CHAPTER 4

ACCELERATION ANALYSIS

4.1 Time Derivative of a Constant Magnitude Rotating Vector

Fig 4.1a shows a particle at points P_1 and P_2 at an interval of time Δt. The position vector of P in the curvilinear can be represented by the set of coordinates (r_1, θ_1). In Fig.4.1b, various vectors have been redrawn from a common point. The magnitudes of i_r and i_t are equal to 1 and from the geometry, the

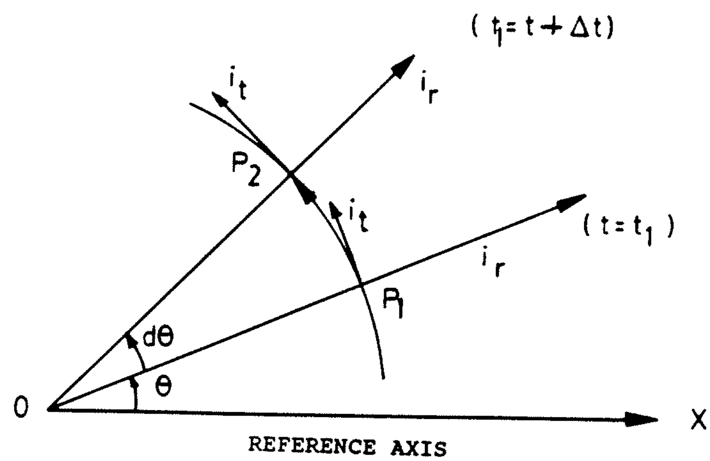

FIG. 4.1(a)

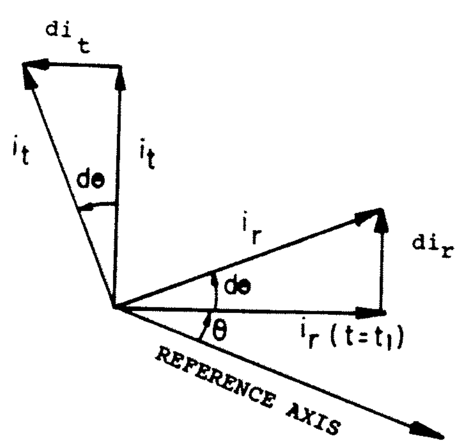

FIG. 4.1(b)

magnitudes of di_r and di_t will be equal to 1 $d\theta$. The direction of di_r will be of i_j and that of di, -i. If $\Delta t \longrightarrow 0$, one can write

$$\frac{di_r}{dt} = \frac{d\theta}{dt} i_j = \dot{\theta} i_j \text{ and} \tag{4.1}$$

$$\frac{di_t}{dt} = \frac{d\theta}{dt} (-i_r) = -\dot{\theta} i_r \text{ and} \tag{4.2}$$

If i_z represents the third unit vector, we can rewrite Eqs. (4.1) and (4.2) as

$$\frac{di_r}{dt} = \underline{\omega} \times i_r \text{ and} \tag{4.3}$$

$$\frac{di_t}{dt} = \underline{\omega} \times i_t \tag{4.4}$$

Here, we have assumed $\underline{\omega} = \dot{\theta} i_k$. Thus, the time derivative of a constant magnitude rotating vector is the cross product of its angular velocity vector and the vector itself. This relationship is valid not only for a unit vector but also for any constant vector. If we have a constant vector **R** attached to the (x,y,z) coordinate system which is rotating with an angular velocity $\underline{\omega}$ as shown in Fig. 4.1c, then

$$\left(\frac{d}{dt}(\mathbf{R}) \right)_{xyz} = \underline{\omega} \times \mathbf{R} \tag{4.5}$$

