```
OR DESCRIPTION OF VARIOUS TERMS USED IN THIS
********************SOFTWARE***********************
TYPE 1 ELSE TYPE ANY NUMBER
AM-MAGNITUDE OF VECTOR A
BM-MAGNITUDE OF VECTOR B
CM-MAGNITUDE OF VECTOR C
A(1)-X COMPONENT OF VECTOR A
A(2)-Y COMPONENT OF VECTOR A
B(1)-X COMPONENT OF VECTOR B
B(2)-Y COMPONENT OF VECTOR B
C(1)-X COMPONENT OF VECTOR C
C(2)-Y COMPONENT OF VECTOR C
THETAA-ANGLE FOR VECTOR A
THETAB-ANGLE FOR VECTOR B
THETAC-ANGLE FOR VECTOR C
AF-VECTOR A DENOTING THE FIRST SOLUTION
AS-VECTOR A DENOTING THE SECOND SOLUTION
BF-VECTOR B DENOTING THE FIRST SOLUTION
BS-VECTOR B DENOTING THE SECOND SOLUTION
OTHER TERMS FOLLOW SIMILARLY
****************************************************
*****************************************************
***************SELECT FROM THE MAIN MENU*************
*****************************************************
*****************************************************
ALL INPUT ANGLES IN DEGREES
CASE NUMBER
1-CASE1
2-CASE2A
3-CASE2B
4-CASE2C
5-SUM OF TWO VECTORS-TWO DIMENSIONAL
6-DIFFERENCE OF TWO VECTORS-TWO DIMENSIONAL
7-CROSS PRODUCT-THREE DIMENSIONAL
8-DOT PRODUCT-TWO DIMENSIONAL
9-RECTANGULAR TO POLAR
10-POLAR TO RECTANGULAR
11-MODIFIED CASE2A
12-ROTATIONAL TRANSFORMATION OF A VECTOR
13-DIVISION AND MULTIPLICATION OF ORDINARY NUMBERS
14-SIMULTANEOUS CROSS PRODUCTS OF SEVERAL
     VECTORS IN SUMMATION FORM
15-SUMMATION OF N VECTORS IN POLAR FORM
16-SIMULTANEOUS DOT PRODUCTS OF N VECTORS
     IN POLAR FORM
17-RECTANGULAR TO POLAR CONVERSION
     OR VICE VERSA FOR N VECTORS
18-SOLUTION OF [A]{X}={B}, TWO EQUATIONS
100-STOP
```

FIG. 2.25 COMPUTER PROGRAM

respect to the reference set of coordinates. An example of this was shown in Eq.(2.12).

The addition or subtraction of ordinary numbers can also be done using programs 5 and 6 respectively. For example, if we want to add 5.132 and 6.059. We can add these by expressing it in a polar form as

$$X \angle\theta = 5.132 \angle\theta + 6.059 \angle\theta$$

where θ can be any number such as zero or 30 etc. The result (X) will be shown in the magnitude of the output. The subtraction can be performed in a similar manner. Program 13 can be used for multiplication or division of ordinary numbers. In this program we have

$$X = \frac{A \times B \times C}{D \times E \times F}$$

Suppose we want to divide 9.132 by 2.416. We can substitute A = 9.132, D = 2.416 and C = B = E = F = 1. This program will be useful in the velocity and acceleration analyses where, for example, the normal acceleration is given by ω^2 r. We can substitute A = B = ω, C = r, and D = E = F = 1. Thus several combination of numbers can be multiplied or divided without using a calculator. One can use program 15 where several vectors expressed in the polar form can be added by using it only once. This program is especially useful in the acceleration analysis.

A given kinematic problem may require the use of several of these programs. After finishing all the computations, one can get back to the system by typing 100. This software can be obtained from the **first author** at a very reasonable cost.

Example 2.14

A crank-slider mechanism is shown in Fig. 2.26. This type of

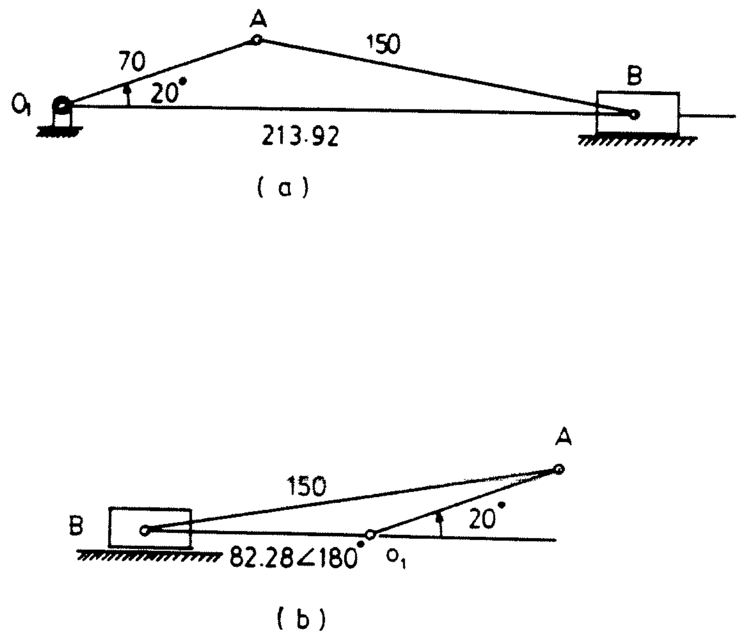

FIG. 2.26 TWO SOLUTIONS OF A CRANK SLIDER MECHANISM

mechanism is commonly used in gasoline or diesel engines. Find the two solutions for this mechanisms corresponding to

$$\mathbf{R}_{AO_1} = 70\underline{/20} \text{ cm and } R_{BA} = 150 \text{ cm}$$

Solution

We start with writing the vector equation

$$\mathbf{R}_{AO_1} = \mathbf{R}_{BO_1} + \mathbf{R}_{AB}$$

$$70 \underline{/20} = x \underline{/0} + 150 \angle\theta_{AB}$$

We recognize that it is case 2b discussed earlier. Using the computer program, we obtain the following two solutions.

$$70 \underline{/20} = 213.855\underline{/0} + 150 \underline{/170.816} \qquad \text{(I)}$$

$$70 \underline{/20} = 82.299\underline{/180} + 150 \underline{/9.184} \qquad \text{(II)}$$

These solutions are shown graphically in Figs. 2.26(a) and 2.26(b).

Example 2.15

An offset slider-crank mechanism is shown in Fig. 2.27. The parameters given are: $R_{AO_1} = 75$; $R_{BA} = 150$; $\mathbf{R}_{CB} = 125 \angle 180$; and $\mathbf{R}_{O_1C} = 20 \angle 90$. Find θ_{AO_1} and θ_{AB} .

Solution

The vector equation in this case will be

$$\mathbf{R}_{O_1A} = \mathbf{R}_{BA} + \mathbf{R}_{CB} + \mathbf{R}_{O_1C} \qquad \text{(a)}$$

Substituting the known parameters from above we can rewrite the above equation as

$$75 \angle\theta_{O_1A} = 150 \angle\theta_{BA} + 125 \angle 180 + 20 \angle 90 \qquad \text{(b)}$$

We can add the last two known vectors using the vector summation program and get

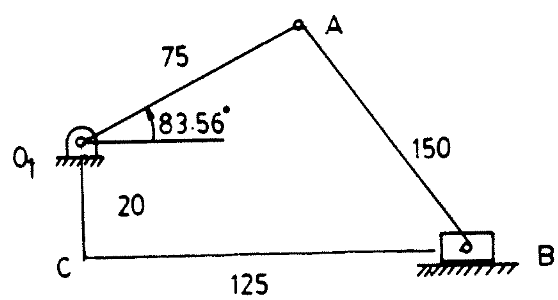

FIG. 2.27(a)

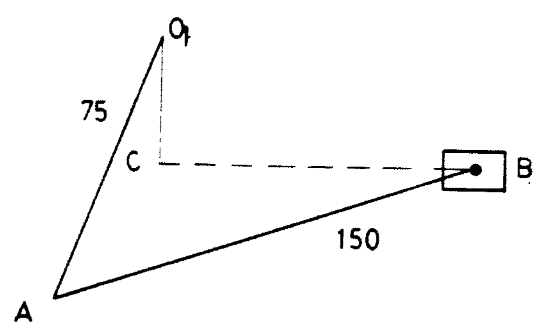

FIG. 2.27(b)

$$75 \angle\theta_{O_1A} + 150 \angle(\theta_{BA} + 180) = 126.590 \angle 170.910$$

This equation corresponds to case 2c. The two solutions are

$$75 \angle 263.476 + 150 \angle 140.943 = 126.590 \angle 170.910 \quad \text{(I)}$$

and

$$75 \angle 78.344 + 150 \angle 200.877 = 126.590 \angle 170.910 \quad \text{(I)}$$

From the first set of solutions we have

$$\theta_{O_1A} = 263.476, \text{ and}$$

$$(\theta_{BA} + 180) = 140.943 \quad \text{(a)}$$

or

$$\theta_{BA} = 140.943 - 180 = -39.057 = 360 - 39.057$$

$$= 320.943$$

From this we can obtain θ_{AB} as

$$\theta_{AB} = \theta_{BA} + 180 = 320.943 + 180 = 140.943$$

which is same as given in Eq. (a). Similarly, we can obtain θ_{AO_1} by writing

$$\theta_{AO_1} = \theta_{O_1A} + 180 = 263.476 + 180 = 443.476$$

$$= 443.476 - 360 = 83.476.$$

It should be noted that whenever the solutions were not between 0^0 to 360^0, we have added or subtracted 360^0 to bring it within this range. We can write the second set of solution as

$$\theta_{O_1A} = 78.344 \text{ and}$$

$$\theta_{BA} + 180 = 200.877.$$

Therefore

$$\theta_{BA} = 200.877 - 180 = 20.877.$$

These solutions are shown in Fig. 2.27b.

Example 2.16

For the mechanism shown in Fig. 2.28(a), the following

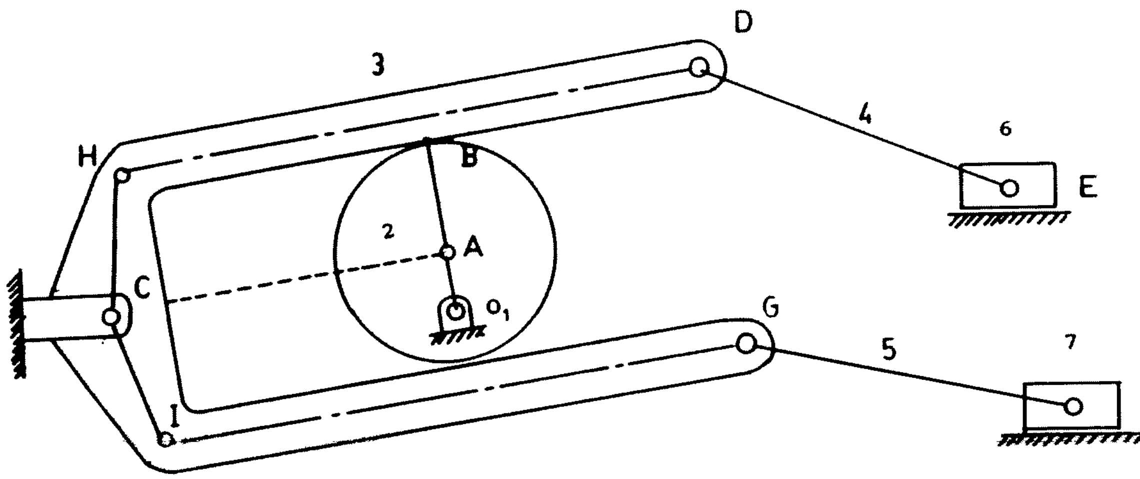

FIG. 2.28(a)

vectors and parameters are given:

$$R_{CO_1} = 20 \angle 190,\quad R_{AO_1} = 2\angle 100,\quad R_{IC} = 7 \angle 280$$

$$R_{BO_1} = 7 \angle 100,\quad R_{ED} = 20\angle 330,\quad R_{IG} = 40 \angle 15.709$$

$$R_{FG} = 20\angle 330,\quad R_{HC} = 7\angle 100,\quad R_{DH} = 40 \angle 15.709$$

The other dimensions are given in the figure. Find R_{DC}, R_{EC} and R_{FC}.

