$$\mathbf{V}_{P_3} = \mathbf{V}_{CA} + \mathbf{V}_{P_3C}$$
$$= 18.957 \angle 175.694^0 + 8.087 \angle 225^0$$
$$= 24.992 \angle 189.967 \qquad \text{(e)}$$

An important fact that should be noted is that when two links rotate due to a rolling contact, the directions of the rotations are always opposite if the point of contact lies between the axes of rotations. On the other hand, if the point of contact does not lie between the line joining the axes of rotations then the directions of rotations will be same.

Case (b)

We will use Eq. (3.32) in this case. If we write this equation with the numerical values for the orientations of various vectors and the known magnitude, we will have

$$V_{CA} \angle 175.694^0 + y \angle 45^0 = \mathbf{V}_{P_4B} = 25 \angle 190^0$$

We can find the magnitudes of the unknown vectors using case 2a. The values obtained are

$$\mathbf{V}_{CA} = 18.943 \quad \angle 175.694^0, \text{ and}$$
$$y = -8.096 \quad \angle 45^0$$

Substituting the values of y and V_{P_3/P_4} in Eq. (3.33) we get

$$-8.096 = V_{P_3C} - 6 \qquad \text{(f)}$$

From this, we obtain $V_{P_3C} = -2.096$. Thus the vector $\mathbf{V}_{P_3C}$ will be given by

$$\mathbf{V}_{P_3C} = -2.096 \angle 45^0 = 2.096 \angle 225^0$$

The direction of rotation of the link 3 can be judged from Fig. 3.15. It is clockwise again. An interesting fact to note is that $\mathbf{V}_{CA}$ even in this case, is same as in part (a) and so will be ω_2 which depends only on $\mathbf{V}_{CA}$ and not on any other velocity. Whether there is a rolling or sliding contact at P, the angular velocity of the link 2 will be the same. However due to the slip condition at P, the velocity difference $\mathbf{V}_{P_3C}$ is now 2.096 instead of 8.087. The direction has to be the same as earlier. Since the magnitude has decreased now therefore the angular velocity will be reduced and it can be calculated as

$$\omega_3 = \frac{V_{P_3C}}{R_{P_3C}} = \frac{2.096}{1} = 2.096 \text{ cw}$$

If we are only interested in the angular velocity of the link 2 then the information that $\mathbf{V}_{P_3/P_4} = 6 \angle 45^0$ is not required, but we should know the path of C_2 and C_4 as shown in Fig. 3.14. We can use Eq. (3.35) in this case.

we can write

$$\mathbf{R}_{CB} = \mathbf{R}_{CP} + \mathbf{R}_{PB}$$
$$= 1 \angle 135^0 + 5 \angle 100^0 = 5.847 \angle 105.702^0$$

Now we will use Eq. (3.35) with

$$\theta_{C_2/C_4} = \theta_{CP} - 90^0$$
$$= 135^0 - 90^0 = 45^0.$$

After substituting appropriate numerical values, we will have

$$V_{C_2A} \angle 175.694^0 = 5 \times 5.847 \angle (105.702^0 + 90^0) + V_{C_2/C_4} \angle 45^0$$

This can be rewritten as

$$V_{C_2A} \angle 175.694^0 = 5 \times 5.847 \angle (105.702^0 + 90^0) + V_{C_2/C_4} \angle 45^0$$

Solving this, we obtain

$$\mathbf{V}_{C_2A} = 18.9196 \angle 175.694^0, \text{ and}$$

$$\mathbf{V}_{C_2/C_4} = 13.124 \angle 45^0$$

Now one can calculate ω_2, which will be

$$\omega_2 = \frac{18.9196}{5.646} = 3.351 \text{ ccw}$$

3.6 The Link-By-Link Method of Velocity Analysis: The Graphical Method

The graphical method is generally found to be the most reliable method though it may lack accuracy at times. It does give a very clear picture about the velocities involved. As we have discussed earlier, the angular relationship between a position difference vector, say $\mathbf{R}_{BA}$, and the velocity difference vector $\mathbf{V}_{BA}$, is that $\mathbf{V}_{BA}$ is always rotated 90^0 in the direction of rotation of the link on which these two points (A and B) are located at. This was the first relationship and the second relationship is that the magnitude of $\mathbf{V}_{BA}$ is equal to the product of the magnitude of the angular velocity of the link and the magnitude of the position difference vector. These two relationships can be expressed as

$$|\mathbf{V}_{BA}| = |\omega| \; |\mathbf{R}_{BA}| \tag{3.36}$$

and

$$\theta_{\mathbf{V}_{BA}} = \theta_{\mathbf{R}_{BA}} + 90^0 \quad \text{if } \omega \text{ is the ccw.} \tag{3.37}$$

$$= \theta_{\mathbf{R}_{BA}} - 90^0 \quad \text{if } \omega \text{ is the cw.} \tag{3.38}$$

To understand this method clearly, let us take an example of a link shown in Fig. 3.16 where the absolute velocity of the point **A** is completely known and only the direction of the absolute velocity of the point B is known. We are required to find, $\mathbf{V}_C$ and ω of this link.

As a first step we write the velocity difference equation which in this case will be

$$\overset{*\checkmark}{\mathbf{V}_B} = \overset{\checkmark\checkmark}{\mathbf{V}_A} + \overset{*\checkmark}{\mathbf{V}_{BA}} \tag{3.39}$$

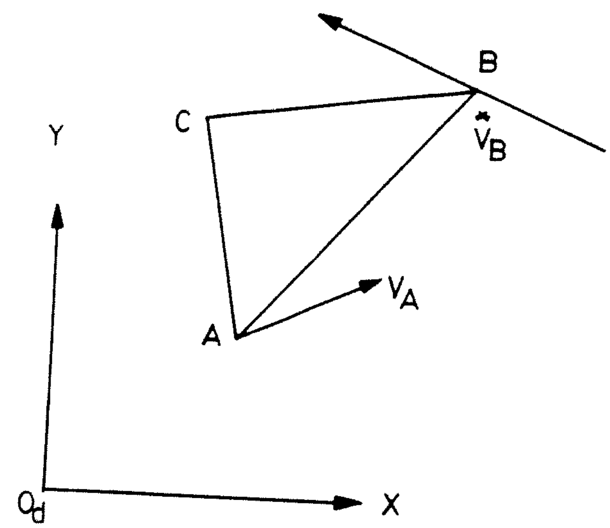

FIG. 3.16(a)

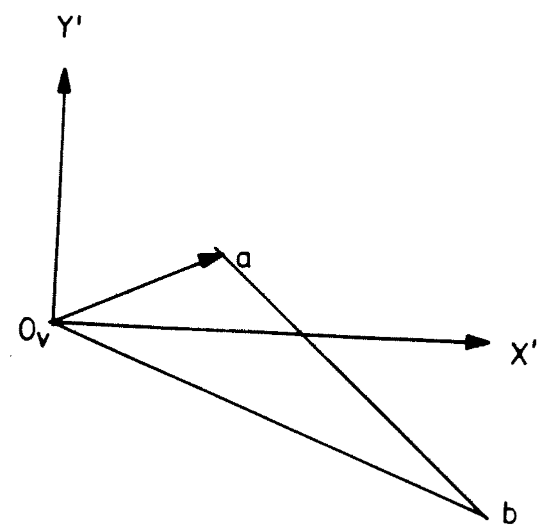

FIG. 3.16(b)

This equation is in the form of case 2a modified. The graphical method of solving case 2a and case 2a modified are identical in the graphical analysis and it was already discussed in Chapter 2.

In the second step we set up another set of axes and its origin is labelled as O_v where V represents the velocity and this set of axes are referred to as the velocity diagram; Fig. 3.16a is called the displacement diagram. The displacement diagram contains all the position vectors and position difference vectors.

In the velocity diagram we solve Eq. (3.39) graphically. Since the velocity difference vector $\mathbf{V}_{BA}$ is always perpendicular ($\pm\ 90^0$) to $\mathbf{R}_{BA}$, we can easily draw its ($\mathbf{V}_{BA}$) direction by referring to $\mathbf{R}_{BA}$ in the displacement diagram. This is the reason why, preferably, these two diagrams are drawn on the same page.

In the third step, we draw the vector $\mathbf{V}_A$ in the velocity diagram using an appropriate scale. The vectors in the displacement and the velocity diagrams are almost always drawn to different scales. Therefore, it is a good idea to show the scales on each of the diagrams. The tip of $\mathbf{V}_A$ is labelled with a small letter 'a'. As discussed in Chapter 2, we draw a line parallel to $\overset{*\surd}{\mathbf{V}}_B$ from the tip of $\mathbf{V}_A$ and a line parallel to $\overset{*\surd}{\mathbf{V}}_{BA}$ from the tail of $\mathbf{V}_A$. The point of intersection is labelled as 'b'. Now we are in a position to draw the arrows in the triangle O_V a b. In Fig. 3.16b, various vectors are labelled after their identifications. The magnitudes and directions of $\mathbf{V}_{BA}$ and $\mathbf{V}_B$ can be scaled from this diagram. Note that the absolute velocity vector $\mathbf{V}_A$ or $\mathbf{V}_B$ has its tail at the origin and the tip at the respective lower case letter. But, in the case of the velocity difference $\mathbf{V}_{BA}$, this vector extends between points 'a' and 'b' in the velocity diagram. The points 'a' and 'b', in the velocity diagram, are called images of the points A and B in the displacement diagram. Every point in the displacement diagram has an image in the velocity diagram. If a point in the displacement diagram is stationary i.e. its velocity has zero magnitude then its image will be at O_V, the origin in the velocity diagram. The reason for this is that the magnitude of the velocity of a point is represented by the distance between O_V and the image of the point in the velocity diagram. Therefore, if the point has no velocity then it must be coincident with O_v.