4.2 Acceleration of a Moving Point

The position vector of a particle in motion as shown in Fig. 4.1a can be written as

$$R_p = ri_r \tag{4.6}$$

The velocity and acceleration equations of this particle are

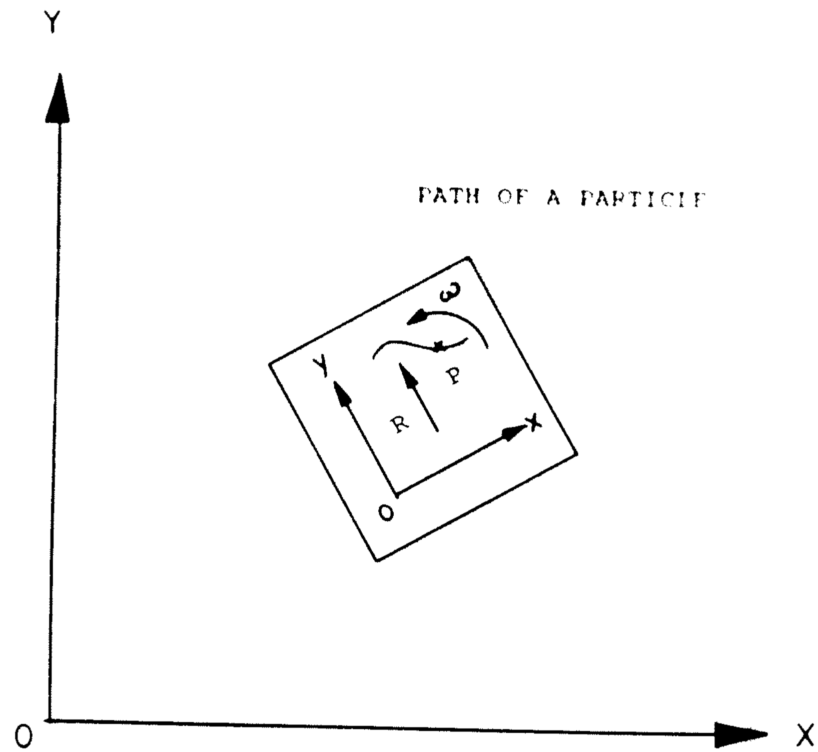

FIG. 4.1(c)

obtained by differentiation of this equation as

$$\mathbf{V}_p = \dot{\mathbf{R}}_P = \dot{r}\ i_r + r\ \frac{d}{dt}\ (i_r)$$

$$= \dot{r}\ i_r + r(\underline{\omega} \times i_r)$$

$$= \dot{r}\ i_r + r\ \omega\ i_t \qquad (4.7)$$

$$\mathbf{A}_p = \ddot{\mathbf{R}}_p = \{\ \ddot{r}\ i_r + \dot{r}\{\underline{\omega} \times i_r\ )\} + [\dot{r}(\underline{\omega} \times i_r\ ) + r(\dot{\underline{\omega}} \times i_r) + r\{\underline{\omega} \times (\underline{\omega} \times i_r\ )\}]$$

$$= \{\ \ddot{r}\ i_r + \dot{r}\ \omega\ i_t\ \} + \{\ \dot{r}\ \omega\ i_t + r\ \alpha\ i_t - \omega^2 r\ i_r\}$$

$$= (\ \ddot{r} - \omega^2 r)\ i_r + (r\ \alpha + 2\ \dot{r}\ \omega\ )\ i_t \qquad .....(4.8)$$

In the above equations, ω and α are the first and second derivatives of θ. Both the $\underline{\omega}$ and $\underline{\alpha}$ vectors will be perpendicular to the plane of the paper. An important point to note is that there is no rigid link connecting O to P_1 but it is the position vector $\mathbf{R}_{p_1}$ which goes through the rotation as well as variation in its magnitude as the particle moves along the curve. These equations have been written under the assumptions

$$\underline{\omega} = \dot{\theta}\ i_z \quad \text{and} \qquad (4.9)$$

$$\underline{\alpha} = \dot{\omega}\ i_z \qquad (4.10)$$

Eqs. (4.7) and (4.8) take simple forms when the trajectory of the particle is a circle or a straight line. For circular trajectories we will have $\dot{r} = 0$ and $\ddot{r} = 0$. If we locate the point O in Fig. 4.1a at its center then using Eqs. (4.7) and (4.8) we get