Solution

We can obtain the solution by writing the following equations

$$R_{AC} = R_{O_1C} + R_{AO_1} = 20\angle(190 - 180) + 2 \angle 100$$

$$= 20.099 \angle 15.709$$

The vectors R_{DH} and R_{AC} should be parallel for the link two to rotate. Therefore we can write

$$R_{DC} = R_{HC} + R_{DH} = 7 \angle 100 + 40 \angle 15.709$$

$$= 41.284 \angle 25.419$$

Similarly, we can obtain

$$R_{GC} = R_{IC} + R_{GI} = 7 \angle(100 + 180) + 40 \angle 15.709$$

$$= 39.916 \angle 5.660$$

Finally, we obtain R_{EC} and R_{FC} by writing

$$R_{EC} = R_{DC} + R_{ED} = 39.916\angle 25.419 + 20\angle 330 = 53.847\angle 7.613$$

$$R_{FC} = R_{GC} + R_{FG} = 39.916\angle 5.660 + 20\angle 330 = 57.363\angle 353.932$$

Example 2.17

The following vectors and parameters are given for the mechanism in Fig. 2.29(a).

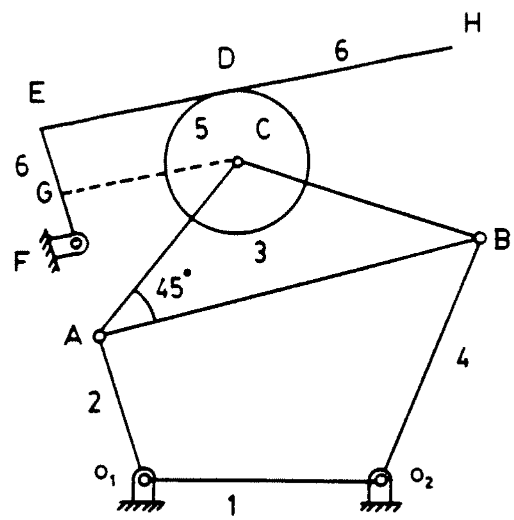

FIG. 2.29a

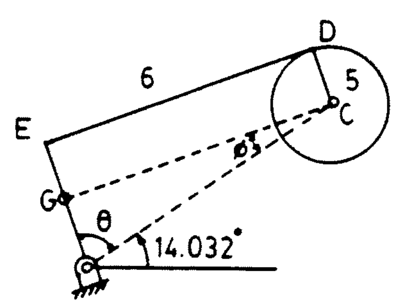

FIG. 2.29b

$$\mathbf{R}_{AO_1} = 2.5\ \underline{/150}; \quad \mathbf{R}_{AO_2} = 9.250\ \underline{/172.306}$$

$$\mathbf{R}_{CO_1} = 8.669\ \underline{/95.567}; \quad \mathbf{R}_{FO_1} = 10\ \underline{/135}$$

$$R_{EG} = R_{DC} = 2; \ \underline{/GED} = 90^0; \ R_{HE} = 10$$

Find $\mathbf{R}_{DF}$ and $\mathbf{R}_{HF}$.

Solution

We obtain $\mathbf{R}_{CF}$

$$\mathbf{R}_{CF} = \mathbf{R}_{CO_1} - \mathbf{R}_{FO_1} = 8.669\ \underline{/95.567} - 10\ \underline{/135}$$

$$= 6.422\ \underline{/14.032}$$

From the triangle GCF (refer to Fig. 2.29(b))

$$\phi = \text{Sin}^{-1}\left(\frac{GF}{FC}\right) = \text{Sin}^{-1}\left(\frac{0.5}{6.424}\right) = 4^0$$

Therefore $\theta = 86^0$

Now we can write

$$\mathbf{R}_{HF} = \mathbf{R}_{EF} + \mathbf{R}_{HE}$$

$$= 2.5\ \underline{/(86 + 14.032)} + 10\ \underline{/10.032}$$

$$= 10.308\ \underline{/24.068}$$

We can also obtain $\mathbf{R}_{DF}$ by writing the equation

$$\mathbf{R}_{DF} = \mathbf{R}_{CF} + \mathbf{R}_{DC} = 6.422\ \underline{/14.032} + 2\ \underline{/100.032}$$

$$= 6.858\ \underline{/30.945}$$

In this equation $\mathbf{R}_{DC}$ is parallel to $\mathbf{R}_{EF}$. We can solve $\mathbf{R}_{CO_1}$ from the figure using the method used in Example 2.13 rather than assuming from the problem statement.

Example 2.18

Find $\mathbf{R}_{CB}$ and $\mathbf{R}_{CO_1}$ in the mechanism shown in Fig. 2.30(a). Use the following data in your calculations:

$$\mathbf{R}_{AO_2} = 5\ \underline{/45}; \quad \mathbf{R}_{O_2O_1} = 20\ \underline{/180}; \quad R_{BA} = 7\ \underline{/80.881}$$

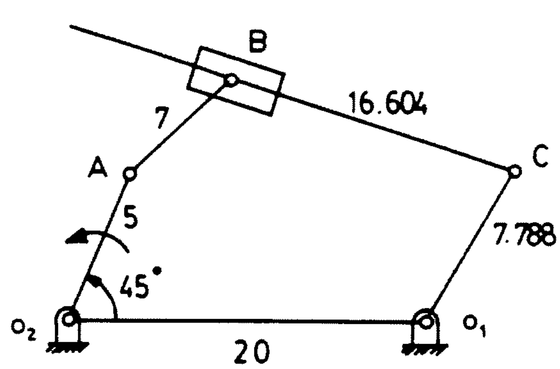

FIG. 2.30a

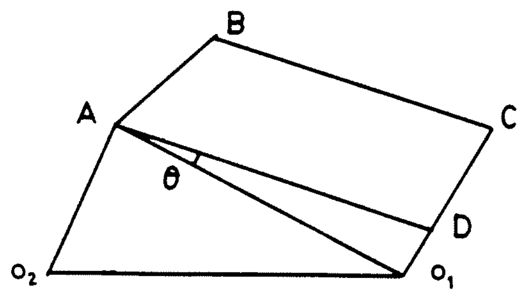

FIG. 2.30b

Solution

We can write

$$\mathbf{R}_{AO_1} = \mathbf{R}_{O_2O_1} + \mathbf{R}_{AO_2}$$

$$= 20 \angle 180 + 5 \angle 45 = 16.840 \angle 167.881$$

From Fig. 2.30(b), we can write the relationship

$$\theta = \mathrm{Sin}^{-1}\left(\frac{O_1D}{AO_1}\right) = \mathrm{Sin}^{-1}\left(\frac{0.788}{16.840}\right) = 3^0$$

In addition, vectors $\mathbf{R}_{BC}$ and $\mathbf{R}_{AD}$ are parallel.

Therefore

$$\theta_{O_1A} = \theta_{AO_1} + 180 = (167.881 + 180) = 347.881$$

$$\theta_{DA} = \theta_{O_1A} + 3 = 347.881 + 3 = 350.881$$

$$\theta_{CO_1} = \theta_{AD} - 90 = (350.881 - 180) - 90 = 80.881$$

Since vectors $\mathbf{R}_{CB}$ and $\mathbf{R}_{DA}$ are parallel,

$$\theta_{CB} = \theta_{DA} = 350.881$$

Therefore the vector $R_{CB} = 16.604 \underline{/350.881}$

2.8 Loop Closure Equation

For the closed loop mechanisms, the loop closure equation is quite useful in analyzing the positions of various points on various links as links take up different orientations during the motion. Vast number of mechanisms are derived from the four bar mechanism shown in Fig.2.31. We shall study this mechanism when the link l_1 (the crank) moves in a counter-clockwise direction. As this link moves, the other links go through a cyclic motion. The link l_2 is called the coupler and l_3 the follower.

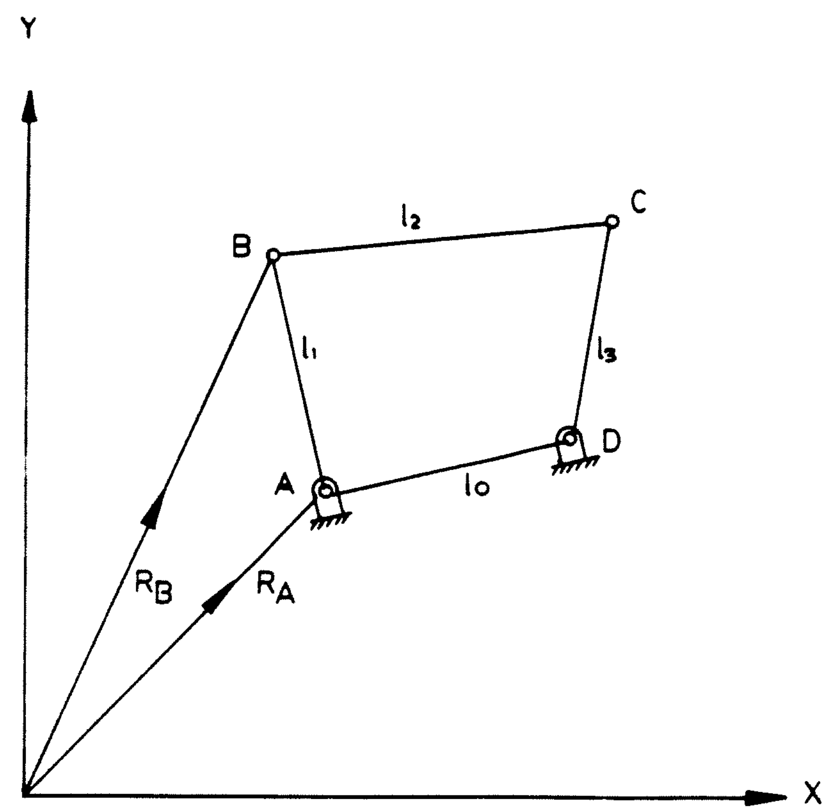

FIG. 2.31

Physically there is no fourth link; the links l_1 and l_3 are connected to the ground through a hinged type of connection. Therefore we can call the ground as l_0 , a frame, which does not move. In the present configuration we can write

$$\mathbf{R}_B = \mathbf{R}_A + \mathbf{R}_{BA} \text{ , and} \tag{2.81}$$

$$\mathbf{R}_C = \mathbf{R}_B + \mathbf{R}_{CB} \tag{2.82}$$

Substituting for $\underline{R}_B$ in Eq. (2.82) from Eq. (2.81) we get

$$\mathbf{R}_C = \mathbf{R}_A + \mathbf{R}_{BA} + \mathbf{R}_{CB} \tag{2.83}$$

Similarly we can write expressions for $\mathbf{R}_D$ and $\mathbf{R}_A$ as

$$\mathbf{R}_D = \mathbf{R}_C + \mathbf{R}_{DC}$$

$$= \mathbf{R}_A + \mathbf{R}_{BA} + \mathbf{R}_{CB} + \mathbf{R}_{DC}\text{, and} \tag{2.84}$$

$$\mathbf{R}_A = \mathbf{R}_D + \mathbf{R}_{AD}$$

$$= \mathbf{R}_A + \mathbf{R}_{BA} + \mathbf{R}_{CB} + \mathbf{R}_{BC} + \mathbf{R}_{AD} \tag{2.85}$$

Cancelling $\mathbf{R}_A$ from both sides we can write

$$\mathbf{R}_{BA} + \mathbf{R}_{CB} + \mathbf{R}_{BC} + \mathbf{R}_{AD} = 0 \tag{2.86}$$

Eq. (2.86) is called the loop closure equation and it is the summation of the position-difference vectors of the various end points of the links. As the crank rotates, various other links take up different orientations; thus the orientations of these vectors change with time. An important point to note is that the magnitude of these vectors do not change with time because the link lengths remain constant during the motion.

The loop-closure equation shows the relationship between the angular configurations of various links and the dependence of the orientations of links l_2 and l_3 on l_1. The rotation of the crank l_1 causes the other links to rotate and we will see in the subsequent chapters on velocity and acceleration analyses that the differentiations of the loop-closure equation leads to the velocity and acceleration relationships between the links.