Now we would like to know the magnitude and direction of ω, the angular velocity of the link. To do this, first of all we will use Eq. (3.36). Thus $|\omega|$ can be calculated using

$$|\omega| = \frac{|\mathbf{V}_{BA}|}{|\mathbf{R}_{BA}|} = \frac{(ab)}{|\mathbf{R}_{BA}|}$$

The distances (ab) and $|\mathbf{R}_{BA}|$ are to be scaled from the velocity and displacement diagrams respectively. The direction of rotation can be judged from either Fig. 3.16c or Fig. 3.16d. In both of

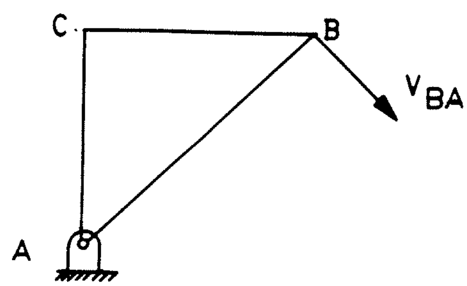

FIG. 3.16(c)

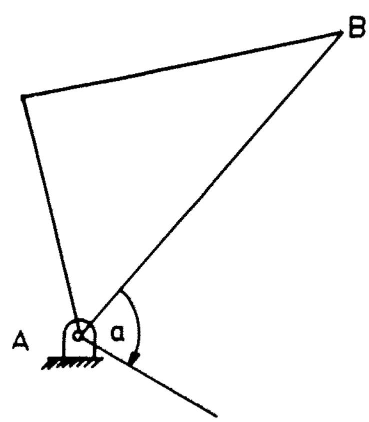

FIG. 3.16(d)

these figures, a fictitious hinge is used at A. In Fig. 3.16d, the velocity of the point B as seen by an observer at A will be as shown by $\mathbf{V}_{BA}$ which has been transferred from the velocity diagram. The observer at A will not feel his own velocity, $\mathbf{V}_A$. To him the velocity of B will be $\mathbf{V}_{BA}$. So he will experience the link rotating in the clockwise direction. Similarly if we transfer the vector $\vec{ab}$ to the displacement diagram such that 'a' coincides with A then the rotation of B is quite clearly clockwise. Thus we completely know ω.

To find out $\mathbf{V}_C$, We can use several methods. In the first method we can write

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA} = \overset{\surd\surd}{\mathbf{V}_A} + \overset{\surd\surd}{\omega} \times \overset{\surd\surd}{\mathbf{R}_{CA}}$$

We already know $\mathbf{V}_A$, ω, and $\mathbf{R}_{CA}$, so $\mathbf{V}_C$ can be calculated. In the second method, we write

$$\overset{**}{\mathbf{V}_C} = \overset{\surd\surd}{\mathbf{V}_A} + \overset{*\surd}{\mathbf{V}_{CA}} = \overset{\surd\surd}{\mathbf{V}_B} + \overset{*\surd}{\mathbf{V}_{CB}} \tag{3.41}$$

If we use the first equality, we can add the vector $\mathbf{V}_{CA}$ to $\mathbf{V}_A$ by drawing a line parallel to the direction of $\mathbf{V}_{CA}$ at 'a' as shown in Fig. 3.17. Next we will use the second equality and draw a line parallel to the direction of $\mathbf{V}_{CB}$ from the point b. The point of intersection of these two newly drawn lines will be 'c'. If we join the origin to 'c', we will get $\mathbf{V}_C$. We can check Eq. (3.41) from Fig. 3.17. In the triangle $O_V$ac we can clearly see that $\mathbf{V}_C$ is equal to the sum of $\mathbf{V}_A$ and $\mathbf{V}_{CA}$. Similarly, in the triangle $O_V$bc, $\mathbf{V}_C$ is the sum of $\mathbf{V}_B$ and $\mathbf{V}_{CB}$.

In the third method which is the quickest method, we draw a line from the point 'a' (shown in Fig. 3.18) at an angle α shown in the displacement diagram. Similarly, we draw another line which makes angle β from the line ab. The point of intersection

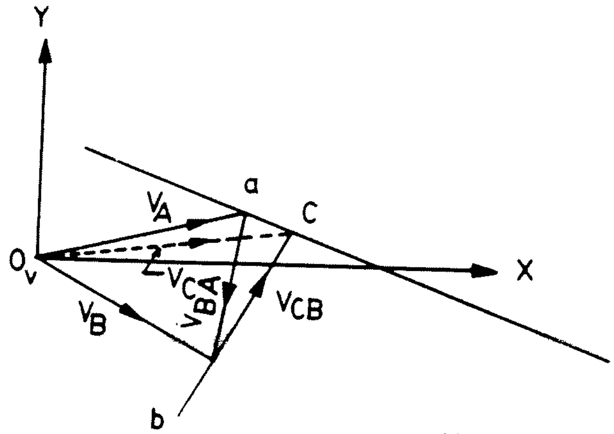

FIG. 3.17

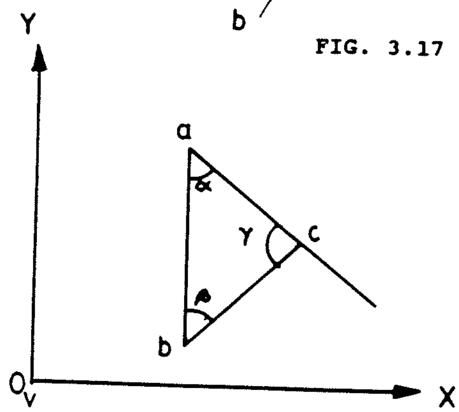

FIG. 3.18(a)

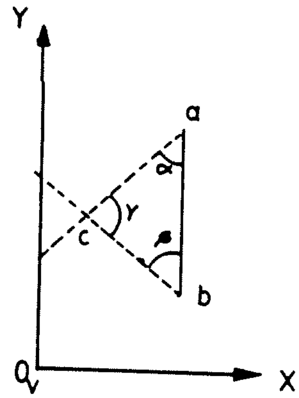

FIG. 3.18(b)

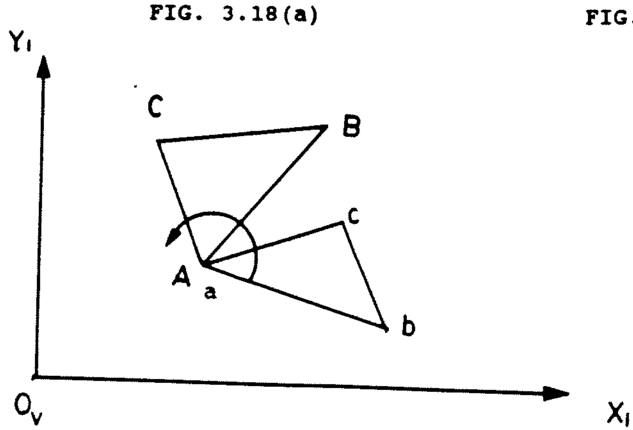

FIG. 3.19

will be the point C. Fig. 3.18 can be drawn only after drawing Fig. 3.18c. The velocity $\mathbf{V}_C$ is obtained by joining O_V to c. The triangle abc is called the image of the link ABC. These two triangles are similar because ab, bc, and ca are the graphical representation of the vectors $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$ and $\mathbf{V}_{CA}$ respectively.

Since each of these velocity difference vectors are perpendicular to the corresponding position difference vectors, i.e. $\mathbf{V}_{AB}$ is perpendicular to $\mathbf{R}_{AB}$, the triangles abc and ABC will be similar. In fact, if we transfer the triangle abc to the displacement diagram as shown in Fig. 3.19, we can see that the image has rotated 90^0 in the clockwise direction i.e. in the rotational direction of the link. While drawing the velocity image an important point should be remembered about the incorrect solution shown in Fig. 3.18b. If we rotate the triangle ABC, we should only get C at the proper relative location with respect to the line ab. Another way to obtain the correct location of c is that if a person travels from A to B in the displacement diagram, he will find the point on his left. This fact will be sufficient to locate the point c in the velocity image. i.e. the angles α and β can be drawn on the proper side of the line ab.