$$\mathbf{V}_p = o\ i_r + r\omega i_t = r\ |\omega|\ i_t \qquad (4.11)$$

$$\mathbf{A}_p = (-\omega^2 r) i_r + (r\alpha) i_t \qquad (4.12)$$

These vectors are shown in Figs 4.2a and 4.2b. For the velocity,

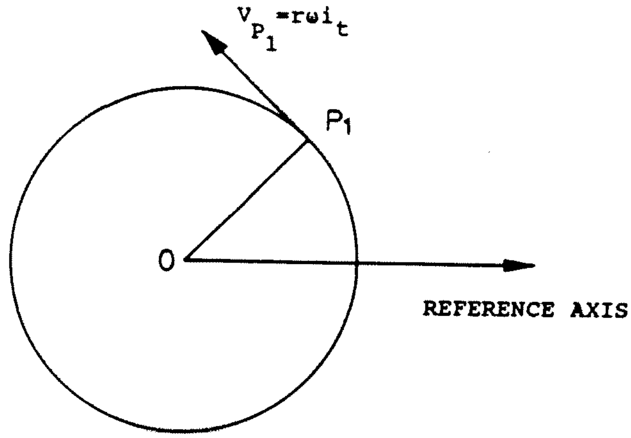

FIG. 4.2(a)

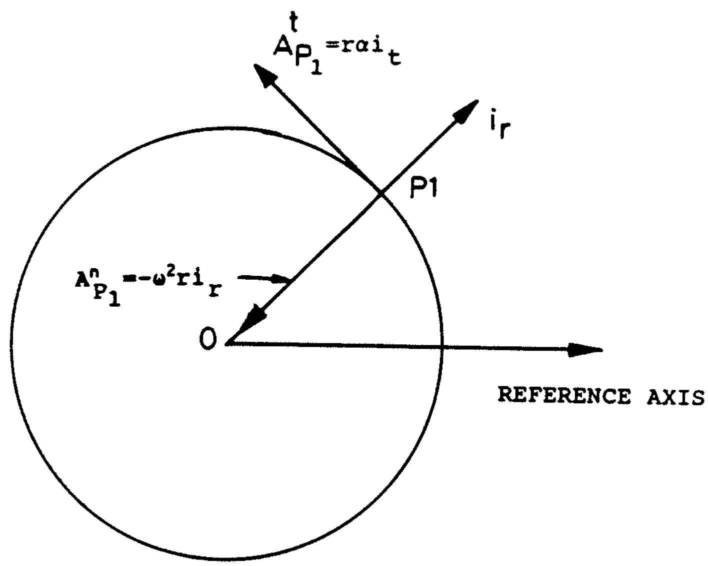

FIG. 4.2(b)

there is only one component along the tangent whereas for the acceleration there are two components, one along the tangent and the other towards the center (it is also called the centripetal acceleration). It shows that even if the particle moves with a constant angular velocity i.e $\alpha = 0$, still there will be a normal component of the acceleration.

On the other hand, if the trajectory is a straight line as shown in Fig. 4.3a, we can always align our reference axis to coincide with the trajectory as shown in the Fig 4.3b. If we do so, we can write

$$\dot{\theta} = \ddot{\theta} = 0 \quad \text{or } \dot{\theta}\ (\underline{\omega}) = \underline{\alpha} = \underline{0} \tag{4.13}$$

Now if we substitute Eq. (4.13) in Eqs (4.7) and (4.8) we would get

$$\mathbf{V}_p = \dot{r}\, i_r = V i_r \quad \text{and} \tag{4.14}$$

$$\mathbf{A}_p = (\ddot{r})\, i_r = a\, i_r \tag{4.15}$$

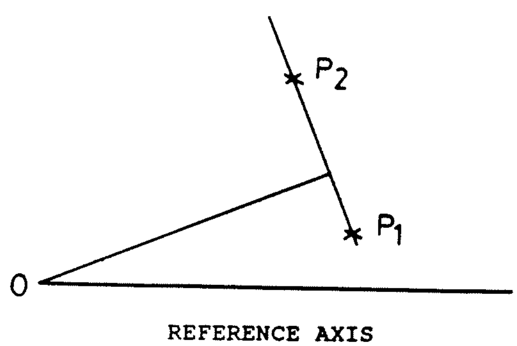

FIG. 4.3(a)