2.9 Absolute and Apparent Displacements, Displacement-Difference Between Points

Fig. 2.32 shows a point moving along a curved path and its locations at two different instants of time, $t = t_1$ and $t=t_2$, are shown by points P_1 and P_2 respectively. This point is observed from a moving coordinate system whose locations at these instants of time are also shown. The absolute displacement of the point is given by

$$\Delta R_{P/O} = R_{P_2 O} - R_{P_1 O} \tag{2.87}$$

It is the difference of the position vectors with respect to the inertial coordinate system. Similarly, the apparent displacement

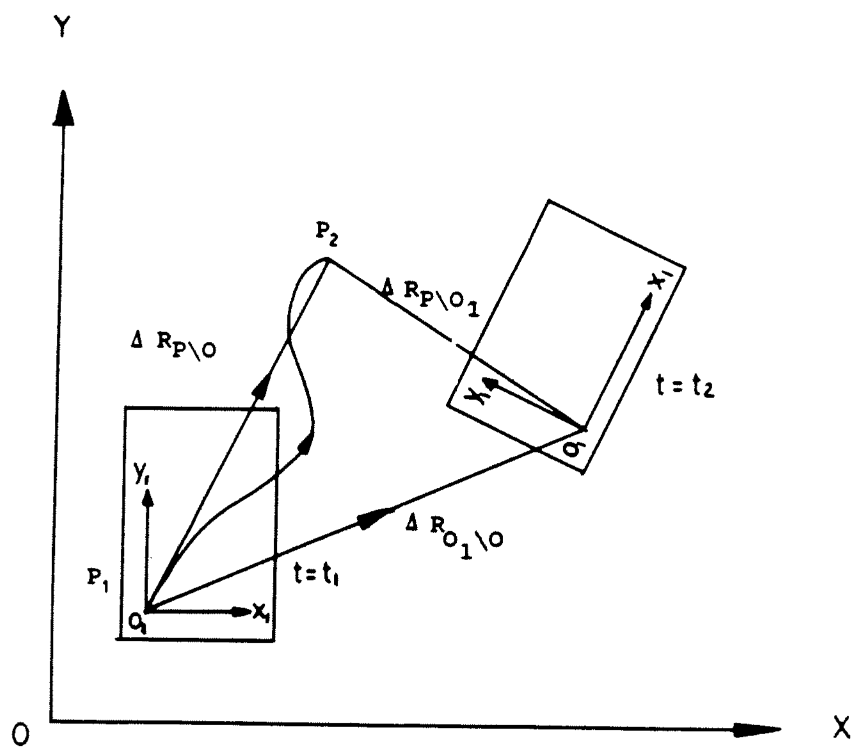

FIG. 2.32 ABSOLUTE AND APPARENT DISPLACEMENTS OF P

will be given by

$$\Delta R_{P/1} = R_{P_2 O_1} \tag{2.88}$$

The relationship between the apparent and absolute displacements is given by

$$\Delta R_{P/O} = \Delta R_{O_1/O} + \Delta R_{P/1} \tag{2.89}$$

This equation relates the absolute displacement of a moving point in the two dimensional space to its apparent displacement with respect to a moving coordinate system. It is an extremely useful relationship and will be used again in the chapters on velocity and acceleration analyses.

2.10 Translation and Rotation

The locations of two links P_1Q_1 and L_1M_1 at a certain instant of time $t = t_1$, is shown in the Fig. 2.33. These links move and their locations are also shown at $t = t_2$ as P_2Q_2 and L_2M_2 respectively. In this figure the orientation of the link P Q has not changed therefore $\Delta R_{P_1} = \Delta R_{Q_1}$. If we take any other point on this link then we will find that its displacement will also be equal to ΔR_{P_1} or ΔR_{Q_1} . This link has undergone translation where all the points have equal displacement vectors i.e., the displacement vectors of all the points are equal in magnitude as well as direction. This is not true in the case of link L_1M_1 where the orientation of the link has changed in the final position. This was possible because of the difference in the displacement vectors of various points. This link has undergone both a translation and a rotation. The total displacement can be divided into two parts. In the translation, the link takes up an orientation shown by $L_2M_2^*$ and then rotates motion the

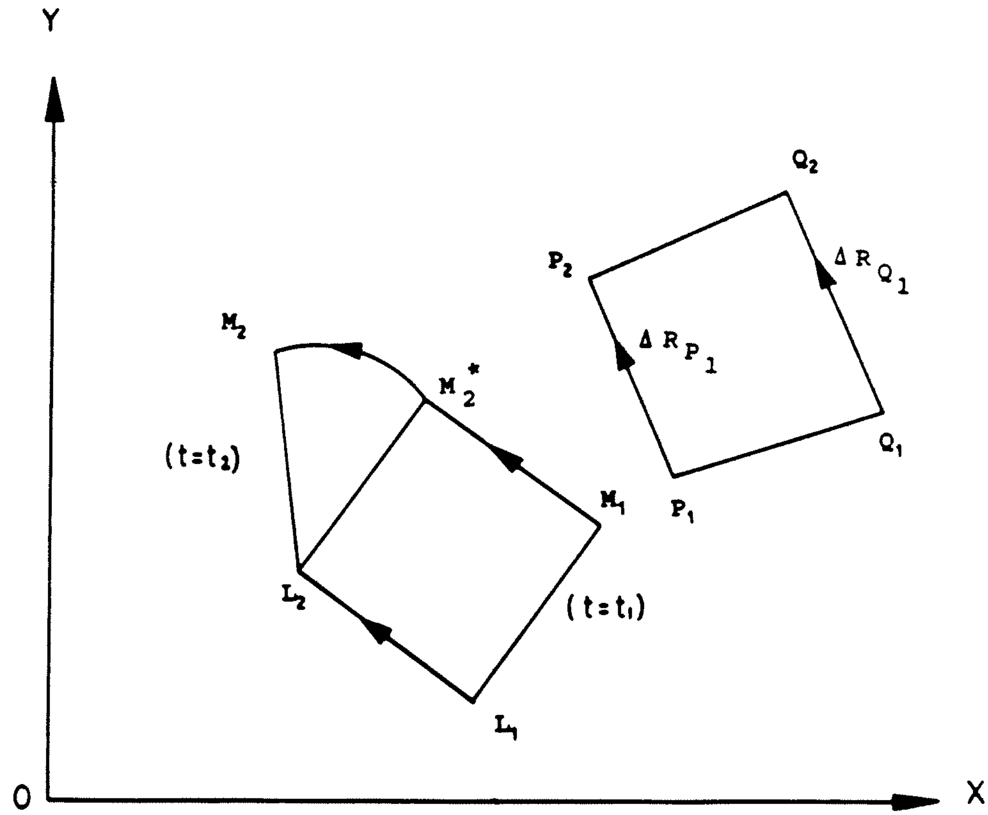

FIG. 2.33 DISPLACEMENT DIFFERENCE BETWEEEN TWO POINTS AND ROTATION

displacements of various points on the link are different and this rotation part accounts for the difference in displacements.

2.12 Maximum and Minimum Transmission Angle and Crank Angle

We have defined the transmission angle in Chapter 1 and for a certain four bar mechanism is shown in Figs. 2.34(a) and 2.34(b). We mentioned earlier that if the Grashoff's criteria is satisfied then the smallest link can revolve through 360^0. For these types of mechanisms only, the extreme values of the transmission angle occurs when the links l_0 and l_1 are alligned as shown in these two figures. Referring to the Figs. 2.34(a) and 2.34(b) we can calculate γ_{max} and γ_{min} as

$$\gamma_{max} = \text{Cos}^{-1}\left\{\frac{l_2^2 + l_3^2 - (l_0 + l_1)^2}{2\, l_2\, l_3}\right\} \tag{2.90}$$

$$\gamma_{min} = \text{Cos}^{-1}\left\{\frac{l_2^2 + l_3^2 - (l_0 - l_1)^2}{2\, l_2\, l_3}\right\} \tag{2.91}$$

On the other hand, if the link lengths are such that they do not satisfy the Grashoff's criteria or is not of the crank-rocker or double rocker type, then the crank would not be able to go through 360^0 but the crank angle will be limited within a certain minimum and maximum values. These values can be easily calculated from Figs. 2.34c and 2.34d as

$$(\theta_1)_{max} = \text{Cos}^{-1}\left\{\frac{l_0^2 + l_1^2 - (l_3 + l_2)^2}{2\, l_0\, l_1}\right\} \quad \text{and} \tag{2.92}$$

$$(\theta_1)_{min} = \text{Cos}^{-1}\left\{\frac{l_0^2 + l_1^2 - (l_3 - l_2)^2}{2\, l_0\, l_1}\right\} \tag{2.93}$$

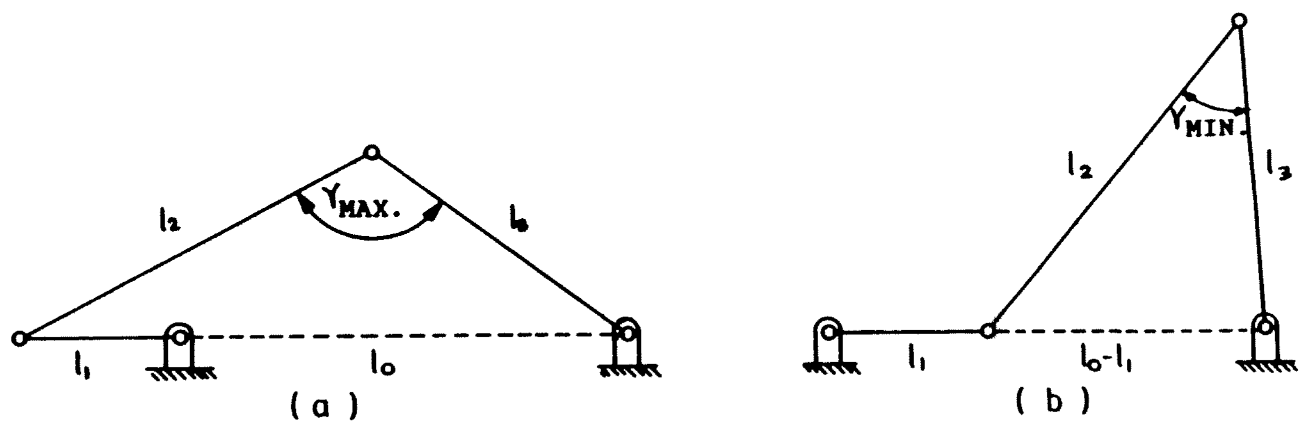

FIG. 2.34 MINIMUM AND MAXIMUM CRANK ANGLE OF FOUR BAR MECHANISMS

(a) MAXIMUM TRANSMISSION ANGLE POSITION

(b) MINIMUM TRANSMISSION ANGLE POSITION

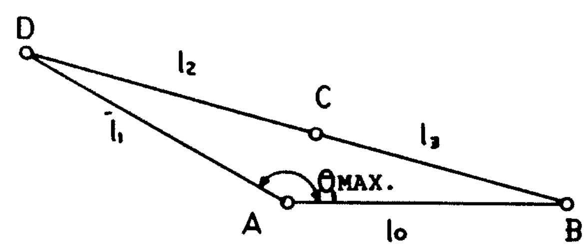

FIG. 2.34(c)

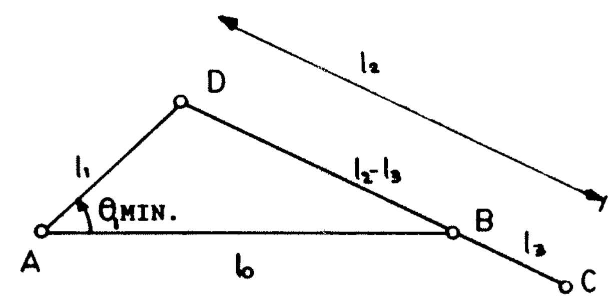

FIG. 2.34(d)

It is quite obvious from these two figures that $\gamma_{max} = 180^0$ and $\gamma_{min} = 0^0$ under these conditions. In other words, the links l_2 and l_3 are alligned along a straight line. In Eqs. (2.90) and (2.91), the links l_0 and l_1 were alligned along a straight line.

CHAPTER 3

VELOCITY ANALYSIS

3.1 Absolute Velocity

Fig. 3.1 shows the curved trajectory of a moving point P at two instants of time. The displacement of this point will be given by ΔR_p as shown in this figure and the average velocity during this interval Δt will be

$$\mathbf{V}_p = \frac{\Delta \mathbf{R}_p}{\Delta t} \tag{3.1}$$

The instantaneous velocity therefore will be

$$\mathbf{V}_p = \underset{\Delta t \to 0}{\mathrm{Lt}} \frac{\Delta \mathbf{R}_p}{\Delta t} \tag{3.2}$$

and its direction will be along the tangent to the curve at P. This is because as t tends to zero, the direction of $\Delta \mathbf{R}_p$ will approach the tangent.