3.7 Derivation of Velocity Equations by Differentiating Displacement Equations

It is possible to obtain the velocity equations by differentiating the displacement equations. For instance, in the Example 3.2, the velocity of the point B_3 can be obtained by writing

$$\mathbf{R}_{B_3} = \mathbf{R}_{ED} + \mathbf{R}_{AE} + \mathbf{R}_{B_3A} \tag{3.42}$$

Differentiating this equation we get

$$\dot{R}_{B_3} = 0 + \omega_2 \times R_{AE} + \omega_3 \times R_{B_3A} \qquad (3.43)$$

where the differentiation rule for a vector **R** whose magnitude and direction change with time is given by

$$\frac{d}{dt}(R) = \frac{d}{dt}(R\ \hat{R}) = \frac{dR}{dt}\hat{R} + R\frac{d\hat{R}}{dt} = \frac{dR}{dt}\hat{R} + R\ (\omega \times \hat{R}) \qquad (3.44)$$

Fig. 3.20 shows an example of such a vector, here we have $V_{A_3} = \frac{d}{dt}(R_{A_3O}\ \hat{R}_{A_3O})$. In this figure, the slider is moving outwards relative to the link 2 which is rotating. At some other instant of time, the magnitude of R_{A_3O} will be different and the orientation of $\hat{R}_{A_3O}$ will also be different. On the other hand, if **R** represents the position vector of the point A_2 then

$$\frac{d}{dt}(R_{A_2}) = \frac{dR_{A_2}}{dt}\hat{R}_{A_2} + R_{A_2}(\omega_2 \times \hat{R}_{A_2})$$

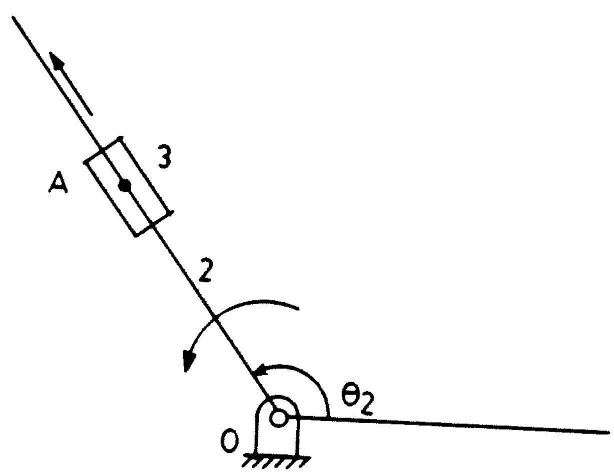

FIG. 3.20

$$= 0 \times \hat{R}_{A_2} + R_{A_2} (\omega_2 \times \hat{\mathbf{R}}_{A_2})$$

$$= R_{A_2} (\omega_2 \times \hat{\mathbf{R}}_{A_2})$$

If we get back to Fig. 3.15, we can also write

$$\mathbf{R}_{B_3} = \mathbf{R}_{B_4D} + \mathbf{R}_{B_3/B_4} \tag{3.46}$$

where the vector $\mathbf{R}_{B_3/B_4}$ has zero magnitude at the present instant of time but the time derivative of its magnitude is not zero. Using Eq. (3.44) we can write

$$\frac{d}{dt}(\mathbf{R}_{B_3}) = \omega_4 \times \mathbf{R}_{B_4D} + \frac{d}{dt}\left(R_{B_3/B_4}\right) \hat{R}_{B_3/B_4} + R_{B_3B_4} \frac{d}{dt}\left(\hat{R}_{B_3/B_4}\right)$$

$$= \omega_4 \times \mathbf{R}_{B_4D} + V_{B_3/B_4} \hat{R}_{B_3/B_4} + 0 \frac{d}{dt}\left(\hat{R}_{B_3/B_4}\right)$$

$$= \omega_4 \times \mathbf{R}_{B_4D} + V_{B_3/B_4} \hat{R}_{B_3/B_4} \tag{3.47}$$

The direction of $\hat{R}_{B_3/B_4}$ is obtained by kinematic inversion of this mechanism where the link 4 is considered as the frame.

If one likes to define the displacement equations in terms of complex numbers one can rewrite Eq. (3.46) as

$$\mathbf{R}_{B_3} = R_{B_4D}\, e^{j\theta_{B_4D}} + R_{B_3/B_4}\, e^{j\theta_{B_3/B_4}} \tag{3.48}$$

The differentiation rule for a complex vector $\mathbf{R} = R\, e^{j\theta}$ can be given as

$$\frac{d}{dt}(\mathbf{R}) = \dot{R}\, e^{j\theta} + R\, j\dot{\theta}\, e^{j\theta} \tag{3.49}$$

Therefore differentiating Eq. (3.48) we get

$$\frac{d}{dt}(\mathbf{R}_{B_3}) = \dot{R}_{B_4D}\, e^{j\theta_{B_4D}} + R_{B_4D} \frac{d}{dt}\left(e^{j\theta_{B_3/B_4}}\right)$$

$$+ \dot{R}_{B_3/B_4} e^{j\left(\theta_{B_3/B_4}\right)} + R_{B_3/B_4} \frac{d}{dt} \left(e^{j\theta_{B_3/B_4}}\right)$$

$$= 0\ e^{j\theta_{B_4D}} + R_{B_4D}\ j\dot{\theta}_{B_4D}\ e^{j\theta_{B_4D}} + V_{B_3/B_4} e^{j\left(\theta_{B_3/B_4}\right)}$$

$$+ 0\ j\dot{\theta}\ e^{j\theta_{B_3/B_4}}$$

$$= R_{B_4D}\ \dot{\theta}_{B_4D}\ e^{j\pi/2}\ e^{j\theta_{B_4D}} + V_{B_3/B_4} e^{j\theta_{B_3/B_4}}$$

$$= R_{B_4D}\ \omega_4\ e^{j\left(\theta_{B_4D}+\pi/2\right)} + V_{B_3/B_4} e^{j\left(\theta_{B_3/B_4}\right)} \tag{3.50}$$

Now we can compare Eqs. (3.47) and (3.50) and see that they are the same. For solving numerical problems, it is advisable to write an equation similar to Eq.(3.43) which is of general nature for velocity analysis. To find the apparent path, there are two steps: (a) fix the link (the guiding link) on which the apparent path of a moving point is to be known; for example in the Fig. 3.15, the link 4 was fixed, and (b) allow small displacements in links and judge the direction of motion of the guided link. The apparent path of any moving point can be known by carrying out the displacement analysis of the inverted mechanisms but for judging the direction of motion of points, one can easily see the apparent paths by allowing small perturbations of the inverted mechanism. Complete displacement analysis is not necessary.

3.8 Apparent Angular Velocity

Suppose there are two links 1 and 2 which rotate with angular velocities ω_1 and ω_2 respectively. To an observer on link 1, the angular velocity of the link 2 will be equal to $(\omega_2 - \omega_1)$. For example, if the link 2 is rotating with 100 rad/s clockwise and link 1 with 50 rad/s counter-clockwise, the apparent angular velocity $\omega_{2/1}$ will be {100k -(-50k)} = 150k. Note that ω_2 or ω_1 are all defined with respect to the stationary frame. In the apparent angular velocity both the links are specified in the subscript.

3.9 The Instantaneous Center of Velocity

When a link rotates relative to another parallel link then it is always possible to find two coincident points when viewed from the normal direction to these links where the velocities of these points are equal. In particular, if one of the links is a fixed link or frame, the velocities of these points will be equal to zero. For example, if we look at the mechanism in the Fig.3.21a, the points A_2 and A_1 are coincident and stationary points. On the other hand, B_2 and B_3 are on links 2 and 3 which have equal velocities. A pin joint or a rolling contact also imposes such a condition. These pairs of coincident points having equal velocities are called poles. If there are n links in a mechanism then the total number of poles, N, is given by the equation

$$N = \frac{n(n - 1)}{2} \tag{3.50}$$

which is the number of possible combinations of links taken two at a time.

Although these poles exist, one has to know the locations of these points in order to carry out the velocity analysis. These locations are known with the help of the **Aronhold-Kennedy** Theorem. According to this theorem the three instant centers shared by three rigid bodies in relative motion to one another, all lie on a straight line. It should be added here that these three links need not intersect each other in the sketch. This is because

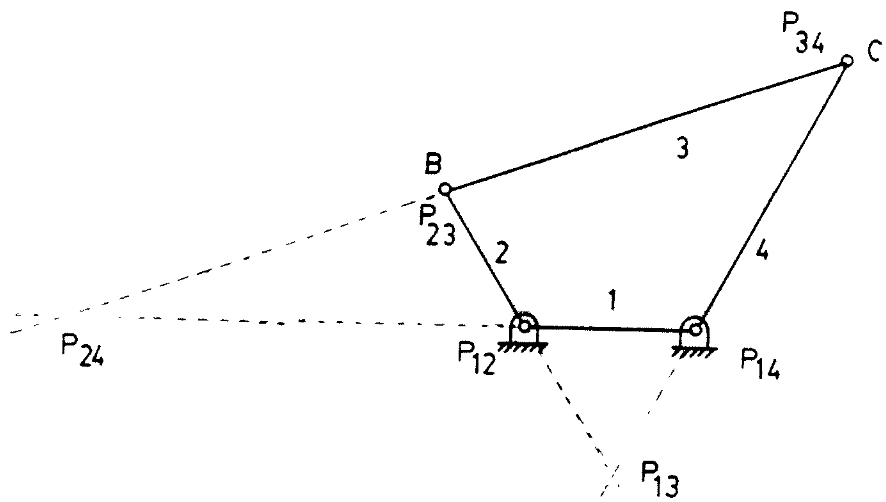

FIG. 3.21(a)

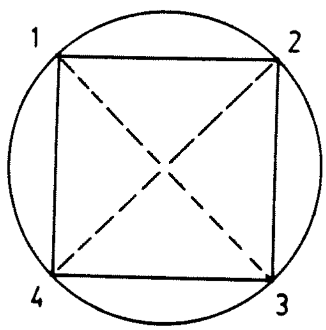

FIG. 3.21(b)

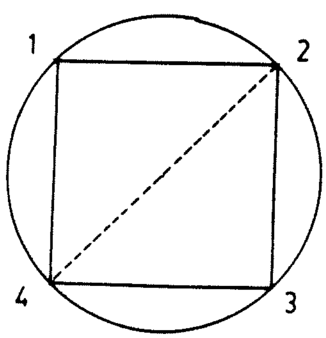

FIG. 3.21(c)

