had been previously covered in their introductory Fluid Mechanics course. A surprising number of them owned no fluid mechanics textbook in which they might quickly find eqn [2], and virtually none of them had any idea what the equation governing an orifice might look like. They struggled through this difficulties on their own, though not happily.

Having the basic theory and their data-set in hand, the students then sought to computationally reproduce (model) what they had observed, in four ways: (i) Using the exact analytical solution that describes the drainage of the first reservoir. (No analytical solution exist for the hydrographs observable in the tanks beyond the first one), (ii) Using two numerical solutions to equations [1] and [2], executed in MS-Excel®, (iii) Using a modern drag-and-drop icon-based simulation package known as Stella®, and (iv) Statistically, via non-linear OLS curve-fitting. The first three methods were presented in the course as 'Deterministic Modelling', the last as an example of 'Non-deterministic Modelling'. These approaches will now be elaborated upon:

Modelling Approaches

(i) Physical Modelling

This component was described in part under 'Method' (above). Low heads corrections to orifice behaviour were not required of the students, although a couple of the more perspicacious ones did ask if they should include this effect. A great deal is known about the physical modelling of

hydraulic phenomenon occurring in open channel (rivers) and over structures (such as spillways), usually based on Froude scaling laws. It does not appear that much work has been done on inferring the behaviour real reservoirs using model reservoirs. This may be investigated at a future date, especially with regard to the trapping of model sediment. At this stage of course development the students were not required to make any such inferences; the outcomes of this laboratory work were taken and used at face value.

(ii) Numerical Modelling

There are many well-known numerical schemes for solving both ordinary differential equations (Liengme 1997, Orvis 1987) and partial differential equations (Hansen 1992, Hansen and Droste 1990, Olsthoorn 1985, Townsend *et al* 1991) that can be executed very efficiently in spreadsheet programs such as Excel® (see also Wolff 1995). In this case the method of Euler as well as Heun's improvement upon it (Chapra and Canale 1988) were applied to the following non-linear first-order ordinary differential equation (for the hydrologic theory see also Bedient and Huber 1992):

$$\frac{dh}{dt} = \frac{Q_{in}}{A_R} \frac{ph^q}{A_R} = f(h, t) \quad [3a]$$

$$A_R = mh^n \quad [3b]$$

where:

h = depth above the orifice (L),

Q_{in} = inflow to the tank (L^3/T),

p and q = empirical parameters governing the outflow hydraulic (an orifice in this case, so $q=0.5$),

A_R = surface area of the reservoir, single-valued in this case (L^2),

m and n = empirical parameters relating A_R to h, equal to unity in this case.

These algorithms can be efficiently executed in the tabular form for which spreadsheets are famous (see Table 1a and 1b). Modern desktop computer CPUs are so fast that there now seems to be little interest in the relative efficiency of the algorithms used to solve many (but not all) civil engineering problems. In addition, what is more important in an educational setting is that (i) the students implement the relevant mathematics personally and 'pseudo-manually' (not using black-box software), and that (ii) students are able to implement the mathematics efficiently. It seems that an excessive amount of time can be spent debugging conventional code, necessitating teaching fewer numerical methods and giving fewer problems. Aspect (ii) is an important consideration when teaching engineering students because their academic load is quite heavy, and especially so if a variety of techniques are being taught within a single course in numerical methods.

Table 1a. Table for executing level-pool reservoir routing, using the Euler method to solve equation [3].

Time (sec)	Inflow Q_{in} (cm ³ /s)	Head in tank 1 (m)	Outflow Q_{out} (cm ³ /s)	Area A_R (cm ²)	Slope $f(h, t)$	$h(t+\Delta t)$ (cm)
0	0	33.020	122.918	294.6	-0.41723	32.186
2	0.00	32.186	121.355	294.6	-0.41193	31.362
etc		31.362				etc

Table 1b. Tabular execution of level-pool reservoir routing, using the Euler-Heun method to solve equation [3].