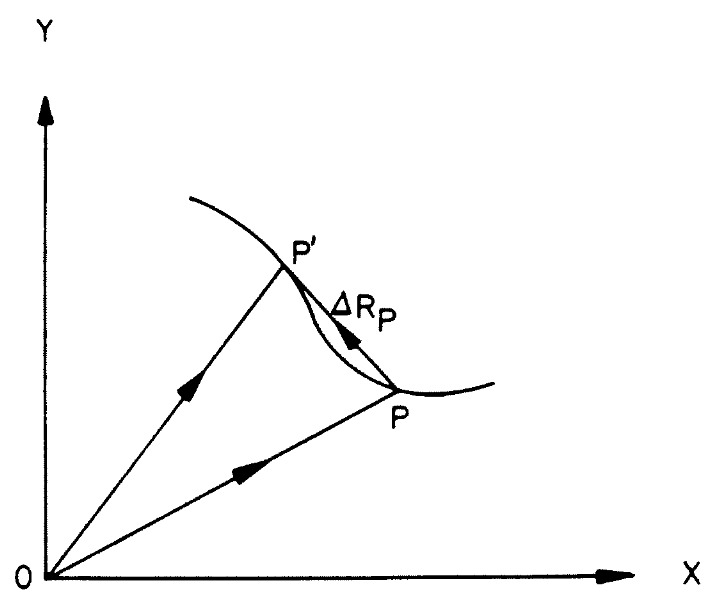

FIG. 3.1 CURVED TRAJECTORY OF A POINT AND INSTANTANEOUS VELOCITY

3.2 Angular Velocity and Velocity Difference

Fig. 3.2 shows the locations of a rigid link PQ at two instants of time t_1 and t_2. Since the orientations of the link at these instants of time are different, the motion of the link can be assumed to be a combination of translation and rotation. ΔR_p shows the translation and $\mathbf{R}_{Q''Q'}$ is the displacement of Q due to the rotation about P'. This is the difference of the displacements between the points Q and P and we will refer the vector $\mathbf{R}_{Q''Q'}$ as ΔR_{QP}. Using the triangle QQ'Q" we can write

$$\Delta R_Q = \mathbf{R}_{Q'Q} + \mathbf{R}_{Q''Q'} = \mathbf{R}_{P'P} + \mathbf{R}_{Q''Q'}$$

$$= \Delta R_P + \Delta R_{QP} \tag{3.3}$$

If $(t_2 - t_1) = \Delta t$ is infinitesimal then we can also write

$$\lim_{\Delta t \to 0} \frac{\Delta R_Q}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta R_P}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta R_{QP}}{\Delta t}$$

or

$$\mathbf{V}_Q = \mathbf{V}_P + \mathbf{V}_{QP} \tag{3.4}$$

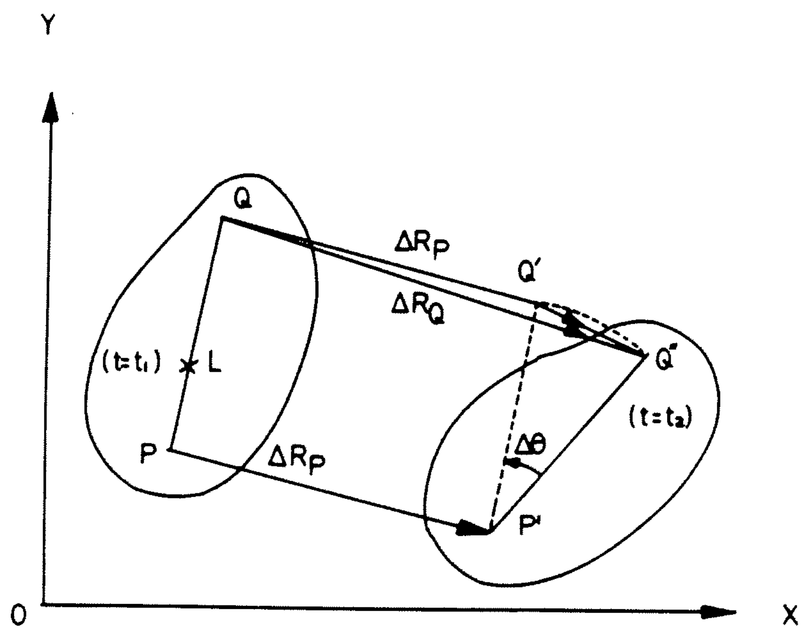

FIG. 3.2 ROTATION AND ANGULAR VELOCITY

Now we want to obtain the magnitude and direction of $\mathbf{V}_{QP}$. If we refer to Fig. 3.3 where the line P'S bisects the angle $\Delta\theta$, we can write $Q'Q'' = 2Q'S = 2Q'P' \sin(\frac{\Delta\theta}{2})$.

Thus, the magnitude of the vector $\mathbf{V}_{QP}$ using the definition above will be

$$|\mathbf{V}_{QP}| = \underset{\Delta t \to 0}{Lt} |Q'P' \frac{\Delta\theta}{\Delta t}| = Q'P'|\omega| \tag{3.5}$$

where the magnitude angular velocity ω is defined as

$$|\omega| = |\underset{\Delta t \to 0}{Lt} \frac{\Delta\theta}{\Delta t}| = |\frac{d\theta}{dt}|$$

and its direction is along the instantaneous axis of rotation which is along the Z axis in this case. If the rotation of the body is counter-clockwise then ω will have positive magnitude. For example, if the link at any instant of time is rotating with 5 radians per second counter-clockwise then we would write

$$\omega = 0\ i + 0\ j + 5\ k$$

The ω vector is always normal to the plane of rotation. Returning back to Eq.(3.4), the direction of $\mathbf{V}_{QP}$ will be tangential to the circle (along the unit vector τ) as shown in Fig. 3.3 Thus, using Eq. (3.5) we can write

$$\begin{aligned} \mathbf{V}_{QP} &= \omega\ R_{Q'P'}\ \tau = \omega \times \mathbf{R}_{Q'P'} \\ &= \omega \times \mathbf{R}_{QP} \end{aligned} \tag{3.6}$$

This equation indicates that the difference between the velocities is due to the rotation of the link provided the points lie on the link. If we take another point L (refer to Fig. 3.2) then using Eq. (3.6) we can write

$$\mathbf{V}_{LP} = \omega \times \mathbf{R}_{LP}$$

It is worth mentioning here that the term relative velocity

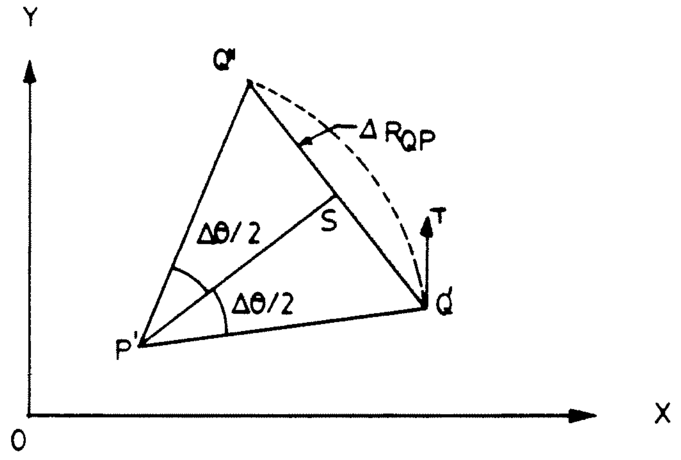

FIG. 3.3 DISPLACEMENT DIFFERENCE AND VELOCITY DIFFERENCE

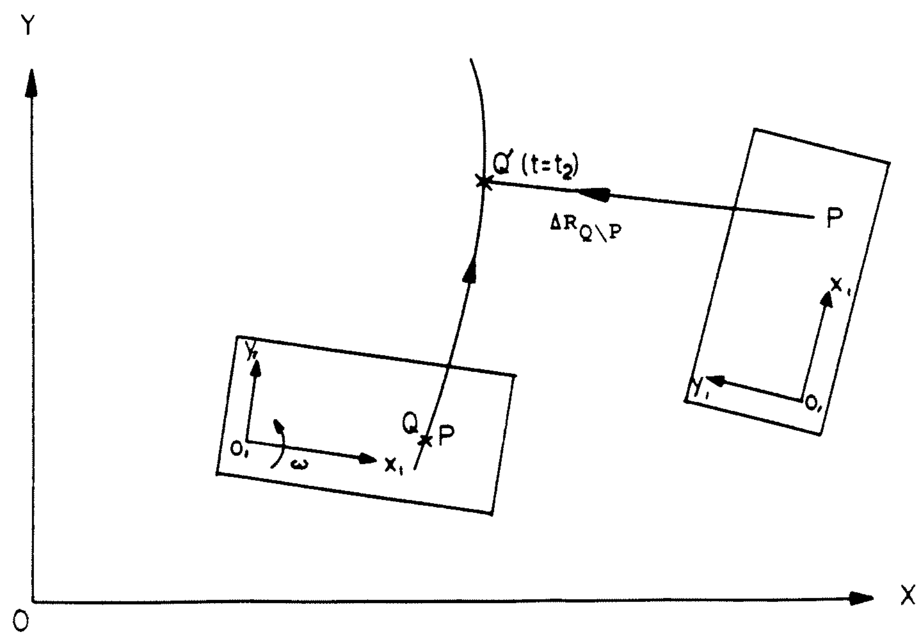

FIG. 3.4 APPARENT DISPLACEMENT AND APPARENT VELOCITY

commonly used in the kinematics area can be sub-divided into, (a) velocity difference and (b) apparent-velocity. The distinction between these two can be understood from Figs. 3.2 and 3.4. In Fig. 3.2, the velocity difference $\mathbf{V}_{QP}$ is due to the rotation of the link where the points Q and P are both on the same link. In Fig. 3.4, the point Q is not on the link and moves along a curved path. At a certain instant of time when viewed from the normal direction, it appears coincident with a point P on the moving link. The absolute velocity of Q can be observed from the X - O - Y system (a stationary system) whereas its velocity as observed from the (x_1- o_1- y_1) system (a moving system) will be apparent due to the motion of the link. The apparent velocity is based on the apparent displacement ($\Delta\mathbf{R}_{Q/P}$) as discussed in the previous chapter. Thus the apparent velocity of Q with respect to P will be referred to in this book as

$$\mathbf{V}_{Q/P} = \underset{\Delta t \to 0}{\mathrm{Lt}} \quad \frac{\Delta\mathbf{R}_{Q/P}}{\Delta t} \tag{3.7}$$

rather than $\mathbf{V}_{QP}$. The apparent velocity $V_{Q/P}$ does not involve ω, the angular velocity of the moving link.

3.3 The Apparent Velocity of a Moving Point

In analyzing mechanisms, sometimes it is convenient to observe or find apparent displacements rather than absolute displacements. In such cases, the absolute velocity is calculated from the apparent velocity. In Fig. 3.5a, the absolute displacement of a point P not attached to the link 2, is shown along with its apparent displacement $\Delta\mathbf{R}_{P/1}$. Here, this point is constrained to move along a curvilinear path on this link. Clearly, its path on the absolute coordinate system will be

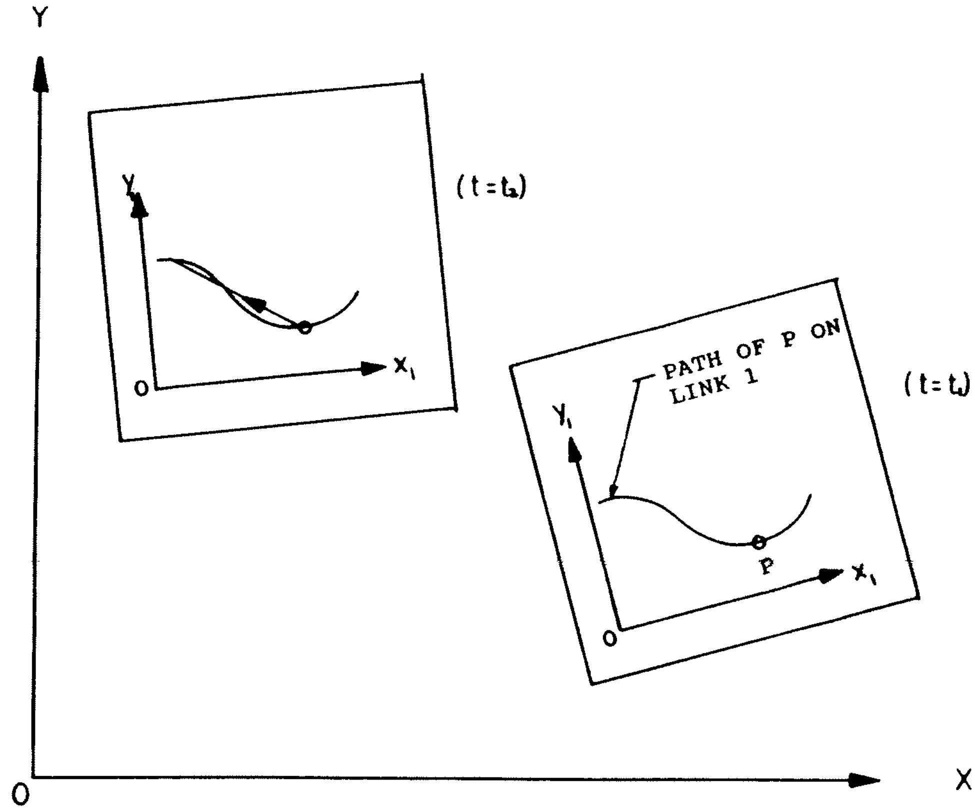

FIG 3.5(a) APPARENT VELOCITY OF A POINT P

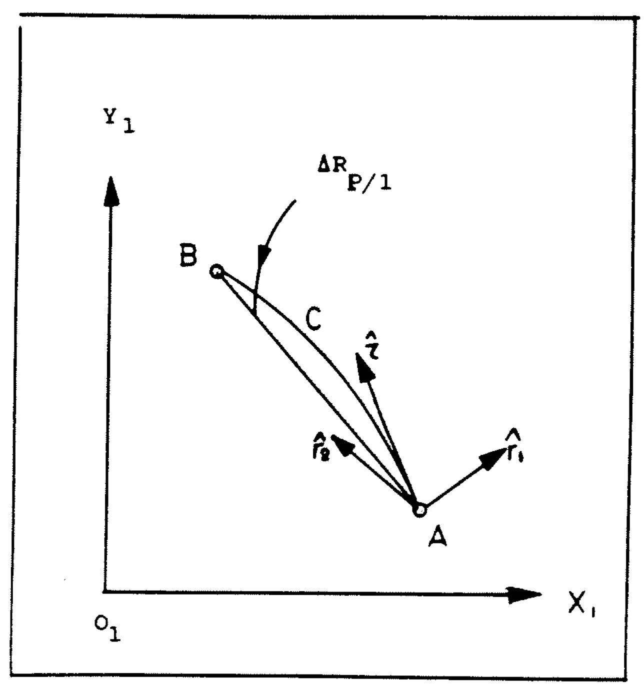

FIG.3.5(b)

different from this one. Since P is moving with respect to P_1 we can write

$$\mathbf{V}_P = \mathbf{V}_{O_1} + \mathbf{V}_{P_1O_1} + \mathbf{V}_{P/P_1} \tag{3.8}$$

$$= \mathbf{V}_{O_1} + \omega \times \mathbf{R}_{P_1O_1} + \mathbf{V}_{P/P_1} \tag{3.9}$$