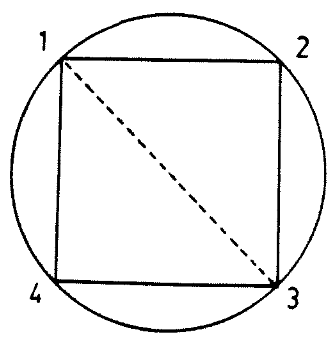

FIG. 3.21(d)

actually all of these links are in parallel planes extended to infinity in any or all directions. This can be understood from Figs. 3.22a and 3.22b. In Fig. 3.22a links 2 and 3 rotate in opposite directions. According to the Kennedy's Theorem the poles of these links (links 1, 2, and 3) should be on a straight line. Here, the notation of the pole of links 1 and 2 is P_{12}. One can use similar notations for other poles. Poles P_{12} and P_{13} will have zero velocity, so these will be called stationary poles. All poles where link 1, the frame, is not involved will be called the moving poles. Now suppose we take a point C which is in the common portion of the extended links. The direction of $\mathbf{V}_{C_2}$ and $\mathbf{V}_{C_3}$ will never be the same whatever the angular velocities ω_2 and ω_3 be. On the other hand, by adjusting the magnitudes of ω_2 and ω_3, the magnitudes of $\mathbf{V}_{C_2}$ and $\mathbf{V}_{C_3}$ can be made equal. The locus of points where the directions of these velocities will be the same, is the line connecting P_{12} and P_{13}. But, both the magnitude and direction will be equal at only one point for a given combination of ω_2 and ω_3. This point is shown as P_{23} in Fig. 3.22a. If the moving links have opposite rotational directions then the moving pole will be between these stationary poles as shown in this figure. On the other hand, if they are rotating in opposite directions then it will be on the line connecting the two stationary poles but the line has to be extended beyond these points as shown in Fig. 3.22b.

The use of this theorem can be understood by taking the example of the four-bar mechanism in shown Fig. 3.21a. A circle is drawn and numbers 1 to 4 are written at equal angular spacings indicating the four links. The obvious poles are located by

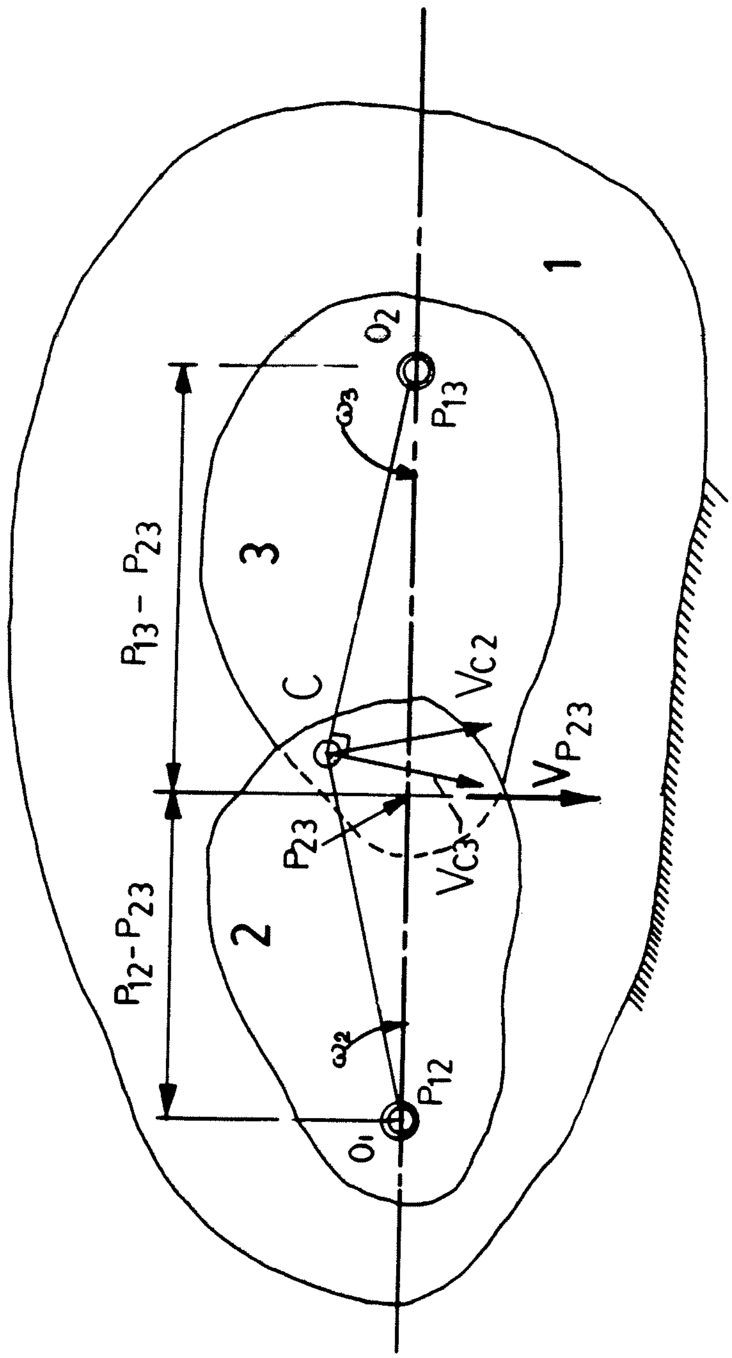
1
2
3
O_1
O_2
P_{12}
P_{13}
P_{23}
C
ω_2
ω_3
V_{C2}
V_{C3}
$V_{P_{23}}$
$P_{13} - P_{23}$
$P_{12} - P_{23}$

FIG. 3.22(a)

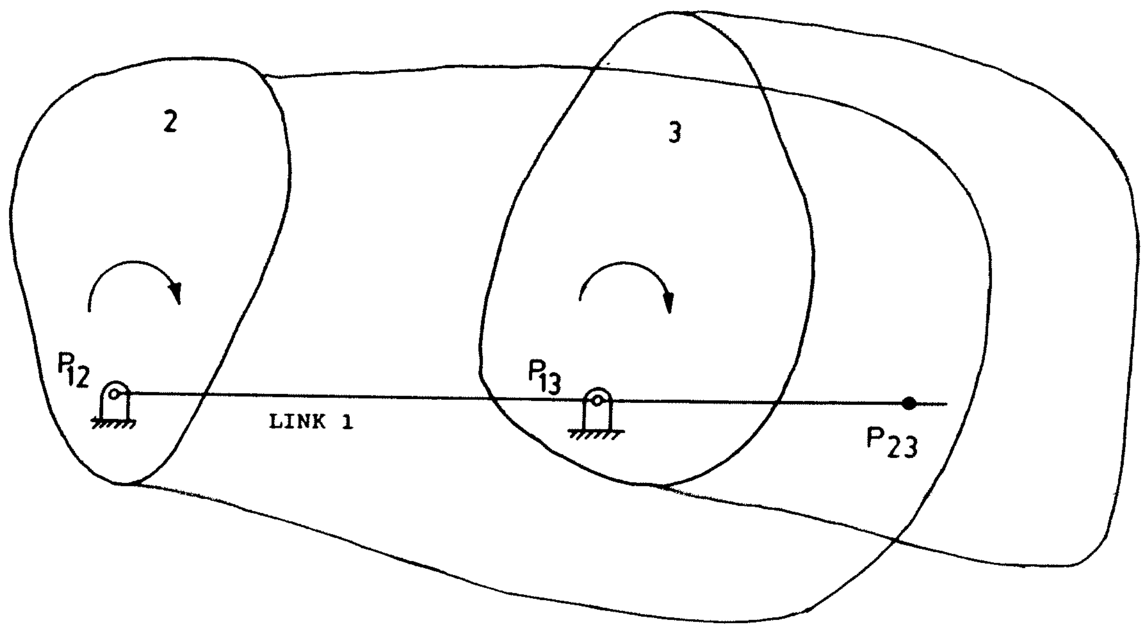

FIG. 3.22(b)

inspection such as those at the pinned joints and the corresponding links are joined by solid lines. The unknown poles are indicated by dashed lines. In the next step, one of the dashed lines is considered at a time as shown in Fig. 3.21c. This dashed line should be common to two triangles which have all other sides as solid lines. Only under such a condition the unknown pole shown by the dashed line can be located. Using the notation P_{12} to indicate the pole for the links 1 and 2, this theorem states that P_{12}, P_{14} and P_{24} must lie on a straight line. Therefore, we extend the line joining P_{12} and P_{14} in both the direction as show in Fig. 3.21a. Using a similar reasoning we extend the line joining P_{23} and P_{34}. In this way, we locate P_{24} as the point of intersection of these two lines. The other unknown pole P_{13} is located using a similar construction as shown in Fig 3.21c. Another point to note is that the instant centers

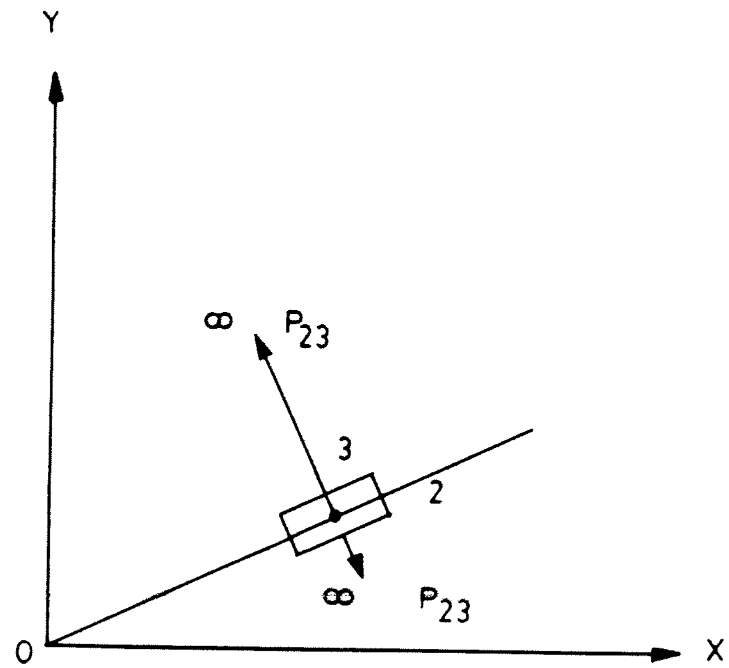

FIG. 3.23

for a slider moving on a link as shown in Fig. 3.23, exists at ∞ on either side of the link 2.