Time (sec)	Inflow Q_{in} (cm ³ /s)	Head in tank 1 (cm)	Outflow Q_{out} (cm ³ /s)	Area A_R (cm ²)	1 st slope $f(h, t)$	Revised Q_{out}	Revised A_R	2 nd slope $f(h, t)$	average \bar{f}	$h(t+\Delta t)$
0	0	32.020	122.918	294.6	-0.41723	121.355	294.6	0.41193	0.41458	32.191
2	0.00	32.191	121.365	294.6	-0.41196	119.801	294.6	0.40665	0.40931	31.372
etc		31.372								etc

The outcome of the above tables (i.e. the outflow hydrographs) for tank one becomes the inflow to tank two, etc.

(iii) Analytical Solution (1st tank)

The closed form solution for the volume that has exited the top tank after time t is (Appendix C):

$$V = C_D A_0 \sqrt{2g} \left(t \sqrt{h_0} - \frac{t^2}{2\beta} \right) \quad [4]$$

Volumes computed using eqn [4] could therefore be compared with the volumes for tank 1 measured in the laboratory. This comparison helped the students realize the limitations on the accuracy of their physical measurements.

(iv) Statistically

An important component of CIVL4720 is the use of OLS curve fitting and non-linear transformations to describe processes non-deterministically. It can be shown that, for the top tank, the head at any time 't' is (see Appendix C):

$$h_2 = \left[\sqrt{h_1} - \frac{t}{\beta} \right]^2 \quad [5a]$$

where h_1 is the initial h (imposed, about 0.32 m in this case). The students needed to realize that they should regress $\sqrt{h_2}$ versus time t . The parameter β contained quantities that the students measured with calipers or a tape measure (see Appendix C):

$$\beta = \frac{2A_R}{C_D A_0 \sqrt{2g}} \quad [5b]$$

so that the regression result, arising from the hydraulic behaviour (about 30 data points collected over 20 minutes), could be compared to an independent estimate of β found using actual length measurements.

(v) Using Systems Simulation Software (Stella[®])

Stella[®] is a systems-simulation software package that can be adapted to almost any time-dependent process (HPS 2000). The examples that come with this software include such things as the manufacture/distribution of beverages, the

ecology of a deer population, and the waiting time of patients going to a hospital Emergency Room. The package is equally comfortable with, and equally easy-to-use, whether the movement of money or of water is being studied. The examples provided by HPS show a highly non-trivial level of detail with respect to the sub-processes considered. The software is easy to learn and is directed at documenting and formalizing one's understanding of processes and sub-process interaction (the company motto is "Because understanding cannot be memorized"). Stella[®] uses four main icons to describe a system: (i) a stock-taker (a box-shaped icon representing the concept of inventory) in which 'stuff' may be accumulated or decremented, (ii) a convertor, (a circle icon, perhaps representing a brain) in which governing formula and functions can be invoked, (iii) a flow controller (a valve-on-pipe icon), which usually connects the stock icons, and (iv) a conveyor (line or curve with arrow-head) which can be thought of as a telephone line that either informs conveyors and flow-controllers of the present status of various quantities, or sends instructions to these icons regarding how to control releases (often via formula outcomes executed in convertors).

The icons are dragged onto the screen and connected intuitively. Equations that govern the details of the processes are added later. The package has utilities for plotting and recording outcomes that are invoked in the same manner. Figure 2 shows how the first author set up the routing problem in Stella.

Outcomes

We were reasonably pleased with the students' reports, especially considering that this was the first time that this experiment had been attempted. The requirements as to what the report had to contain were stated in too general a fashion. Instructions to "compare outcomes" were generally not well-executed by the students. In future they will be told exactly what graphs to prepare.

It was assumed that the general idea of modelling a physically-observable phenomenon using theories, as compared to approximations to theories, was already understood. This was apparently not uniformly the case. Many students seemed to treat all of the outcomes, including the physical modelling effort, as having completely equal validity and significance, in that many students did not seem to treat the experimental data as the basis for all other comparisons. They were also weak in their appreciation of (i) the role of errors in their physical measurements on the outcomes, and of (ii) the idea that parameter estimates arising from modelling efforts might not be perfect, and might in fact be honed or calibrated in order to improve the agreement between computed and observed outcomes.