For small apparent displacements one can write (refer to Fig. 3.5b)

$$\Delta \mathbf{R}_{P/1} = (\text{chord AB})\ \hat{r}_2 = (\text{arc ACB})\ \hat{r}_2$$

In this figure $\hat{r}_1$ is perpendicular to $\hat{r}_2$. It is obvious that as $\Delta t \to 0$ $\hat{r}_2$ will coincide with $\hat{\tau}$, a unit vector tangential to the curve at A. If we represent arc ACB as ΔS then we can write

$$\mathbf{V}_{P/P1} = \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta S}{\Delta t}\right) \hat{\tau}$$

$$= \left(\frac{ds}{dt}\right) \hat{\tau} \tag{3.10}$$

Using Eq. (3.10), we can rewrite Eq. (3.9) as

$$\mathbf{V}_P = \mathbf{V}_{O_1} + \omega \times \mathbf{R}_{P_1O_1} + \left(\frac{ds}{dt}\right) \hat{\tau} \tag{3.11}$$

Example 3.1:

To clearly understand each of the terms on the right hand side of Eq.(3.11), let us take an example of a man jogging on a ship shown in Fig. (3.6). The man, at this instant, is represented by a moving point P and he is at a point marked P_1 on the deck. He jogs with a speed of 10 km/hour; therefore his apparent velocity at this particular instant will be

$$\mathbf{V}_{P/P} = 10\ \hat{\tau} = -10\ i_1$$

The other information provided to us are: $\mathbf{V}_{O_1} = 0.20\ (\text{m/s}) \angle 120^0$; $\omega_1 = 5k$ (rad/sec); and $\mathbf{R}_{P_1O_1} = 100\ \text{m} \angle 70^0$. In this figure, the absolute coordinate system (X - 0 - Y) is on the shore. One can conveniently use the subroutines discussed in Chapter 2 to do this

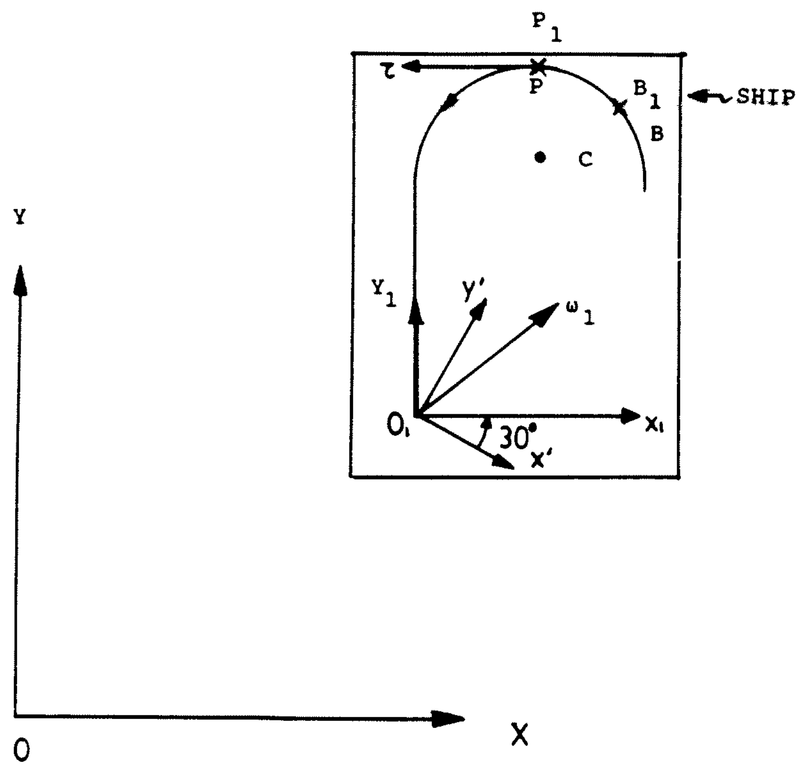

FIG. 3.6 VELOCITY ANALYSIS OF POINTS IN DIFFERENT COORDINATE SYSTEMS

problem. To use the subroutines, $\mathbf{R}_{P_1 O_1}$ is at first converted into the cartesian coordinates. So we can express the vectors as

$$\mathbf{R}_{P_1 O_1} = 34.158\ i_1 + 93.986\ j_1 + 0\ k$$

$$\omega_1 = 0\ i_1 + 0\ j_1 + 5\ k_1$$

and their cross product using Eq. (2.38) will be

$$\begin{vmatrix} i_1 & j_1 & k_1 \\ 0 & 0 & 5 \\ 34.158 & 93.986 & 0 \end{vmatrix} = -469.93\ i_1 + 170.79\ j_1 + 0\ k_1$$

We can add the second and third terms now and their sum will be $(-469.93 i_1 + 170.79 j_1 + 0 k_1) + (-10\ i_1) = -479.93 i_1 + 170.79 j_1 + 0 k_1$ We can convert this vector into the absolute coordinate system by writing in the form

$$\begin{vmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{vmatrix} \begin{vmatrix} -479.93 \\ 170.79 \end{vmatrix} = \begin{vmatrix} -501.007 \\ -92.162 \end{vmatrix}$$

$$= -501.007i - 92.162j$$

Now, we add the term 1 in the cartesian form and obtain

$$\mathbf{V}_P = (-0.100i + 0.173j) + (-501.007i - 92.162j)$$

$$= -501.107i - 91.989j$$

$$= 509.480 \angle(190.398)$$

We can find $\mathbf{V}_P$ by a second method also where we can convert all the vectors in the absolute coordinate system to start with, and obtain the results. If we do this we will have

$$\mathbf{R}_{P_1 O_1} = \begin{vmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{vmatrix} \begin{vmatrix} 34.158 \\ 93.986 \end{vmatrix} = -17.432i + 98.470j,$$

$$\omega = 5k_1 = 5k$$

$$\omega \times \mathbf{R}_{P_1 O_1} = \begin{vmatrix} i & j & k \\ 0 & 0 & 5 \\ -17.432 & 98.470 & 0 \end{vmatrix} = -492.35i - 87.16j \qquad (3.12)$$

and

$$10\ \tau = -10i_1 + oj_1 = \begin{vmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{vmatrix} \begin{vmatrix} -10 \\ 0 \end{vmatrix} = -8.69i - 5.00j$$

Thus the summation of all the vectors will be

$$\mathbf{V}_P = (-0.100i + 0.173j) + (-492.35i - 87.16j) + (-8.69i - 5.00j)$$
$$= -501.14\ i - 91.987\ j$$

which is the same result as before. In the graphical method to be discussed now, the conversion of vectors $\mathbf{R}_{P_1 O_1}$ and ω_1 from the $(x_1 - o_1 - y_1)$ into absolute absolute coordinate system is done very easily mentally, so the graphical method appears to be quite efficient. Another advantage of the graphical method is that it helps in visualizing the problem very clearly. The disadvantage of the graphical method is that, if in the problems to be solved the magnitudes of various terms to be added or subtracted are of very different order, then this method lacks in accuracy. In these cases it is very difficult to choose a convenient scale for plotting these vectors. An important point to note is that for linkage motions in parallel planes, the ω vector or the angular acceleration vector α to be discussed in the next chapter, are always perpendicular to the plane. So the effect of the cross product of ω or α with any other vector will be to rotate that vector by 90^0 in the direction of ω. The cross product can be done very easily mentally rather than as shown in Eq. (3.12). In this particular case we have

$$\omega \times \mathbf{R}_{P_1 O_1} = 5k \times (-17.432i + 98.470j)$$

This cross product, in the polar form, will be

$$= 5k \times 100 \angle 70^0$$

$$= 500 \angle (70^0 + 90^0)$$

$$= 500 \angle 160^0$$

We simply multiply the magnitudes of the two vectors which are 5 and 100 and add 90^0 to the angle made by the vector $\mathbf{R}_{P_1 O_1}$ with respect to the reference axis. On the other hand, if the ω was clockwise i.e. if it was represented by -5k then we would write it as

$$-5k \times 100 \angle 90 = 500 \angle (70 - 90)$$

$$= 500 \angle -20 = 500 \angle 340$$

If we remember this fact then we would reduce the computations either in the velocity or acceleration analysis drastically. To add or subtract two vectors analytically, one must calculate the cartesian components, but in the graphical method it is very easily done as explained in the Chapter 2.

An observer on the ship will only see the apparent velocity expressed in the third term, whereas an observer on the shore will see the resultant of all the three terms. The second term is due to the velocity difference and the cause of this is the angular velocity ω of the ship. On the other hand, the third term is due to the man jogging on the ship and this is due to the ability of man to run; there is no ω involved in this third term which is called the apparent velocity of the man. The first term arises due to the horse power of the engine of the ship, ocean currents, wind etc. The source of ω are also these causes. $\mathbf{V}_{O_1}$ is the absolute velocity of the point O_1. The term ' **Relative Velocity** ' is sometimes used to describe the difference in velocities of two points without making any distinction about the cause of this difference in velocities. Thus, both the apparent-velocity and the velocity-difference can be called as the Relative Velocity. These concepts can be understood more clearly by taking examples of two mechanisms shown in Figs. 3.7 and 3.8. In Fig. 3.7a, links 2, and 4 can rotate about points A and F respectively because of the pinned connection. As shown here two separate motors will be required to rotate these two links. If we assemble link 3 using the two pins shown in the diagram, we would get a mechanism as shown in Fig. 3.8b. Now we need only one motor which can drive the mechanism and may be attached to the link 2. The motion to the link 4 is transmitted through link 3 which acts as a coupling link between links 2 and 4. To derive this mechanism only one motion is required because this mechanism has only one degree of freedom. In the mechanism shown in Fig. 3.8, the link 3 is connected to link 2 through a pin but it has a sliding connection with the link 4 rather than a pinned connection in Fig.3.7. The velocity analysis of these types of mechanisms can be easily done by applying Eq. (3.11) to each of these links and using equations of constraints which describe the types of connections between the links. This method is called the link-by-link method.