The following are the steps for the velocity analysis using the instant center method:

1. Locate all the instant centers using the Aronhold-Kennedy Theorem.
2. Starting with the link whose velocities are given, obtain the velocities of the various poles for this link. For example, if ω_2 for the four bar mechanism is known then obtain the velocity of the pole P_{24}.
3. Since the velocities of two points on any link are sufficient to determine its angular velocity and the velocities of other points; use the velocity of the pole determined above and the stationary pole of this link to determine its angular velocity. In this example, the velocities at P_{24} and P_{14} are known now. The point P_{14} is the axis of rotation for the

link 4. Thus the angular velocity of the link 4 can be determined. As mentioned earlier, that there are two kinds of poles at a given instant of time; the first kind are poles which have non zero velocities such as P_{23}, P_{34} etc. and the others are stationary, and these are the poles of the moving links with the frame. It is these stationary poles which act as the axis of rotation for the moving links. For each link we may have several moving poles but only one stationary pole.

Just like in the link-by-link method, even here we start at a link, say link i, where either the velocity of one of the moving poles is known or its angular velocity is known. Then one can calculate the velocities of all its moving poles using its stationary pole as the axis of rotation. Next, we find the angular velocity of the next link, the link i + 1 because at this stage we already know the velocity of $P_{i,i+1}$ and we also know the location of $P_{i,i+1}$. Thus we can calculate the angular velocity of link i + 1, and then knowing this we can calculate the velocity of any point on this link including the velocity of another moving pole, $P_{i+1,i+2}$. Now we are in a position to repeat the calculation process for the link i + 2. This method of velocity analysis can also be carried out either graphically or analytically.

3.10 The Analytical Method for the Instant Center

In the previous section, the graphical method was discussed but the analytical method is more accurate and equally easy to use. The first step in this method is to obtain the position difference vectors between the stationary poles and the corresponding points whose velocities are to be determined. For example, suppose we are given ω_2 and we would like to know ω_4 of the previous problem. At first, we have to locate all the stationary and moving poles. In this case the parameters i.e. the angular velocity of the link 2 is given, so we should find the location of the moving pole P_{24} first. After locating it, we would calculate

$$\mathbf{V}_{24} = \omega_2 \times \mathbf{R}_{P_{24}P_{12}} = \omega_2 \times \mathbf{R}_{P_{24}P_{12}} \angle (\theta_{P_{24}P_{12}} \pm 90^0)$$

To find $\mathbf{R}_{P_{24}P_{12}}$ we have to write an equation involving $\mathbf{R}_{P_{24}P_{14}}$. This can be easily done if we consider the vector triangle equation

$$\overset{\surd\surd}{\mathbf{R}_{P_{34}P_{14}}} = \overset{*\surd}{\mathbf{R}_{P_{24}P_{14}}} + \overset{*\surd}{\mathbf{R}_{P_{34}P_{24}}} \tag{3.51}$$

The directions of each of the vectors are known because the direction of $\mathbf{R}_{P_{34},P_{14}}$, $\mathbf{R}_{P_{24},P_{14}}$, and $\mathbf{R}_{P_{34},P_{24}}$, correspond to those of $\mathbf{R}_{CD}$, $\mathbf{R}_{AD}$ and $\mathbf{R}_{CB}$ respectively. One can use the method used for case 2a to obtain the magnitudes of each of these vectors. Once we know $\mathbf{R}_{P_{24}P_{14}}$, we can calculate ω_4 using

$$\omega_4 = \frac{|\mathbf{V}_{P_{24}}|}{|\mathbf{R}_{P_{24},P_{14}}|}$$

where the direction of ω_4 is known by considering a link hinged at P_{14} as shown in Fig. 3.21a.. The direction of $\mathbf{V}_{P_{24}}$ located at P_{24}.

Example 3.3: For the mechanism shown in Fig. 3.24, the angular velocity of link 2 is given as ω_2 = 40 rad/sec (ccw). Find $\mathbf{V}_{B_4}$,

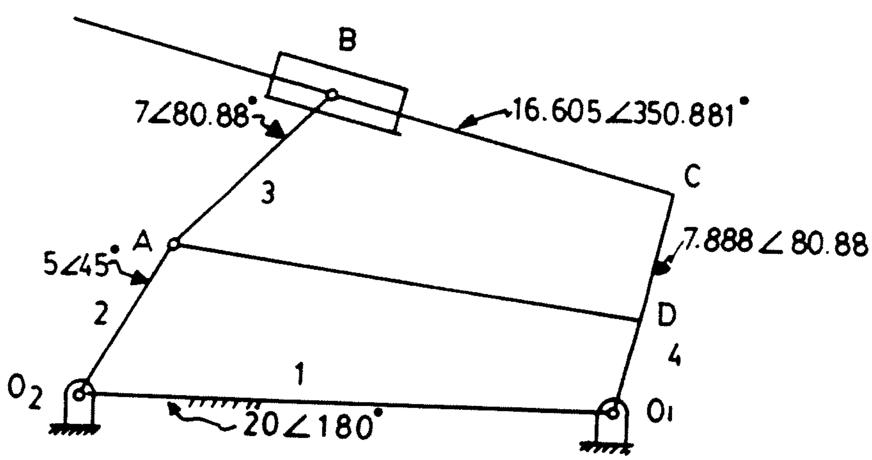

FIG. 3.24 EXAMPLE 3.3

$\mathbf{V}_{B_3}$, ω_3 and ω_4.

Solution

<u>LINK 2</u>

$$\mathbf{V}_A = \omega_2 R_{AO_2} \angle(\theta_{AO_2} + 90^0) = (40 \times 5) \angle(45^0 + 90^0)$$

$$= 200 \angle 135^0$$

$$\mathbf{R}_{B_4O_1} = \mathbf{R}_{CO_1} + \mathbf{R}_{B_4C} = 7.888 \angle 80.881^0 + 16.604 \angle 170.881^0$$

$$\mathbf{R}_{B_4O_1} = 18.382 \angle 145.470^0$$

<u>LINK 4</u>

$$\mathbf{V}_{B_4O_1} = \omega_4 \times \mathbf{R}_{B_4O_1} = (\omega_4 \mathbf{R}_{B_4O_1}) \angle(145.470 + 90)$$

$$= (\omega_4 \mathbf{R}_{B_4O_1}) \angle 235.47^0 \qquad \text{(a)}$$

$$\mathbf{V}_{B_3} = \mathbf{V}_{B_4} + \mathbf{V}_{B_3/B_4} \qquad \text{(b)}$$

<u>LINK 3</u>

$$\mathbf{V}_{B_3} = \mathbf{V}_A + \omega_3 \times \mathbf{R}_{B_3A}$$

$$= \mathbf{V}_{O_2} + \mathbf{V}_{AO_2} + \omega_3 \times \mathbf{R}_{B_3A}$$

$$= 0 + \mathbf{V}_A + \omega_3 \times \mathbf{R}_{B_3A}$$

$$= 200\angle 115^0 + (\omega_3 7)\ \angle(80.881^0 + 90^0)$$

$$= 200\angle 135^0 + (7\omega_3)\ \angle 170.881^0 \qquad \text{(C)}$$

Substituting for $\mathbf{V}_{B_4} = \mathbf{V}_{B_{O_1}O}$ from Eq.(a) and $\mathbf{V}_{B_3}$ from Eq.(c) into Eq.(b), we get

$$200\angle 135^0 + 7\omega_3\ \angle 170.881^0 = (\omega_4 8.382)\ \angle 235.47^0 + V_{B_3/B_4}\angle 170.881^0 \qquad \text{(d)}$$

We also have the relationship

$$\omega_3 = \omega_4 \qquad \text{(e)}$$

Using Eq.(e) one can rewrite Eq.(d) as

$$200\angle 135 = (v_{B_3/B_4} - 7\omega_3)\ \angle 170.881 + (18.382\omega_3)\ \angle 235.47$$

$$= x\ \angle 170.881 + (18.382\omega_3)\ \angle 235.47 \qquad \text{(f)}$$

This is of the form of case 2a and its solution will be

$$200\angle 135^0 = 217.735\ \angle 170.881^0 - 129.776\ \angle 235.47^0 \qquad \text{(g)}$$

From Eqs.(f) and (g) we get (after comparing the magnitudes the vectors on the right hand side)

$$x = v_{B_3/B_4} - 7\omega_3 = 217.735 \qquad \text{(h)}$$

and

$$18.382\omega_3 = -129.776$$

or

$$\omega_3 = -7.060 \qquad \text{(i)}$$

Substituting for ω_3 in Eq.(h) we get

$$V_{B_3/B_4} = 217.735 + 7 \times (-7.060) + 7(-9.972)$$

$$=168.315$$

Therefore we can write

$$\mathbf{V}_{B_3/B_4} = 168.315 \angle 170.881^0$$

Substituting the value for ω_4 ($\omega_3 = \omega_4$) in Eq.(a) we get

$$\mathbf{V}_{B_4} = \mathbf{V}_{B_4O_1} = (-7.060)\ (18.382)\ \angle(145.470^0 + 90^0)$$

$$= 129.777 \angle 55.47^0$$

$$\mathbf{V}_{B_3} = \mathbf{V}_{B_4} + \mathbf{V}_{B_3/B_4}$$

$$= 129.777\angle 55.47^0 + 168.315 \angle 170.881^0$$

$$= 162.560 \angle 124.736^0$$

Example 3.4

For the mechanism used in Example 2.16 (Fig.2.28a), the angular velocity of link 2 is given as ω_2 = 30 rad/s (ccw). Find $\mathbf{V}_G$, $\mathbf{V}_D$, $\mathbf{V}_E$, $\mathbf{V}_F$, ω_3, ω_4 and ω_5.