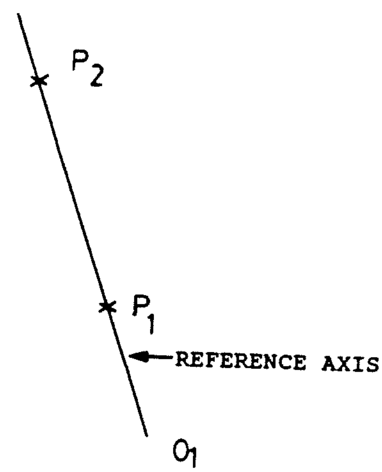

FIG. 4.3(b)

where V and a are the magnitudes of velocity and acceleration of the particle.

In general, the equations of trajectories may be more complicated than that of a straight line or a circle. To use Eqs (4.7) and (4.8), we should know the expressions for $\frac{d\theta}{dt}$, $\frac{d^2\theta}{dt^2}$, $\frac{dr}{dt}$, and $\frac{d^2r}{dt^2}$. The evaluation of these derivatives is very simple if the equation of the curve is given in time-parametric form i.e $r=f_1(t)$ and $\theta = f_2(t)$.

For example, suppose we are given

$$\theta = t^{2/3} \text{ and}$$

$$r = 100\ t^2$$

In this case we will have

$$\dot{r} = 200\ t,$$

$$\ddot{r} = 200,$$

$$\dot{\theta} = 2/3\ t^{-1/3}$$

and

$$\ddot{\theta} = \frac{-2}{9}\ t^{-4/3}$$

All these derivatives can be evaluated at any instant of time and substituted in Eqs (4.7) and (4.8). In another case, the equation of the curve can be given as $r = f(\theta)$. In this case, we would have to use the chain rule of calculus and the following relationships would be valid:

$$\dot{r} = \frac{df(\theta)}{d\theta}\dot{\theta} \text{ and} \qquad (4.16)$$

$$\ddot{r} = \frac{d^2f(\theta)}{d\theta^2}\dot{\theta} + \frac{df(\theta)}{d\theta}\ddot{\theta} \qquad (4.17)$$

Out of four quantities r, $\dot{r}$, $\dot{\theta}$ and $\ddot{\theta}$, if two are known then the other two can be calculated using these two equations. One other possibility that can commonly be encountered is that the equation of the curve is given in terms of cartesian coordinates. This equation can easily be converted into the polar coordinates using $x = r\cos\theta$, and $y = r\sin\theta$.

Finally, in certain situations it may be advantageous to think in terms of normal and tangential components of the acceleration which are in mutually perpendicular directions. If one wants to know these components then, the first thing would be to use Eqs. (4.7) and (4.8) to calculate the accelerations along i_r and i_t, and then resolve these vectors along the radial (towards the center of curvature) and tangential directions using the transformation matrices discussed in Chapter 2. However, if we set up the reference axis passing through the center of curvature, as was done in the case of a circle, the second step of transforming these accelerations into radial and tangential directions can be avoided. In that case, our origin should be at the center of curvature and the reference axes, parallel to the radial and tangential directions. Thus, i_r will be along the radial direction and i_t along the tangential direction.

4.3 The Derivation of the Acceleration Equation Involving Moving Coordinate System

The path of a particle B on a moving coordinate system is shown in the Fig 3.6. The coordinate system $(X_1 - O_1 - Y_1)$ itself is rotating with an angular acceleration $\underline{\alpha}$ and the acceleration of its origin is represented by A_{O_1}. The center of curvature of this path (the apparent path) is C. The actual path of the point B in the (X, O, Y) system, the frame, will be quite different. The purpose of this derivation is to relate the absolute acceleration of B to its apparent acceleration, i.e. the acceleration observed from the (X-O-Y) system. The point B is a moving point, which for the sake of clarity, can be thought of as a person jogging on the deck of a ship floating in the ocean. There is another point B_1 which can be considered as another person standing, not moving, on the deck. The system (X-O-Y) is on the shore and is considered as the inertial coordinate system.