Appendix A presents a statement of how the initial data collection and processing was to be executed. A progress report was required shortly afterwards so as to 'spread out' the work in a more explicit manner (students being notoriously poor at beginning the analysis of fresh data in a timely manner). Appendix B is the statement of what was finally required of each group – a semi-formal report in which all the methods used to model the routing phenomenon were to be compared. Appendix C is a mathematical derivation that was given to the students to aid in their understanding of the behaviour of top tank in the reservoir cascade.

The students found the quality of the still images to be adequate, but only just. Perhaps a video sequence taken at close-range would provide adequate image quality and also permit reading of times from the clock and water levels from the tanks at any desired interval.

Conclusions

Hydrologic routing was successfully used as a vehicle to introduce civil engineering students to the idea that there are many ways to 'simulate' a given phenomenon. Data collection of time-varying water levels with a digital camera was reasonably successful, but might in future be simplified by creating a digital video that could be mounted on the WebCT site for the course. There were some lighting problems and the student groups were somewhat large, but these problems will be remedied in the next offering of the course. Many of the students were very intrigued by the possibilities of the Stella[®] systems simulation software. The new experiment was a qualified success and will be used again in the fall of 2001.

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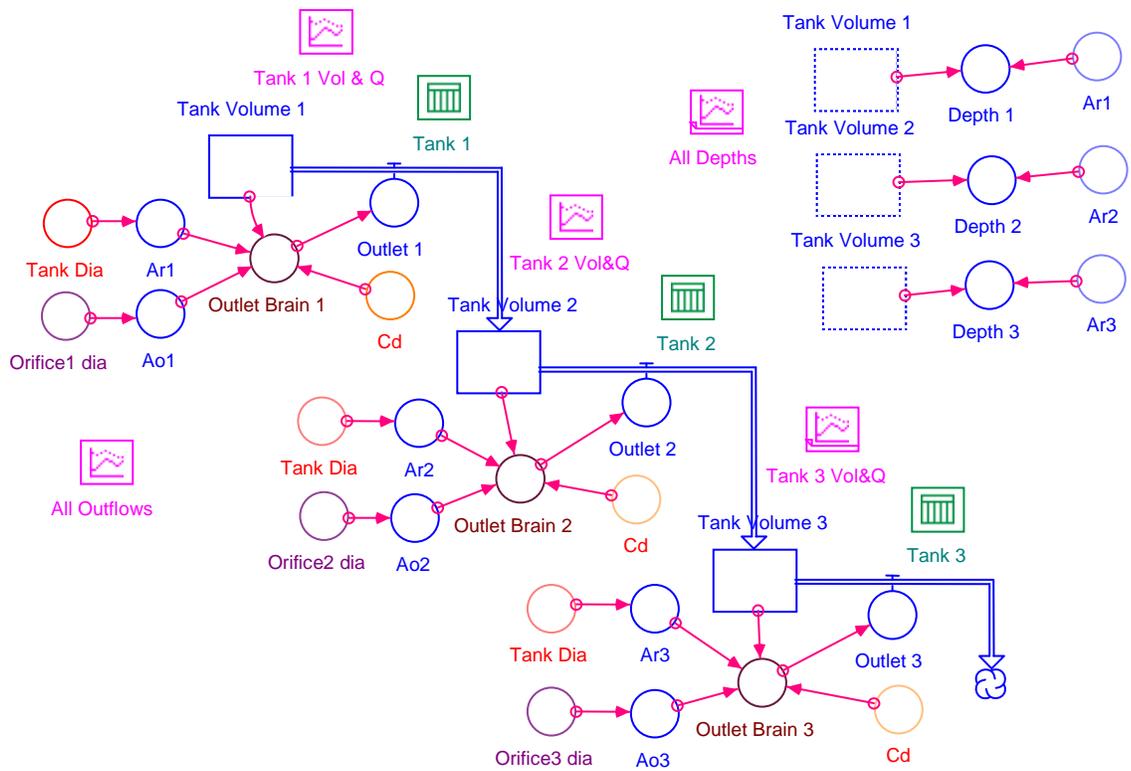


Figure 2. 'Template' for the reservoir cascade model created using Stella[®] systems simulation software. (The template becomes animated during simulation, with the tanks filling and emptying.)