3.4 The Link-By-Link Method of Velocity Analysis: The Analytical Method

To illustrate this method let us consider another example. Suppose the link 2 shown in the Fig. 3.9 rotates with an angular velocity $\omega = 20$ rad/s ccw. We would like to know the following:

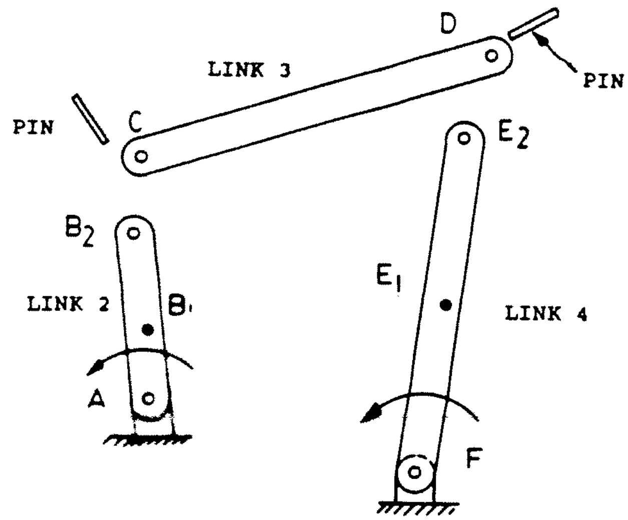

FIG. 3.7(a)

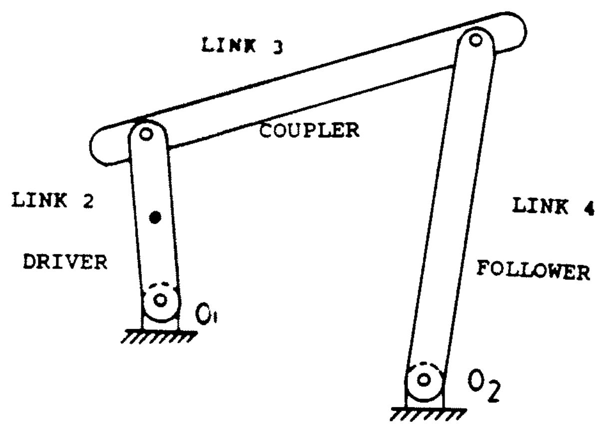

FIG. 3.7(b)

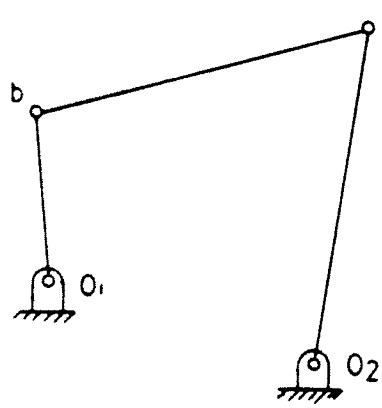

FIG. 3.7(c)

FIG. 3.7 ASSEMBLY OF VARIOUS LINKS OF A FOUR BAR MECHANISM

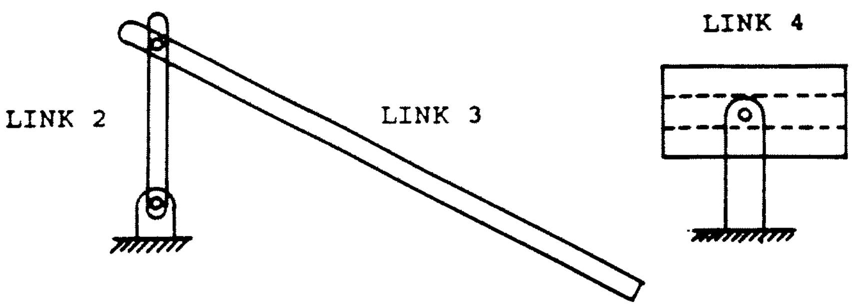

FIG. 3.8(a)

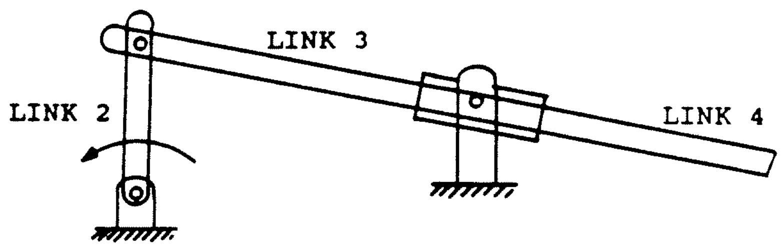

FIG. 3.8(b)

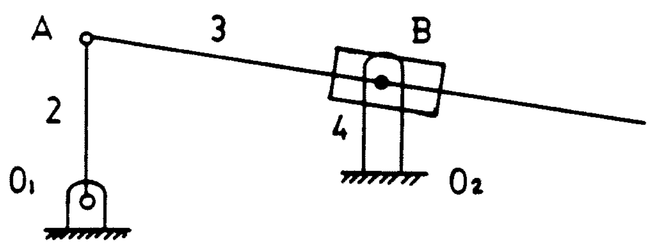

FIG. 3.8(c)

FIG. 3.8 LINE DIAGRAM AND ASSEMBLY OF LINKS OF A MECHANISM

a) $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$

b) $\underline{\omega}_3$ and $\underline{\omega}_4$.

3.4.1 Velocities of Various Points on Link 2

We will start with the link 2 whose angular velocity is given to us and apply Eq. (3.11). This equation, in the present case, will be

$$\mathbf{V}_A = \mathbf{V}_{O_1} + \mathbf{V}_{AO_1}$$

$$\mathbf{V}_A = \mathbf{V}_{O_1} + \omega \times \mathbf{R}_{AO_1} \quad = 0 + 2k \times 2.5 < 150$$

$$= 50 \angle (150 + 90) = 50 \angle 240$$

In Fig. 3.9, the direction of the vector $\mathbf{V}_{AO_1}$ is obtained by rotating $\mathbf{R}_{AO_1}$, in the direction of ω_2 by 90^0 and multiplying the magnitudes of ω_2 and $\mathbf{R}_{AO_2}$. If we were also interested in knowing the velocity-difference of another point J shown in Fig. 3.9.b on this link, we would have to rotate the vector $\mathbf{R}_{JO_1}$ also by 90^0 at the point J. This would give us the direction of $\mathbf{V}_{JO_1}$ and its magnitude will be $|\omega|$ times $|\mathbf{R}_{JO_1}|$. The tip of $\mathbf{V}_{JO_1}$ will lie on the line joining the tip of $\mathbf{V}_{AO_1}$ and $\mathbf{O}_1$. All the velocity difference vectors are parallel because they are all perpendicular to $\mathbf{R}_{AO_1}$.

3.4.2 Velocities of Various Points on Link 3

Since there is a pinned connection between links 2 and 3, the velocity of the point A on link 3 will be same as the velocity of the point A on link 2. Representing the number of the link as the subscript of the point, we can express this constraint equation as

$$\mathbf{V}_{A_3} = \mathbf{V}_{A_2} \qquad (3.13)$$

The links 2 and 3 can rotate relative to each other. Now we can

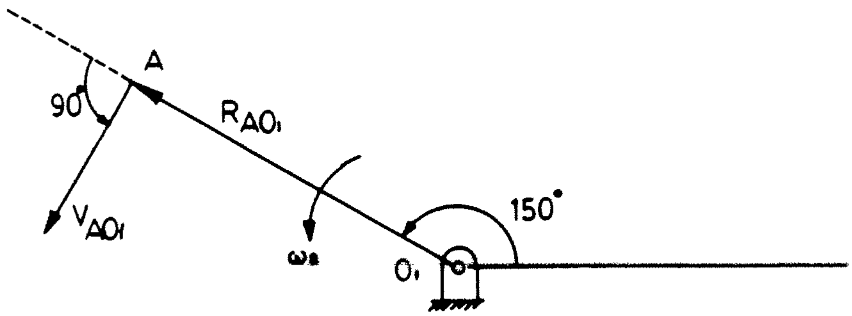

FIG. 3.9(a)

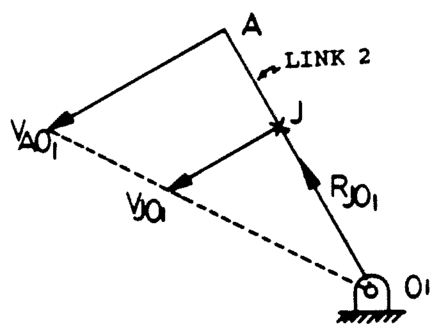

FIG. 3.9(b)

FIG. 3.9 VELOCITY DIFFERENCE BETWEEN POINTS A AND J write the equations for velocities for various points on the link 3. If we use Eq. (3.12) again, we can write

$$\mathbf{V}_{B_3} = \mathbf{V}_{A_3} + \mathbf{V}_{B_3A_3} = \mathbf{V}_{A_3} + \omega \times \mathbf{R}_{B_3A_3} \tag{3.14}$$

We have already determined $\mathbf{V}_A = \mathbf{V}_{A_2} = 50 \angle 240$ and using the first equality in Eq. (3.14) we obtain

$$\mathbf{V}_{B_3} = 50 \angle 240 + \mathbf{V}_{B_3A_3} \tag{3.15}$$

3.4.3 Velocities of Various Points on Link 4

Because of the pinned connection between links 3 and 4, we can write a constraint equation similar to Eq. (3.13) which will be

$$\mathbf{V}_{B_4} = \mathbf{V}_{B_3} \tag{3.16}$$

We also have an additional velocity difference equation for this link as

$$\begin{aligned} \mathbf{V}_{B_4} &= \mathbf{V}_{O_2} + \mathbf{V}_{B_4O_2} = \mathbf{V}_{O_2} + \omega_4 \times \mathbf{R}_{B_4O_2} \\ &= 0 + \mathbf{V}_{B_4O_2} = 0 + \omega_4 \times \mathbf{R}_{B_4O_2} \end{aligned} \tag{3.17}$$

Now let us see as to what do we know about the magnitudes and directions of various terms in Eqs. (3.14) or (3.15), and (3.17). We have seen in Fig. 3.9b that the velocity difference vector ($\mathbf{V}_{AO1}$) is always perpendicular to the position difference vector ($\mathbf{R}_{AO1}$). The fact is that the velocity analysis is carried out only after the displacement analysis where the position-difference vectors are obtained as solutions. Therefore the direction of the velocity-difference vector is known at the outset of the velocity analysis. For example in the present case $\mathbf{R}_{B_3A_3} = 10 \angle 34.75^0$, and $\mathbf{R}_{B_4O_2} = 7 \angle 97.809^0$.

Thus we can write

$$\mathbf{V}_{B_3A_3} = V_{B_3A_3} \angle (34.75 + 90) \quad \text{if } \omega_3 \text{ is ccw} \qquad (3.18)$$

or

$$\mathbf{V}_{B_3A_3} = V_{B_3A_3} \angle (34.75 - 90) \quad \text{if } \omega_3 \text{ is cw} \qquad (3.19)$$

Similarly, we can write

$$\mathbf{V}_{B_4O_2} = V_{B_4O_2} \angle (97.809 + 90) \quad \text{if } \omega_4 \text{ is ccw}$$

$$= V_{B_4O_2} \angle (97.809 - 90) \quad \text{if } \omega_4 \text{ is cw}$$

In this book we will assume that the unknown angular velocities are always counter-clockwise thus add 90^0 in Eqs. (3.18) or (3.19) and solve for the corresponding magnitudes $V_{B_3A_3}$ and $V_{B_4O_2}$ by a method to be explained now. If any of the magnitudes turn out to be negative numbers then the corresponding ω has to be clockwise and so 90^0 in Eqs. (3.18) or (3.19) will have to be subtracted instead of added. With this in mind, let us go back and substitute the numerical values in Eqs. (3.15) and (3.17). Using Eq. (3.16) we get

$$\overset{**}{\mathbf{V}_{B_3}} = 50 \angle 240 + \mathbf{V}_{B_3A_3} \angle 245.75 \qquad (3.20)$$

$$= 50 \angle 240 + \overset{*\checkmark}{\mathbf{V}_{B_3A_3}} \qquad (3.21)$$

Also,

$$\overset{**}{\mathbf{V}_{B_3}} = \overset{**}{\mathbf{V}_{B_4}} = 0 + \mathbf{V}_{B_4O_2} \angle 187.809 \qquad (3.22)$$

or

$$\overset{**}{\mathbf{V}_{B_3}} = \overset{*\checkmark}{\mathbf{V}_{B_4O_2}} \qquad (3.23)$$

Since the left hand side of Eqs. (3.21) and (3.23) are the same, we can equate the right hand sides of two and write

$$\overset{*\checkmark}{\mathbf{V}_{B_4O_2}} = 50 \angle 240 + \overset{*\checkmark}{\mathbf{V}_{B_3A_3}} \qquad (3.24a)$$

$$\mathbf{V}_{B_4O_2} \angle 187.809 = 50 \angle 240 + \mathbf{V}_{B_4A_3} \angle 124.75 \qquad (3.24b)$$