Now we will derive the acceleration equation for B, the person who is jogging. In this situation B_1 is a fixed point with respect to (X_1- O_1 -Y_1) system but B is a moving point. Therefore, we can write

$$\frac{d}{dt}(x_{B_1}) = 0$$

$$\frac{d}{dt}(x_B) = x_{B/B_1}$$

$$x_B = x_{B_1}$$

$$Y_B = Y_{B_1}, \text{ and}$$

$$\mathbf{R}_{B_1 O_1} = \mathbf{R}_{B O_1} = (x_B\, i_1 + y_B\, j_1) = (x_{B_1} i + y_1 j) \qquad (4.16)$$

The position vector of B_1 will be

$$\mathbf{R}_{B_1} = \mathbf{R}_{O_1} + \mathbf{R}_{B_1 O_1} + \mathbf{R}_{B/B_1}$$

$$\mathbf{R}_{B_1} = \mathbf{R}_{O_1} + (x_B i_1 + y_B j_1) \qquad (4.17)$$

Differentiating this equation once, we get

$$V_{B_1} = \dot{R}_{B_1} = \dot{R}_{O_1} + \frac{d}{dt}(x_B i_1 + y_B j_1)$$

$$= \dot{R}_{O_1} + (\dot{x}_B i_1 + \dot{y}_B j_1) + (\frac{di_1}{dt} x_B + y_B \frac{dj_1}{dt})$$

$$= \dot{R}_{O_1} + (\dot{x}_B i_1 + \dot{y}_B j_1) + \underline{\omega} x(x_B i_1 + y_B j_1) \tag{4.18}$$

$$= (V_{O_1})_{XYZ} + (V_{B/B1})_{X_1 Y_1 Z_1} + \underline{\omega} \times (R_{BO_1})_{X_1 Y_1 Z_1} \tag{4.19}$$

The first term above is the velocity of point O_1; the second is the velocity of B as seen by B_1 who is stationary with respect to the ship. It is important to note that this velocity of B is seen by a person who can be considered coincident with B. In fact we can rearrange Eq. (4.19) and write

$$V_{B_1} = \{(V_{O_1})_{XYZ} + \underline{\omega} \times (R_{B_1 O_1})_{XYZ} \} + (V_{B/B1})_{X_1 Y_1 Z_1}$$

$$= (V_{B_1})_{XYZ} + (V_{B/B1})_{X_1 Y_1 Z_1}$$

Here, we are defining the velocity of B_1 and then vectorially adding the apparent velocity of B with respect to B_1

Differentiating Eq. (4.18) once again we obtain

$$\dot{V}_{B_1} = A_{B_1} = \ddot{R}_{O_1} + \{(\ddot{x}_B i_1 + \ddot{y}_B j_1) + (\frac{di_1}{dt} \dot{x}_B + \dot{y}_B \frac{dj_1}{dt})\}$$

$$+\{\dot{\underline{\omega}} \times (x_B i_1 + y_B j_1) + \underline{\omega} \times (\dot{x}_B i_1 + \dot{y}_B j_1) + \underline{\omega} \times (x_B \dot{i}_1 + y_B \dot{j}_1)\}$$

$$= \ddot{R}_{O_1} + \{ A_{B/B1} + \omega \times (\dot{x}_B i_1 + \dot{y}_B j_1)\}$$

$$+\{\dot{\underline{\omega}} \times (x_B i_1 + y_B j_1) + \underline{\omega} \times (\dot{x}_B i_1 + \dot{y}_B j_1) + \underline{\omega} \times(\underline{\omega} \times (x_B i_1 + y_B j_1)\} \quad \ldots\ldots\ldots(4.21)$$

$$= \ddot{\mathbf{R}}_{O_1} + \mathbf{A}_{B/B1} + 2\,\omega\, V_{B/B1} \angle\,(\,\theta_{V_{B/B1}} + 90)$$

$$+ \alpha\, R_{BO_1} \angle\,(\theta_{BO_1} + 90) + \omega^2\, R_{BO_1} \angle\,(\theta_{BO} + 180) \qquad \ldots\ldots(4.22)$$

The first is the acceleration of the point O_1; the second term will have two components as explained in the Section 4.1 (Eq. (4.8)) has to be used. We can also use the normal and tangential acceleration concept. The third term is the Coriololis acceleration and is explained in detail in the next section. The fourth term is the tangential component of the acceleration difference of B_1 with respect to O_1. Finally, the fifth term is the normal acceleration difference of B_1 with respect to O_1.