APPENDIX A
LABORATORY ASSIGNMENT:
INITIAL DATA COLLECTION FROM PHYSICAL MODEL TEST.

INTRODUCTION

Select a group leader – he/she will be the recipient of the jpg images, by email. The TA will record this person's name and the names of the people in your group. In this experiment you will record the behaviour of physical model of a reservoir cascade. In this case it will be a stack of three tanks, set up above one another as a cascade of three, placed in series. These reservoirs are cylindrical and each has an orifice in the bottom (centre).



Figure A1. Ensure that the above apparatus is level, so that the tank walls are vertical.

PROCEDURE:

1. Use a black dry-erase marker to write the name of your group leader and the date on the card at the top of the apparatus.
2. Gently 'screw' the rubber plug into the hole in the (empty) top reservoir.
3. Temporarily move the clock away from the stand.
4. Use the hose to fill the top reservoir nearly to the top. Use the tap (hand-valve) in the SE corner of the lab to control the fill-up.
5. Put the clock back on the stand, plug it in, and zero it. Turn on the light and hold it **near** (but not in front of) the clock. The light is important in illuminating the clock face, so as to be clearly seen in the photos (ie. you will use a series of jpg's to read off the times so the clock face needs to be well-lit).
6. Get photographer in position. You need a photo at $t=0$. Measure the initial level in the top tank.
7. Have one person must simultaneously pull the plug and flick the start switch on the clock.
8. Take a sequence of pictures to document what happens with respect to water level variation in the three containers, through time. Note any unusual behaviour.

BEFORE YOU LEAVE:

- Make sure you understand how to read the clock-face. Note that the major divisions are not minutes and the smallest divisions are not seconds.
- Measure all relevant physical dimensions that, in your opinion, affect the behaviour of the cascade. (using a ruler, calipers, etc.) This may necessitate taking the cascade apart. There is a small crescent wrench nearby to enable you to take the top nuts off. Do not attempt to re-assemble the cascade. Please do not drop the acrylic tanks!

INITIAL POST-PROCESSING OF THE DATA:

- i) Find the governing equation for an orifice in a fluid mechanics or hydraulic structures textbook (stated as flow, Q , as a function of depth over the centre of the hole, h). These books are in the TA347 and TC5 sections of our library. (This law was discovered by Evangelista Torricelli *circa* 1640.) Write down the formal reference for the book(s) that you use in academic citation format. Study the part of the text associated with the governing equation and write down all of the assumptions that are built into it. Note: the equation can be derived by applying Bernoulli's eqn between two points: the water level at h and the jet which emerges at atmospheric pressure at velocity V . This result will not, however, show the empirically-determined orifice coefficient C_D .
- ii) The TA will email your group leader the jpg images. Use a paper printout of these jpg's to obtain a series of water levels (heads) and times (by reading the clock in each view). Separate these data-sets by tank. You jpg's can be viewed by MS Photo-editor (installable from Win98) or Windows Paint. Increasing the brightness and contrast (along with the percent magnification) may assist you in reading the times from the clock.
- iii) Convert the heads to outflows using the orifice equation from step (i).
- iv) Plot the three outflow hydrographs (one for each tank) on the same piece of graph paper. Make the ordinate flow in cm^3/s and the abscissa time in minutes.
- v) Submit all of the above by Friday of the same week to the TA, for a preliminary evaluation.