The reason for equating the left hand sides of Eqs. (3.21) and (3.23) was that each of these two equations had three unknowns, so we could not have solved for the unknowns because only three unknowns per vector equation can be solved; but the two unknowns ($\mathbf{V}_{B_3}^{**}$) are common in these two equations. Although there are three unknowns in each equation yet they do not add up to six unknowns because two unknowns, $\mathbf{V}_{B_3}^{**}$, are common. There are a total of four unknowns, and we have two vector equations which are sufficient in number for the unknowns to be solved. In Eq. (3.24a), we have eliminated two unknowns and it is in the case 2a modified form. We can use program 11 to solve Eq. (3.24b). If we do so, we will obtain

$$\mathbf{V}_{B_4A_3} = 44.291 \angle 124.75^0$$

$$\mathbf{V}_{B_4O_2} = 50.730 \angle 187.809^0$$

Since the magnitudes were both positive, the assumed counter-clockwise rotational directions were correct. The magnitudes of the vectors ω_3 and ω_4 can be obtained by

$$\omega_3 = \frac{V_{B_4A_3}}{R_{BA}} = \frac{44.291}{10} = 4.4291 \text{ rad/s} \quad \text{ccw}$$

and

$$\omega_4 = \frac{V_{B_4O_2}}{R_{BO_2}} = \frac{50.730}{7} = 7.247 \text{ rad/s} \quad \text{ccw}$$

The velocity of the point C on the coupler can be obtained by writing

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA} = \mathbf{V}_A + \omega_3 \times \mathbf{R}_{CA}$$

$$= 50 \angle 240^0 + 4.4291k \times 7.5 \angle (79.75^0)$$

$$= 50 \angle 240^0 + 4.4291 \times 7.5 \angle (79.75^0 + 90^0)$$

$$= 68.434 \angle 212.75^0$$

The first two terms above can be added using the programmes for vector sum or case 1. We can also obtain $\mathbf{V}_C$ by writing

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB} = 50.73 \angle 187.809^0 + \omega_3 \times \mathbf{R}_{CB} \qquad (3.25)$$

The vector $\mathbf{R}_{CB}$ can be obtained by

$$\mathbf{R}_{CB} = \mathbf{R}_{AB} + \mathbf{R}_{CA}$$

$$= 10 \angle 214.75^0 + 7.5 \angle 79.75^0 = 7.076 \angle 165^0$$

Substituting the value for $\mathbf{R}_{CB}$ in Eq. (3.25) we get

$$\mathbf{V}_C = 50.73 \angle 187.809^0 + 4.429k \times 7.076 \angle 165^0$$

$$= 50.73 \angle 187.809^0 + 31.339 \angle 255^0 = 69.187 \angle 212.555^0$$

The magnitudes and directions calculated for $\mathbf{V}_C$ using the two methods are quite close. To be precise, we should write $\mathbf{V}_C$ as $\mathbf{V}_{C_3}$ because the velocity difference was based on ω_3. Moving on to the link 5 which has pinned connection with link 3, we can write

$$\mathbf{V}_{C_5} = \mathbf{V}_{C_3} \qquad (3.26)$$

On the other hand, the motion from the link 5 to link 6 is transmitted through a rolling contact at D. The analysis of motions transmitted through such types of contacts is discussed in the next section.

3.5 Velocity Analysis of Mechanisms with Sliding or Rolling Contact

There are two mechanisms shown in Figs. 3.10 and 3.11. In the first of these two, there is a sliding contact whereas, in the second one, there is a rolling contact. When two links have a rolling contact, the absolute velocities of the contact points are equal i.e. their magnitudes as well as directions are the same. For example, in Fig. 3.11, there is a rolling contact and therefore in this case we can write the constraint equation as

$$\mathbf{V}_{P_3} = \mathbf{V}_{P_4} \qquad (3.27)$$

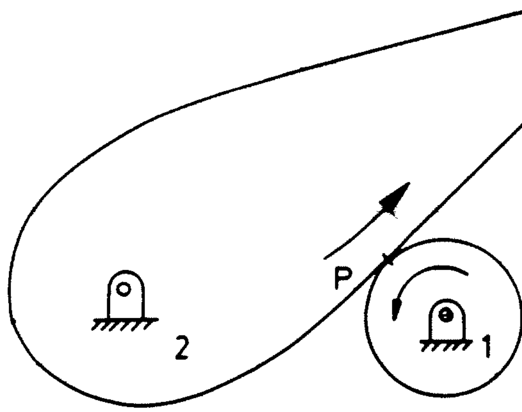

FIG. 3.10 VELOCITY ANALYSIS OF POINTS ON CAMS

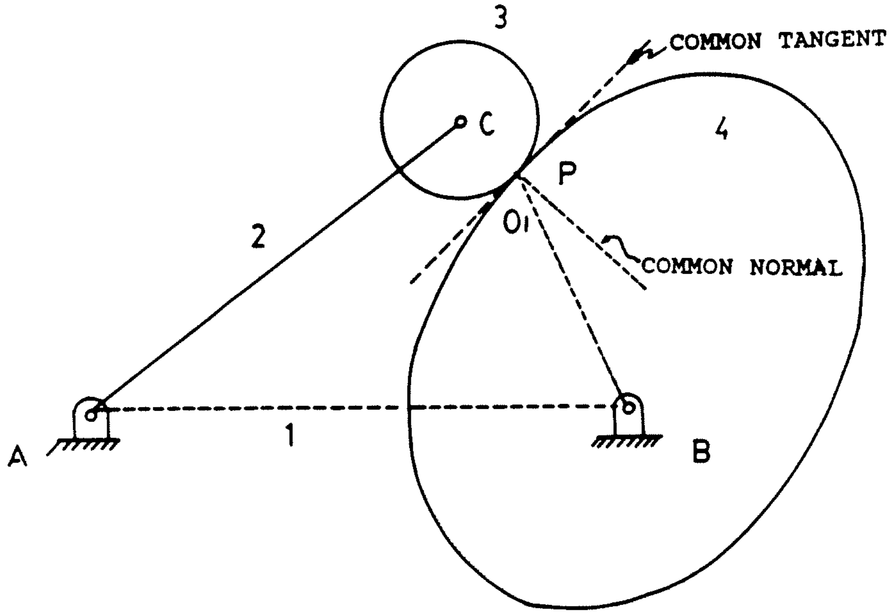

FIG. 3.11

We can obtain $\mathbf{V}_{P_3}$ and $\mathbf{V}_{P_4}$ separately if the angular velocities of both the links are known. For example we can express $\mathbf{V}_{P_3}$ as

$$
\begin{aligned}
\mathbf{V}_{P_3} &= \mathbf{V}_{C_3} + \mathbf{V}_{PC_3} \\
&= \mathbf{V}_{C_2} + \mathbf{V}_{P_3C_3} \qquad (3.28) \\
&= \omega_2 \times \mathbf{R}_{CA} + \omega_3 \times \mathbf{R}_{CP}
\end{aligned}
$$

Similarly, we will have

$$\mathbf{V}_{P_4} = \omega_4 \times \mathbf{R}_{P_4B} \qquad (3.29)$$

The rolling contact is similar to a pin joint where the velocities of the contact points are equal. But, we will see in the next chapter that **for the rolling contact the accelerations of the contact points are not equal whereas for the pin joints they are equal.**

Suppose there is a sliding contact at point P in Fig. 3.12.

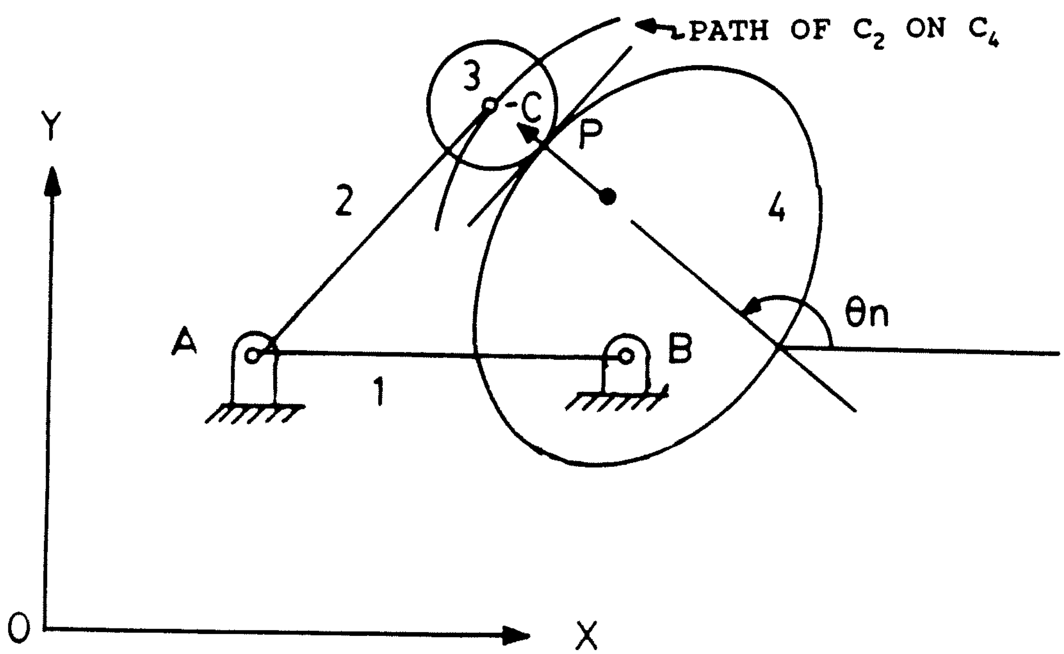

FIG. 3.12

For the links 3 and 4 to be in contact all the time, the constraint equation similar to Eq. (3.27) will be that the velocities only in the common normal direction rather than in all the directions be equal. This can be mathematically written as $V_{P_3} \angle \theta_n = V_{P_4} \angle \theta_n$ where θ_n is the direction of the common normal. Suppose ω_4 is known and we have to find ω_2, ω_3, $\mathbf{V}_{P_2}$ and $\mathbf{V}_{P_3}$.

To solve the problem we will use the directions of various position difference vectors, for example θ_{CA}, which is the direction of the vector $\mathbf{R}_{CA}$ etc. We have already defined the common normal direction as θ_n and therefore the common tangent direction can be represented as $(\theta_n - 90)$. Now we are in a position to write the equation

$$\overset{**}{\mathbf{V}_{P_3}} = \overset{\checkmark\checkmark}{\mathbf{V}_A} + \overset{*\checkmark}{\mathbf{V}_{CA}} + \overset{*\checkmark}{\mathbf{V}_{P_3C}}$$

$$= \mathbf{0} + V_{CA} \angle (\theta_{CA} + 90^0) + V_{P_3C} \angle (\theta_{P_3C} + 90^0) \qquad (3.30)$$

we can also write

$$\overset{**}{V_{P_3}} = \overset{\checkmark\checkmark}{\mathbf{V}_B} + \overset{o\checkmark}{\mathbf{V}_{P_4B}} + \overset{o\checkmark}{V_{P_3/P_4}}$$

$$= \mathbf{0} + \omega_4 R_{P_4B} \angle (\theta_{P_4B} + 90^0) + V_{P_3/P_4} \angle (\theta_n - 90^0)$$

(3.31)

Because of the constraint equation, the only direction in which the apparent motion, $\mathbf{V}_{P_3/P_4}$, can take place will be along the common tangent which is also the direction of the vector $\mathbf{V}_{P_3C}$. Thus equating Eqs. (3.30) and (3.31) and using Eq. (3.32) we get

$$\mathbf{V}_{CA} \angle (\theta_{CA} + 90) + \mathbf{V}_{P_3C} \angle (\theta_{P_3C} + 90^0)$$

$$= V_{P_4B} \angle (\theta_{P_4B} + 90^0) + (V_{P_3/P_4} \angle (\theta_{P_3C} + 90^0$$

This can be rewritten as

$$V_{CA} \angle (\theta_{CA} + 90) + y \angle (\theta_{P_3C} + 90^0) = V_{P_4B} \angle (\theta_{P_4B} + 90^0) \tag{3.32}$$

where

$$y = V_{P_3C} - V_{P_3/P_4} \tag{3.33}$$

The unknowns, V_{CA} and y, in Eq. (3.32) can be obtained from the solutions for case 2a because V_{P_4B} and $\angle (\theta_{P_4B} + 90^0)$ are known. After this, we can obtain the other unknown quantities using the following equations:

$$\omega_2 = \frac{V_{CA}}{R_{CA}} ; \tag{3.34a}$$

$$V_{P_3C} = y + V_{P_3/P_4} ; \tag{3.34b}$$

$$\omega_3 = \frac{V_{P_3C}}{R_{P_3C}} \tag{3.34c}$$

It should be emphasized here that because of the slip condition, this mechanism has 2 degrees of freedom at the contact point of the links 3 and 4; thus ω_4 and V_{P_3/P_4} have to be given before hand. If this is so then V_{P_3C} can be calculated from Eq. (3.34) because y is obtained from Eq. (3.32). However, if we are interested in finding ω_2 and $\mathbf{V}_C$ and not ω_3, we can use an alternate method where we avoid going through the sliding contact point. In this case, we must know the path of C on link 4. This path is easily obtained from the kinematic inversion where the link 4 is fixed and the link 1, which is the frame in the present mechanism, is made mobile by removing the fixed condition at its ends. This kinematic inversion is shown in Fig. 3.13a. To find