We can use $\mathbf{R}_{BO_1} = \mathbf{R}_{BO_1}$ in Eq. (4.17) and arrive at

$$\mathbf{A}_B = \{\, (\mathbf{A}_{O_1})_{XYZ} + \underline{\alpha} X\ (R_{B_1O})_{XYZ} + \underline{\omega}\ x\ (\underline{\omega}\ x\ (\mathbf{R}_{B_1O})_{XYZ}\}$$

$$+ 2\ \underline{\omega}\ x\ (\mathbf{V}_{B/B_1})_{XYZ} + (\mathbf{A}_{B/B1})_{XYZ} \ \ldots(4.23)$$

$$= \mathbf{A}_{B_1} + 2\ \underline{\omega}\ x\ (V_{B/B_1})_{XYZ} + \mathbf{A}_{B/B1} \quad \ldots(4.24)$$

If we compare Eqs. (4.19) and (4.24) we will find that it is not sufficient to add the apparent acceleration of B with respect to B_1, to the acceleration of B_1, to obtain the acceleration of B; one also has to add the Coriolis term. The computations, involved are the cross products of vectors as well as their additions which can easily be done using the software mentioned in Chapter 2. In any case, all the vectors in either Eq. (4.23) or Eq. (4.24) must be expressed in the same coordinate system for adding. One can use the rotational transformation matrices discussed in Chapter 2.

4.4 Coriolis Acceleration

The movement of a slider along a rotating rod is shown in Fig 4.4a. The rod is rotating with uniform angular velocity ω i.e $\alpha = 0$ and the slider moves outwards with a uniform velocity V relative to the rod i.e $V_{P_2/P_1} = v \angle 90^0$ at any time t. We can use the following relationships from planar geometry:

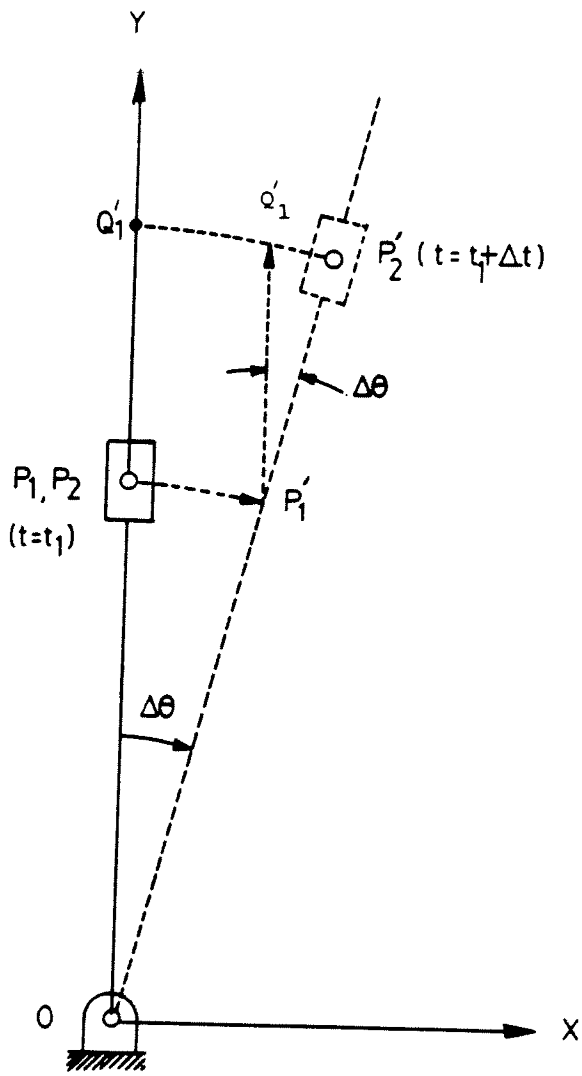

FIG. 4.4(a)

$$\text{Arc } Q_1' P_2' = \text{Arc } Q_1 P_2' - \text{Arc } Q_1 Q_1',$$
$$= \text{Arc } Q_1 P_2' - \text{Arc } P_1 P_1'$$
$$= Q_1O\,\Delta\theta - P_1O\,\Delta\theta = (Q_1O - P_1O)\Delta\theta$$
$$= Q_1P_1\,\Delta\theta$$
$$= (V\,\Delta t)\,\Delta\theta \qquad (4.25)$$