APPENDIX B

CIVL4720 Civil Engineering Computations Part II

Group Assignment¹: COMPARISON OF METHODS FOR MODELLING THE ROUTING PHENOMENON.

Analysis and Report Preparation

1.0 Background and Theory.

Document all the data collected in the lab according to the guidelines provided re: presentation of graphs, equations, tables, and references. Also, document all relevant characteristics of the physical model. State all equations that are relevant to the physics of this problem, along with any limitations or assumptions. State the fundamental differential equation that governs level-pool routing. (One person.)

2.0 Deterministic Approach using Modern Simulation Software.

Use Stella to model the temporal variation in volume, head, and outflow for the various 'components' in the 3-tank cascade. Use your best estimates of the various physical characteristics of the cascade (diameters, circular areas, discharge coefficients) to reproduce the behaviour that you observed in the lab. Present graphical comparisons and discuss. (Two people.)

3.0 Conventional Deterministic Approach.

- a) Present the mathematical basis for the Euler-Heun Method, using nomenclature appropriate to this particular problem. Use this numerical method to simulate the observed behaviour of the cascade (this can be done efficiently in Excel). Present graphical comparisons and discuss. (One person.)
- b) Compare outcomes #2 and #3a, together with a presentation of the effect of changing Δt . (One person.)

4.0 Non-deterministic Approach (Curve-fitting via Statistical Methods).

Use appropriate transformations (where necessary) to obtain OLS-based 'best-fit' equations that describe your experimental data. (In some cases you might want to try to minimize $\sum \varepsilon_i^2$ without the use of a transformation.) Compare constants having physical significance and discuss your results. (One person.)

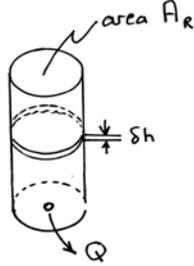
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¹ The names of the people in the groups who submit the first, second, and third-best reports will be published. The name(s) of the person(s) covering each section or doing a given set of tasks must be listed in an appendix. There will be one mark per report unless it is clear that one individual did not properly share the work-load.

Names: _____, _____, _____, _____, _____

APPENDIX C

BEHAVIOUR OF TOP TANK IN THE RESERVOIR CASCADE.



Find an expression for the time it takes to drain a cylindrical tank via an orifice located at depth h :

$$Q \delta t = -A_R \delta h \quad [C-1]$$

Solving for δt and integrating between the limits $h = h_1$ at $t = 0$ and $h = h_2$ at any subsequent time t :

$$\int_0^t dt = - \int_{h_1}^{h_2} \frac{A_R}{Q} dh \quad [C-2]$$

$$Q = C_D A_0 \sqrt{2gh} \quad [C-3]$$

by substitution and integration:
$$t = \frac{2A_R}{C_D A_0 \sqrt{2g}} (\sqrt{h_1} - \sqrt{h_2}) \quad [C-4]$$

or:
$$t = \beta (\sqrt{h_1} - \sqrt{h_2}) \quad [C-5]$$

where h_1 is the initial head and:

$$\beta = \frac{2A_R}{C_D A_0 \sqrt{2g}}$$

From eqn [C-5] this means that:

$$h_2 = \left[\sqrt{h_1} - \frac{t}{\beta} \right]^2 \quad [C-6]$$

Using eqn [C-6] in equation [C-3] gives:
$$Q = C_D A_0 \sqrt{2g} \left(\sqrt{h_1} - \frac{t}{\beta} \right) \quad [C-7]$$

Obviously:
$$\forall = \int Q dt \quad [C-8]$$

From eqn [C-7]:
$$\forall = C_D A_0 \sqrt{2g} \int_{t_1}^{t_2} \left(\sqrt{h_1} - \frac{t}{\beta} \right) dt \quad [C-9]$$

For $t_1 = 0$ and $h_1 = h_0$, and calling t_2 't':
$$\forall = C_D A_0 \sqrt{2g} \left(t \sqrt{h_0} - \frac{t^2}{2\beta} \right) \quad [C-10]$$