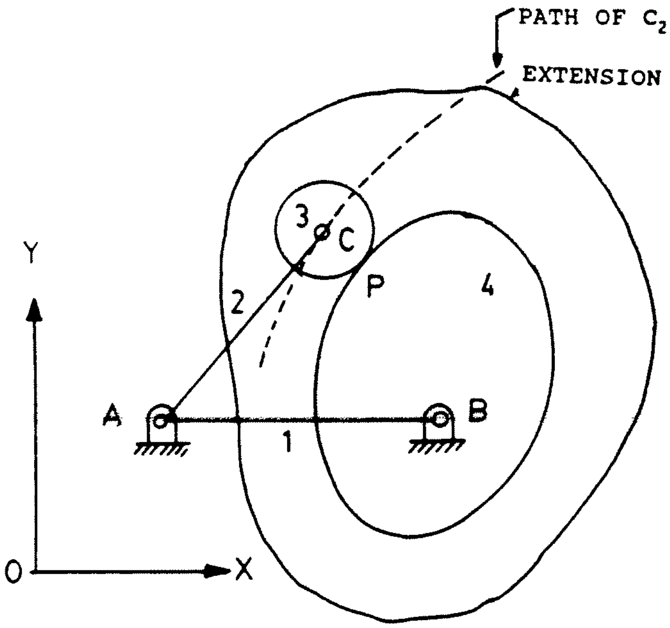

FIG. 3.13(a)

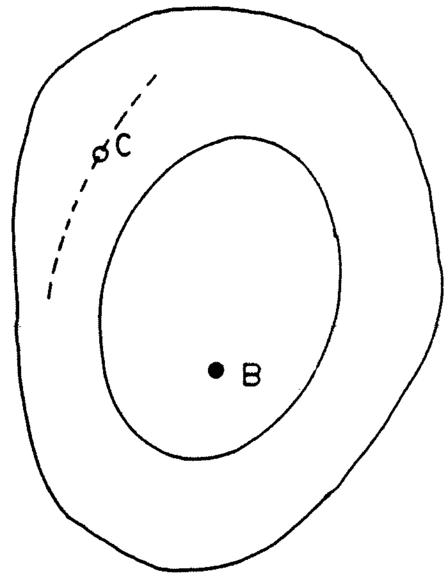

FIG. 3.13(b)

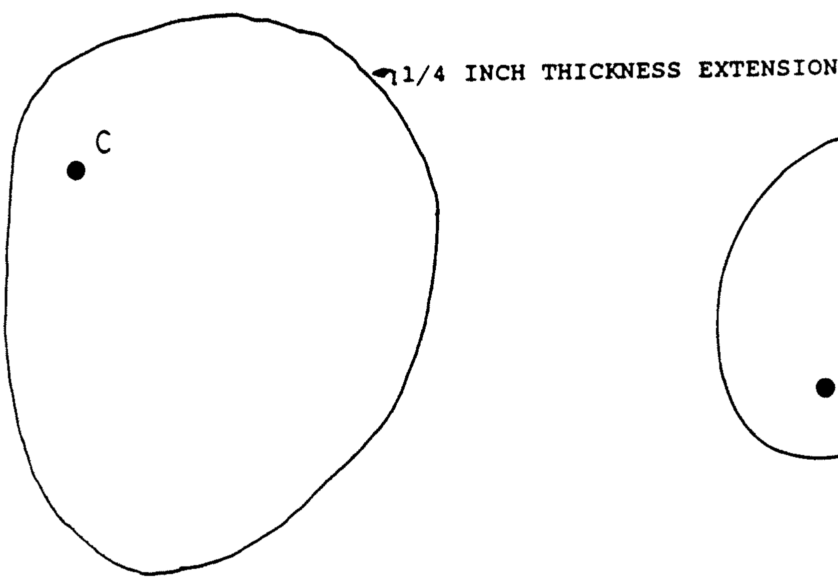

FIG. 3.13(c)

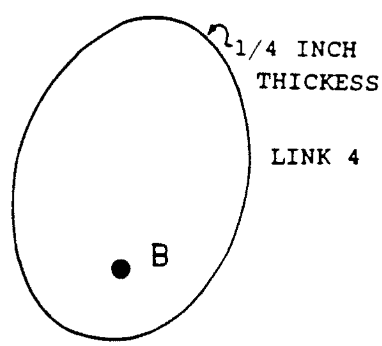

FIG. 3.13(d)

FIG. 3.13 CONCEPT OF EXTENSION OF A LINK the path of C_2 on link 4, we extend this link (link 4) as shown in this figure. How this extension is achieved is shown in Figs. 3.13b to 3.13d. Imagine that links 3, 4, and the extension, are cut-out of a large plate 1/4 inches thick. We can lay the link 4 on the extension and weld them together as shown in Fig. 3.13b. In this way the extension becomes an integral part of the link 3. Now we assemble the mechanism as shown in Fig. 3.13a. If we allow the link 3 to move around link 4, the path of C_2 on C_4 (a point directly below C_2 but on the extension) will be as shown in Fig. 3.13a. If we refer to Fig. 3.14, here link 1 is fixed and link 4

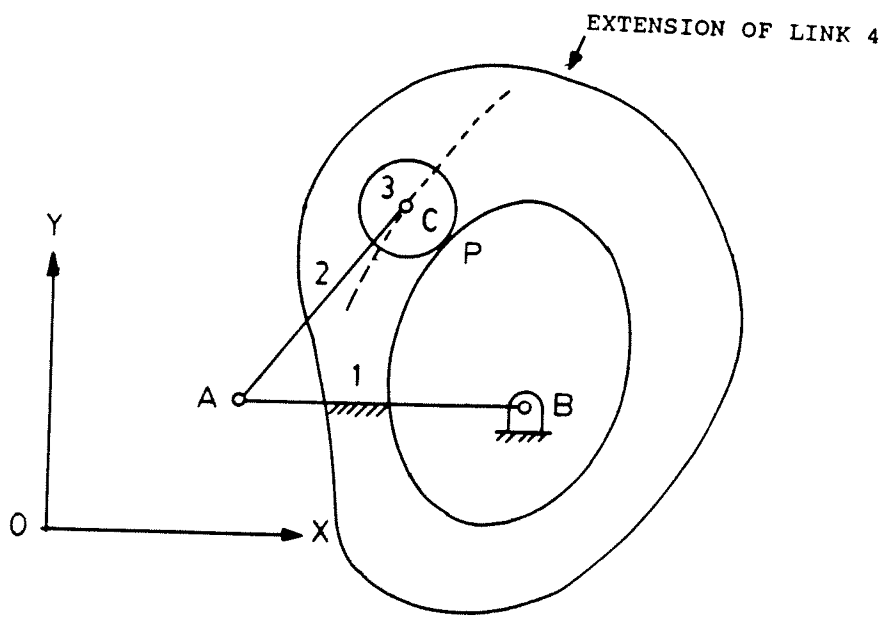

FIG. 3.14

along with its extension can rotate about B. The angular velocities of the link 4 and extension would be the same because they are welded.

To obtain ω_2 we can write

$$\overset{**}{\mathbf{V}_{C_2}} = \mathbf{V}_A + \overset{*\checkmark}{\mathbf{V}_{C_2A}} = \mathbf{V}_B + \overset{\checkmark\checkmark}{V_{C_4B}} + \overset{*\checkmark}{V_{C_2/C_4}}$$

Using the second equality we can write

$$0 + V_{C_4B} \angle (\theta_{C_4B} + 90) + V_{C_2/C_4} \angle \theta_{C_2/C_4} = 0 + V_{C_2A} \angle(\theta_{C_2A} + 90^0)$$

(3.35)

In the Fig. 3.14, θ_{C_2/C_4} is clearly shown. It is along the tangential direction to the path of C_2 on C_4 at this instant of time. If we use the second equality in Eq. (3.35) we can obtain V_{C_2A} and V_{C_2/C_4} using program 11 (case 2a modified). Next, we can obtain ω_2 as given in Eq. (3.34). If the slip velocity or $\mathbf{V}_{P_3}$ is not given, we can not obtain ω_3.

This mechanism has two degrees of freedom which can be ascertained by using Eq. (1.2). There are 3 single-degree-of-freedom joints, one two-degree-of-freedom joint, and there are four links. Thus we require two specifications to completely define the motion. For example, these can be ω_2, and ω_4. These concepts are illustrated in the example given below.

Example 3.2

In Fig. 3.12, find the angular velocities of the links 2 and 3 (a), assuming a rolling contact at P, and ω_4 = 5 rad/s ccw, and (b) if there is a slip at P and $\mathbf{V}_{P_3/P_4}$ = 6 in/s $\angle$ 45^0. Assume ω_4 same as before. Given: $\mathbf{R}_{PB}$ = 5 $\angle$ 100^0, $\mathbf{R}_{CP}$ = 1 $\angle$ 135^0, $\mathbf{R}_{CA}$ =

$5.646 \angle 85.694$; and $\mathbf{R}_{BA} = 2 \angle 0^0$.

Solution

(a) In the case of rolling contact, $\mathbf{V}_{P_3} = \mathbf{V}_{P_4}$. We can also write

$$\overset{**}{\mathbf{V}_{P_3}} = \overset{\checkmark\checkmark}{\mathbf{V}_A} + \overset{*\checkmark}{\mathbf{V}_{CA}} + \overset{*\checkmark}{\mathbf{V}_{P_3C}}$$

$$= \mathbf{0} + V_{CA} \angle (\theta_{CA} + 90^0) + V_{P_3C} \angle (\theta_{P_3C} + 90^0) \qquad \text{(a)}$$

and

$$\overset{**}{\mathbf{V}_{P_3}} = \overset{**}{\mathbf{V}_{P_4}} = \mathbf{V}_B + \mathbf{V}_{P_4B}$$

$$= \mathbf{0} + \omega_4 \text{ k} \times \mathbf{R}_{P_4B} \angle \theta_{P_4B}$$

$$= 5\text{k} \times 5 \angle 100^0$$

$$= 25 \angle 190^0 \qquad \text{(b)}$$

Equating Eqs. (a) and (b), and substituting the values for θ_{CA} and θ_{P_3C} we obtain

$$25 \angle 190 = V_{CA} \angle 175.694^0 + V_{P_3C} \angle 45^0.$$

This equation can be solved using the program 2 which is for the case 2a. The solution of this problem is

$$\mathbf{V}_{CA} = 18.943 \angle 175.694^0, \text{ and}$$

$$\mathbf{V}_{P_3C} = -8.087 \angle 45^0 = 8.087 \angle 225^0$$

The angular velocities can now be obtained as

$$\omega_3 = \frac{V_{P_3C}}{R_{PC}} = \frac{-8.087}{1} = -8.087 \text{ cw} \qquad \text{(c)}$$

and

$$\omega_2 = \frac{V_{CA}}{R_{CA}} = \frac{18.957}{5.646} = 3.358 \text{ ccw} \qquad \text{(d)}$$

The rotational directions of these two links can also be understood from Fig. 3.15 where the links at this particular

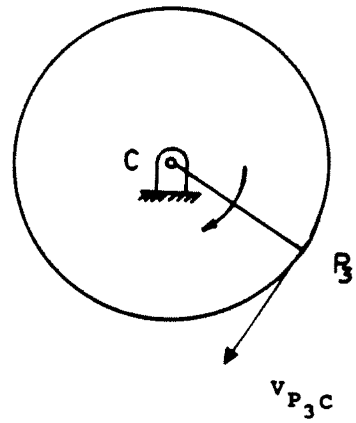

FIG. 3.15

instant are separately shown. The direction of rotation of the link 2 will obviously be counter-clockwise. To understand the direction of rotation of the link 3, we should use a hinged connection on this link at the point which is the second subscript in the velocity difference expression; this is C in $\mathbf{V}_{P_3C}$. Clearly, this link also rotates in the clockwise direction.

Now, it is worth understanding the absolute velocities of various points discussed above. The magnitude of the velocity vector at A will be zero because this point is fixed to the ground. The absolute velocity of the point C will be the vectorial sum, not the algebraic sum, of the velocity of the point A and the velocity difference, $\mathbf{V}_{CA}$. This velocity difference is obviously defined in the vectorial sense because a velocity is a vector quantity which requires specifications of magnitude and direction. Since $\mathbf{V}_A$ is zero, the velocity of the point C, $\mathbf{V}_C$, will be equal to $\mathbf{V}_{CA}$. Thus $\mathbf{V}_C$ can also be given by $18.957 \angle 175.694^0$. If we move on to the link 3, the velocity of the point P_3 will be sums of $\mathbf{V}_A$, $\mathbf{V}_{CA}$ and $\mathbf{V}_{CP_3}$. Since $\mathbf{V}_A$ is zero, we will have