Solution

LINK 2

$$\mathbf{V}_{A_2} = \omega_2\ R_{AO_1}\angle(100^0 + 90^0) = (30 \times 2)\ \angle(100^0 + 90^0)$$

$$= 60 \angle 190^0$$

LINK 3

Fix the guiding link (link 3) and allow the guided link (link 2) to move. The equation for the velocity of the point A can be written as

$$\mathbf{V}_{A_2} = \mathbf{V}_{A_3} + \mathbf{V}_{A_2/A_3}$$

$$60 \angle 190^0 = (\mathbf{V}_C + \mathbf{V}_{A_3C}) + \mathbf{V}_{A_2/A_3}$$

$$60 \angle 190^0 = \underline{0} + V_{A_3C}\ \ \angle(15.709^0 + 90^0) + \mathbf{V}_{A_2/A_3}\ \angle(15.709^0)$$

This corresponds to case 2a. The solution using program 2 is

$$60 \angle 190^0 = 5.968 \angle 105.709^0 + (-59.702) \angle 15.709^0$$

Therefore,

$$\omega_3 = \frac{V_{A_3C}}{R_{A_3C}} = \frac{5.968}{20.099} = 0.297 \text{ rad/s} \quad \text{ccw}$$

$$\mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC} = 0 + (\omega_3 R_{DC}) \angle(25.419^0 + 90^0)$$

$$= (0.297 \times 41.284) \angle(25.419^0 + 90^0)$$

$$= (12.261) \angle 95.660^0$$

LINK 4

$$\mathbf{V}_E = \mathbf{V}_D \quad \mathbf{V}_{ED}$$

$$V_E \angle 180^0 = 12.261 \angle 115.419^0 + (\omega_4 R_{ED}) \angle(330^0 + 90^0)$$

Using program 11 we get

$$11.656 \angle 180^0 = 12.261 \angle(115.419^0) + (-12.787) \angle 60^0$$

$$\mathbf{V}_E = 11.656 \angle 180^0$$

$$\omega_4 = \frac{-12.787}{20} = -0.639 \text{ rad/s cw}$$

LINK 5

$$V_F = \mathbf{V}_G + \mathbf{V}_{FG}$$

$$\mathbf{V}_F \angle 180^0 = 11.868 \angle 95.660^0 + (\omega_5 R_{FG}) \angle(330^0 + 90^0)$$

Using program 11 we obtain

$$7.989 \angle 180^0 = 11.868 \angle 95.660^0 + (-13.637) \angle 60^0$$

$$\omega_5 = \frac{-13.637}{20} = -0.682 \text{ rad/s cw}$$

$$\mathbf{V}_F = 7.989 \angle 180$$

Example 3.5: For the mechanism shown in Fig.2.27a the following data are given:

$\mathbf{R}_{CO_1} = 20 \angle 90^0$; $\mathbf{R}_{BC} = 125 \angle 0^0$; $\mathbf{R}_{AO_1} = 75 \angle 83.596^0$; $\mathbf{R}_{BA} = 150 \angle 211.103^0$ and $\omega_2 = 10$ rad/s (ccw). Find $\mathbf{V}_B$ and ω_3

Solution:

LINK 2

$$\mathbf{V}_A = \mathbf{V}_{O_1} + \mathbf{V}_{AO_1} = 0 + \omega_2 R_{AO_1} \angle(\theta_{AO_1} + 90^0)$$

$$= (10 \times 75) \angle(83.596^0 + 90^0) = 750 \angle 173.596^0$$

LINK 3

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

$$V_B \angle 180^0 = 750 \angle 173.596^0 + (\omega_3 R_{BA}) \angle(211.103^0 + 90^0)$$

$$694.850 \angle 180 = 750 \angle 173.596 + 97.699 \angle 301.103 \text{ (Using program 11)}$$

Therefore,

$$\mathbf{V}_B = 694.850 \angle 180^0$$

$$\omega_3 = 97.699/150 = 0.651 \text{ rad/s} \quad \text{ccw}$$

Example 3.6: For the mechanism shown in Fig. 3.25 the following data are given:

$\mathbf{R}_{AO_2} = 40 \angle 120^0$; $\mathbf{R}_{BA} = 80 \angle 18.704^0$; $\mathbf{R}_{O_4B} = 70 \angle 239.214^0$; $\mathbf{R}_{O_2O_4} = 20 \angle 180^0$; $R_{FE} = 15 \angle 288.584^0$; $R_{EA} = 40 \angle 18.584^0$ and $\omega_2 =$ 10 rad/s (ccw). Find ω_3, ω_4, $\mathbf{V}_E$, $\mathbf{V}_B$ and $\mathbf{V}_F$.

Solution:

LINK BY LINK METHOD

LINK 2

$$\mathbf{V}_A = \mathbf{V}_{O_2} + \mathbf{V}_{AO_2} = 0 + \omega_2 R_{AO_2} \angle(120^0 + 90^0)$$

$$= (10 \times 40) \angle 210^0 = 400 \angle 210^0$$

LINK 3

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} = 400 \angle 210^0 + (\omega_3 R_{BA}) \angle(18.584^0 + 90^0) \tag{a}$$

LINK 4

$$\mathbf{V}_B = \mathbf{V}_{O_4} + \mathbf{V}_{BO_4} = 0 + (\omega_4 R_{BO_4}) \angle(59.214^0 + 90^0)$$

(b)

Equating the above expressions (a) and (b), we get

$$400 \angle 210^0 + (\omega_3\ 80) \angle 108.584^0 = (\omega_4\ 70) \angle 149.214^0 \qquad \text{(c)}$$

It can be solved using program 11 (modified case 2a) and we can write the following :

$$400 \angle 210^0 + 536.143 \angle 108.584^0 = 602.125 \angle 149.214^0 \qquad \text{(d)}$$

$$\omega_3 = \frac{536.143}{80} = 6.702 \text{ rad/s} \quad \text{ccw}$$

$$\omega_4 = \frac{602.125}{70} = 8.602 \text{ rad/s} \quad \text{ccw}$$

Using Eq. (a) we get

$$\mathbf{V}_B = 400 \angle 210^0 + (6.702 \times 80) \angle(18.584^0 + 90^0) = 400 \angle 210^0$$

$$+ \ 536.16 \angle 108.584^0$$

$$= 602.137 \angle 149.213^0$$

we can write

$$\mathbf{V}_E = \mathbf{V}_A + \mathbf{V}_{EA} = 400 \angle 210^0 + (\omega_3 R_{EA}) \angle(18.584^0 + 90^0)$$

$$= 400 \angle 210^0 + 268.08 \angle 108.584^0$$

$$= 435.222 \angle 172.859^0$$

To calculate $\mathbf{V}_F$ we have to know $\mathbf{R}_{FA}$ which can be obtained by writing

$$\mathbf{R}_{FA} = \mathbf{R}_{EA} + \mathbf{R}_{FE} = 40 \angle 18.584^0 + 15 \angle 288.584^0$$

$$= 42.72 \angle 358.028^0$$

Also,

$$\mathbf{V}_F = \mathbf{V}_A + (\omega_3 R_{FA}) \angle(358.028^0 + 90^0)$$

$$= 400 \angle 210^0 + (6.702 \times 42.72) \angle 448.028^0$$

$$= 400 \angle 210^0 + 286.309 \angle 88.028^0$$

$= 347.406 \angle 165.644^0$

Example 3.7: For the mechanism in Fig. 3.25 data is given in Example 3.6 determine ω_3, ω_4, $\mathbf{V}_E$, $\mathbf{V}_B$ and $\mathbf{V}_F$ by the instant center method.

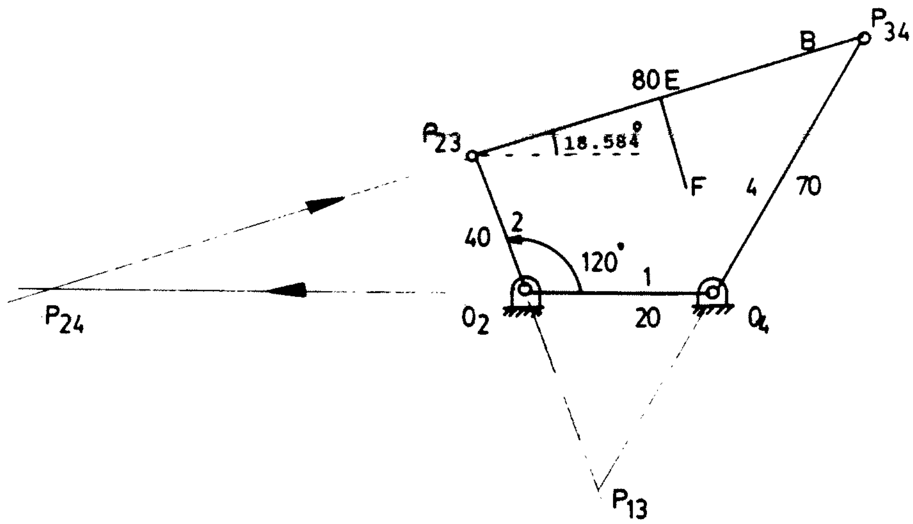

FIG. 3.25(a)

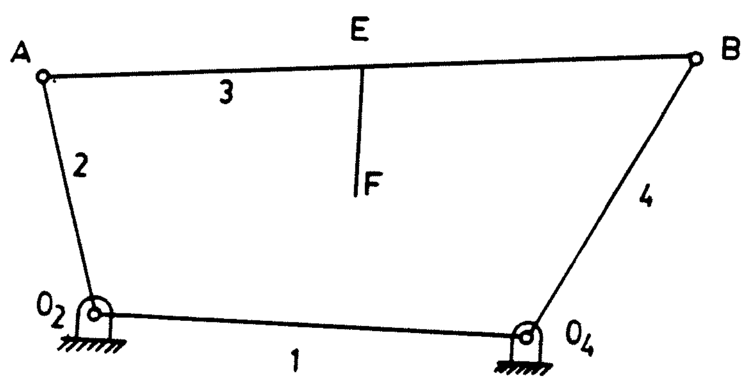

FIG. 3.25 EXAMPLE 3.6

Solution:

LINK 2

Refer to Fig.3.25a, one can obtain various displacement vectors as mentioned below:

$$\mathbf{V}_A = \omega_2 \, R_{AO_2} \angle(120+90)$$

$$= 400 \angle 210$$

$\Delta\ ABP_{13}$

$$\mathbf{R}_{BA} = \mathbf{R}_{P_{13}A} + \mathbf{R}_{BP_{13}}$$

$$80 \angle 18.584^0 = \mathbf{R}_{P_{13}A} \angle 300^0 + \mathbf{R}_{BP_{13}} \angle 59.214^0 \qquad \text{(case 2a)}$$

$$= 59.686 \angle 300^0 + 89.845 \angle 59.214^0 \qquad \text{(Using program 2)}$$

$\Delta\ P_{24}O_2A$

$$\mathbf{R}_{AO_2} = \mathbf{R}_{P_{24}O_2} + R_{AP_{24}}$$

$$40 \angle 120^0 = R_{P_{24}O_2} \angle 180^0 + R_{AP_{24}} \angle 18.584^0$$

$$= 123.029 \angle 180^0 + 108.697 \angle 18.584^0 \qquad \text{(Using program 2)}$$

LINK 3

We can write

$$\frac{V_A}{V_B} = \frac{R_{AP_{24}} \quad \omega_3}{R_{BP_{24}} \quad \omega_3} = \frac{R_{AP_{24}}}{R_{BP_{24}}}$$

or

$$\frac{400}{V_B} = \frac{108.697}{188.697}$$

from which we obtain $V_B = 694.396$. Now we can write the following:

$$\mathbf{R}_{EP_{13}} = \mathbf{R}_{AP_{13}} + \mathbf{R}_{EA} = 59.686 \angle 120^0 + 40 \angle 18.584^0$$

$$= 64.941 \angle 82.86^0$$

$$\mathbf{R}_{FP_{13}} = \mathbf{R}_{EP_{13}} + \mathbf{R}_{FE} = 64.941 \angle 82.86^0 + 15 \angle 288.284^0$$

$$= 51.796 \angle 75.718^0$$

$$\frac{V_E}{V_A} = \frac{R_{EP_{13}}}{R_{AP_{13}}} = \frac{64.941}{59.686} = 1.088,$$

Substituting $V_A = 400$ we get

$$V_E = 435.218$$

Substituting $V_A = 400$ below we get

$$\frac{V_F}{V_A} = \frac{R_{FP_{13}}}{R_{AP_{13}}} = \frac{51.796}{59.686} = 0.868, \quad V_F = 347.123$$

$$\frac{V_B}{V_A} = \frac{R_{BP_{13}}}{R_{AP_{13}}} = \frac{89.845}{59.686} = 1.505,$$

or,

$$V_B = 602.084$$

writing the velocity difference equation

$$\mathbf{V}_{BA} = \mathbf{V}_B - \mathbf{V}_A$$

$$= 602.084 \angle(59.214^0 + 90^0) - 400 \angle 210^0$$

$$= 536.112 \angle 108.581^0$$

Therefore,

$$\omega_3 = \frac{V_{BA}}{R_{BA}} = \frac{536.112}{80} = 6.701 \text{ rad/s ccw, and}$$

$$\omega_4 = \frac{V_{BO_4}}{R_{BO_4}} = \frac{V_B}{R_{BO_4}} = \frac{602.084}{70} = 8.602 \text{ rad/s ccw}$$

Example 3.8: Assuming $\omega_2 = 50$ rad/s cw in the mechanism shown in Fig 3.25, find ω_3, ω_4, $\mathbf{V}_F$ and V_B using the instant center method.

Disp Analysis

In the $\Delta\ AO_2O_4$

$$\mathbf{R}_{O_4A} = \mathbf{R}_{O_4O_2} + \mathbf{R}_{AO_2}$$

$$= 20\ \angle 180^0 + 40\ \angle 120^0 = 52.915\ \angle 139.107^0$$

In the $\Delta\ ABO_4$

$$\overset{\surd\surd}{\mathbf{R}_{O_4A}} = \overset{\surd *}{\mathbf{R}_{BA}} + \overset{\surd *}{\mathbf{R}_{O_4B}}$$

$$52.915\ \angle 319.107 = 80\ \angle\ \theta_{BA} + 70\ \angle\ \theta_{O_4B}$$

$$= 80\ \angle 18.584 + 70\ \angle 239.214$$

In $\Delta\ P_{24}AO_2$ shown in the Fig 3.25a we can write

$$\mathbf{R}_{AO_2} = \mathbf{R}_{P_{24}O_2} + \mathbf{R}_{P_{23}P_{24}}$$

or,

$$40\ \angle 120^0 = x\ \angle 180^0 + y\ \angle 378^0$$

Using program 2 we can write

$$40\ \angle 120^0 = 127.2394\ \angle 180 + 111.7728\ \angle 18.24$$

$$V_{P_{24}} = 6361.97\ \angle 90^0 = \omega_2\ R_{P_{24}O_2}$$

$$= 6361.97\ \angle 90^0$$

In $\Delta\ ABP_{13}$

$$\mathbf{R}_{BA} = \mathbf{R}_{P_{13}A} + \mathbf{R}_{BP_{13}}$$

$$80\ \angle 18.7044 = \mathbf{R}_{P_{13}A}\ \angle 300^0 + \mathbf{R}_{BP_{13}}\ \angle 59.0287$$

$$= 59.2096\ \angle 300 + 89.4932\ \angle 59.0287 \text{ (Using program 2)}$$

For link 2 we have

$$V_A = 40.0 \times 50\ \angle(120-90)$$

Similarly for link 3 we can write

$$\omega_3 = \frac{40 \times 50}{59.2096} = 33.778 \text{ rad/s cw}$$

$$\mathbf{V}_B = \omega_3 \times \mathbf{R}_{BP_{13}} = 33.778 \times 89.4932 \angle(59-90)$$

$$= 3022.90 \angle -31^0$$

$$\omega_4 = \frac{|\mathbf{V}_B|}{|\mathbf{R}_{BO_4}|} = \frac{3022.90}{70} = 43.184 \text{ cw}$$

Alternatively we can calculate the angular velocities in the following sequence:

$$\frac{|V_{P_{24}}|}{|V_B|} = \frac{|\mathbf{R}_{P_{24}O_2}|}{|\mathbf{R}_{B_{04}}|}$$

or,

$$|\mathbf{V}_B| = \frac{|\mathbf{V}_{P_{24}}||\mathbf{R}_{BO_4}|}{|\mathbf{R}_{P_{24}O_4}|} = \frac{6361.95}{147} = 3029.5$$

Therefore,

$$\mathbf{V}_B = 3029.5 \angle 50-90 = 3029.5 \angle -31 = 3029.5 \angle 329$$

$$\omega_4 = \frac{|\mathbf{V}_B|}{|\mathbf{R}_{BO_4}|} = \frac{3029.5}{70} = 43.2785 \text{ rad/s cw}$$

we know that

$$\mathbf{V}_{BA} = \mathbf{V}_B - \mathbf{V}_A$$

$$= 3029.5 \angle 329 - 2000 \angle 120.90$$

$$= 2698.27 \angle 288.81$$

Therefore

$$\omega_3 = \frac{|\mathbf{V}_{BA}|}{|\mathbf{R}_{BA}|} = \frac{2698.27}{80} = 33.728$$

Example 3.9: Assuming ω_2 = 50 rad/s ccw in the Fig 3.25a, find

$\mathbf{V}_E, \mathbf{V}_F$ and $\underline{\omega}_3$, and $\underline{\omega}_4$.

Solution:

The solution is shown in Fig 3.26. The points O_2 and O_4 are located at the origin O_V. The velocity of the point A will be represented by the image 'a' where the line $O_V a$ is perpendicular to R_{AO_2}. The distance $O_V a$ will be equal to $\omega_2 \ R_{AO_2}$. Next we write the following equations for links 3 and 4 for the point B:

$$\mathbf{V}_B = \underbrace{\overset{\checkmark\checkmark}{\mathbf{V}_{O_4}} + \overset{*\checkmark}{\mathbf{V}_{BO_4}}}_{\text{LINK 4}} = \underbrace{\overset{\checkmark\checkmark}{\mathbf{V}_A} + \overset{*\checkmark}{\mathbf{V}_{BA}}}_{\text{LINK 3}}$$

Using the second equality we write

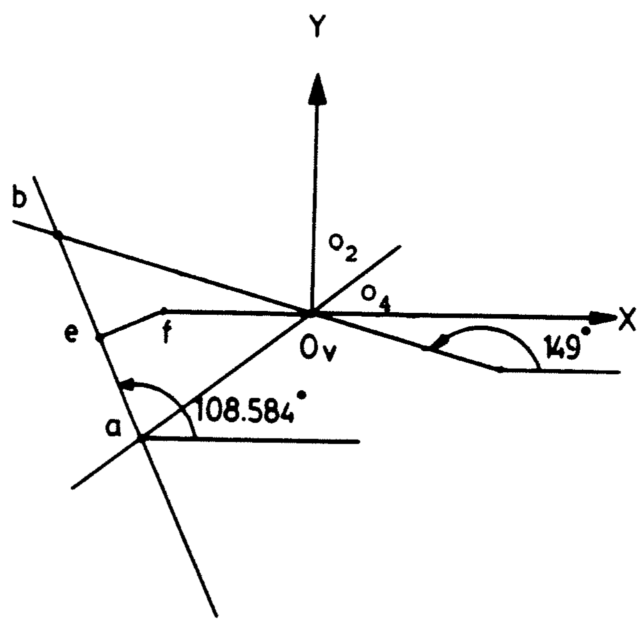

FIG. 3.26 GRAPHICAL SOLUTION IN EXAMPLE 3.9

$$\overset{*\checkmark}{0 + \mathbf{V}_{BO_4}} = \overset{\checkmark\checkmark}{\mathbf{V}_A} + \overset{*\checkmark}{\mathbf{V}_{BA}}$$

Which is in the modified case 2a form.

The graphical solution is obtained by drawing a line parallel to $\overset{*\checkmark}{\mathbf{V}_{BO}}$ from the top of $\mathbf{V}_A$ and another line parallel to $\overset{*\checkmark}{\mathbf{V}_{BA}}$ from the tail of $\mathbf{V}_A$. The point of intersection locates the point b. Now we are in a position to draw the image of the link 3 to locate points e and f.

The velocities of any of the points can be obtained by drawing a vector from O_V to that point.

Example 3.10: The Displacement analysis of the mechanism shown in Fig. 2.29a was performed in Example 2.17. For this mechanism the following are given:

$\mathbf{R}_{AO_1} = 2.5 \angle 150^0$, $\mathbf{R}_{BA} = 10 \angle 214.658^0$, $\mathbf{R}_{BO_2} = 7 \angle 97.713^0$,

$\mathbf{R}_{O_1O_2} = 7 \angle 180^0$, $\mathbf{R}_{CA} = 7.496 \angle 79.826^0$,

$\mathbf{R}_{AO_2} = 9.250 \angle 172.234^0$,

$\mathbf{R}_{DC} = 2 \angle 100.032^0$, $\mathbf{R}_{CF} = 6.422 \angle 14.032^0$,

$\mathbf{R}_{HE} = 10 \angle 10.032^0$,

$\mathbf{R}_{HF} = 10.038 \angle 24.068^0$, $\mathbf{R}_{DF} = 6.858 \angle 30.945^0$, $\omega_2 = 5$ rad/s (ccw),

$\alpha_2 = 7$ rad/s (cw). Determine $\mathbf{V}_{C_3}$, ω_3, ω_5, ω_6, $\mathbf{V}_D$ and $\mathbf{V}_H$.

Solution:

VELOCITY ANALYSIS

LINK 2

$$\mathbf{V}_A = \omega_2 R_{AO1} \angle(\theta_{AO_1} + 90^0) = (5 \times 2.5) \angle(150^0 + 90^0)$$
$$= 12.5 \angle 240^0$$

LINK 3 | LINK 4

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \qquad = \mathbf{V}_{O_2} + \mathbf{V}_{BO_2}$$

$$= 12.5 \angle 240^0 + (\omega_3\ 10) \angle(214.658^0 + 90^0) = (\omega_4 7) \angle(97.713^0 + 90^0)$$

$$= 12.5 \angle 240^0 + 10\ \omega_3 \angle 204.658^0 = 7\ \omega_4 \angle(187.713^0)$$

$$= 12.5 \angle 240^0 + (-33.928) \angle 204.658^0 = (-24.809) \angle(187.713^0)$$

(Using program 11)

Therefore,

$$\omega_3 = \frac{-33.928}{10} = -3.393 \text{ rad/s cw}$$

$$\omega_4 = \frac{-24.809}{7} = -3.544 \text{ rad/s cw}$$

$$\mathbf{V}_{C_3} = \mathbf{V}_A + \mathbf{V}_{CA} = 12.5 \angle 240^0 + (-3.393 \times 7.496) \angle 79.826^0 + 90^0)$$

$$= 12.5 \angle 240^0 + 25.434 \angle 349.826^0$$

$$= 24.238 \angle 320.804^0$$

(Using program 5)

LINK 5

$$V_{C_5} = \mathbf{V}_{C_3} = 24.238 \angle 320.804 \qquad \text{(a)}$$

LINK 6

$$\mathbf{V}_{C_5} = V_{C_6} + V_{C_5/C_6}$$

$$= \omega_6 R_{C_6F} \angle (\theta_{C_6F} + 90^0) + V_{C_5/C_6} \angle \theta_{C_5/C_6}$$

$$= (\omega_6 \times 6.422) \angle(14.032 + 90) + V_{C_5/C_6} \angle 10.032^0$$

Substituting the value for V_{C_5} from Eq. (a) we get

$$24.238 \angle 320.804^0 = 6.422\ \omega_6 \angle 104.032^0 + V_{C_5/C_6} \angle 10.032^0$$

or,

$$24.238 \angle 320.804^0 = (-18.469) \angle 104.032^0 + 14.599 \angle 10.032^0$$

(Using program 2)

Therefore,

$$V_{C_5/C_6} = 14.599 \angle 10.032^0$$

$$\omega_6 = \frac{-18.469}{6.422} = -2.878 \text{ rad/s cw}$$

$$\mathbf{V}_{D_6} = \omega_6 R_{D_6F} \angle(\theta_{D_6F} + 90^0) = (-2.878 \times 6.858) \angle(30.945^0 + 10^0)$$

$$= 19.737 \angle 300.945^0$$

Because of rolling contact at D

$$\mathbf{V}_{D_5} = \mathbf{V}_{D_6} = 19.737 \angle 300.945^0$$

LINK 5

$$V_{D_5C_5} = \mathbf{V}_{D_5} - \mathbf{V}_{C_5}$$

$$= 19.737 \angle 300.945^0 - 24.238 \angle 320.804^0$$

$$= 8.784 \angle 190.560^0 \qquad \text{(Using program 6)}$$

$$\omega_5 = \frac{V_{D_5C_5}}{R_{D_5C_5}} = \frac{8.784}{2} = 4.392 \text{ rad/s ccw}$$

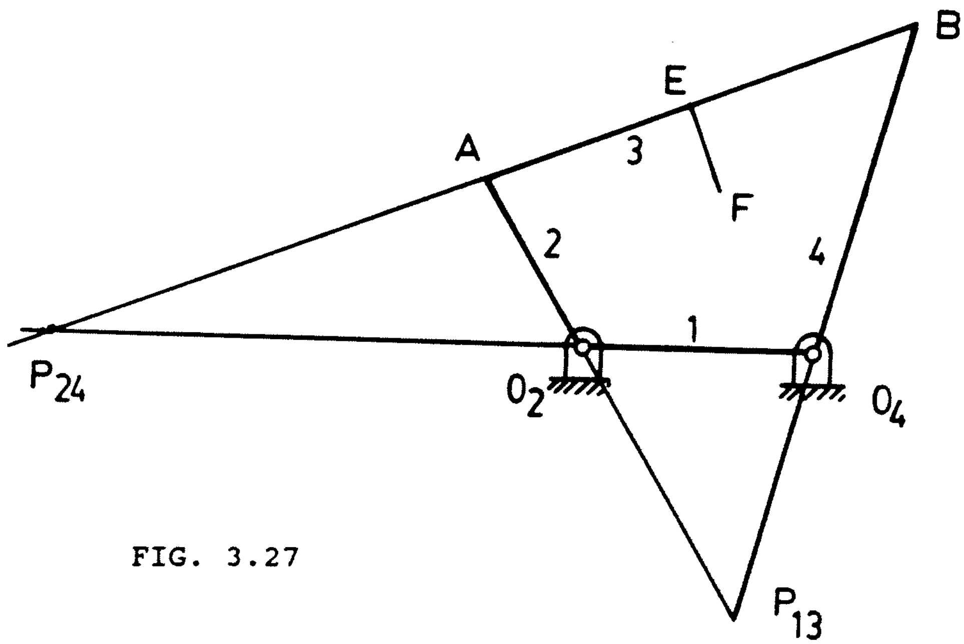

FIG. 3.27