For small angle $\Delta\theta$, arc $Q_1' P_2'$= chord $Q_1' P_2'$. If we analyze the movements of the slider as well as the rod, we will find that the slider moves a distance equal to $P_1 Q_1 = P_1' Q_1'$, in the time Δt when the rod changes its position from Q_1O to P_2' O. Thus if we add the displacements $\mathbf{R}_{P_1'P_2}$ and $\mathbf{R}_{Q_1'P_1}$, we will obtain $\mathbf{R}_{Q_1'P_2}$. These two displacements are due to $\underline{\omega}$ and $\mathbf{V}_{P_2/P_1}$ respectively. The remaining displacements vector $\mathbf{R}_{p_2'q_1}$ is due to the Coriolis acceleration, $\mathbf{A}_c$. If we write the expression for displacement due to this acceleration, we will have

$$\text{chord } Q_1' P_2' = 1/2\, A_c (\Delta t)^2 \qquad (4.26)$$

Equating the right hand side terms of Eqs. (4.25) and (4.26), we will have

$$(1/2)\, A_c(\Delta t)^2 = (V\,\Delta t)\Delta\theta \qquad (4.27)$$

For small time interval i.e $\Delta t \longrightarrow 0$ this equation can be rewritten as

$$A_c = 2V \lim_{\Delta t \longrightarrow 0} \left(\frac{\Delta\theta}{\Delta t}\right) = 2|V|\,\omega \qquad (4.28)$$

In the vector form we can write

$$\mathbf{A}_c = 2\,\underline{\omega} \times \mathbf{V}_{P_2/P_1} \qquad (4.29)$$

The magnitude of the Coriolis acceleration is given by Eq. (4.28) and the direction by Eq. (4.29). To know the direction, we simply rotate the apparent velocity vector V_{P_2/P_1} in the direction of ω as shown in Fig. 4.4b. Another important fact to note is that the Coriolis acceleration can be calculated from the quantities known from the velocity analysis only; they are the angular velocity $\underline{\omega}$ and the apparent velocity V_{P_2/P_1} which were discussed in Chapter 2.

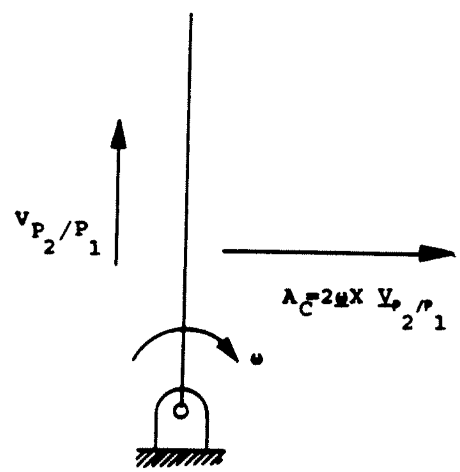

FIG. 4.4(b)

4.5 Direct Contact and Rolling Contact

We have seen in Chapter 3 that there are two kinds relative motions between any two rigid bodies. There exists an apparent slipping velocity or no such slip occurs between the two bodies in direct contact. In rolling contact, no slip is possible which implies that the apparent velocity at such a contact point is zero. Let us derive here an expression for the apparent acceleration of a point in rolling contact.

Figure 4.5 shows a wheel (link 3) in rolling contact with a straight fixed link 2.

We can write

$$\mathbf{V}_{P_3} = \mathbf{V}_{P_2} = 0 \qquad \text{and } \mathbf{V}_{P_3/P_2} = 0$$

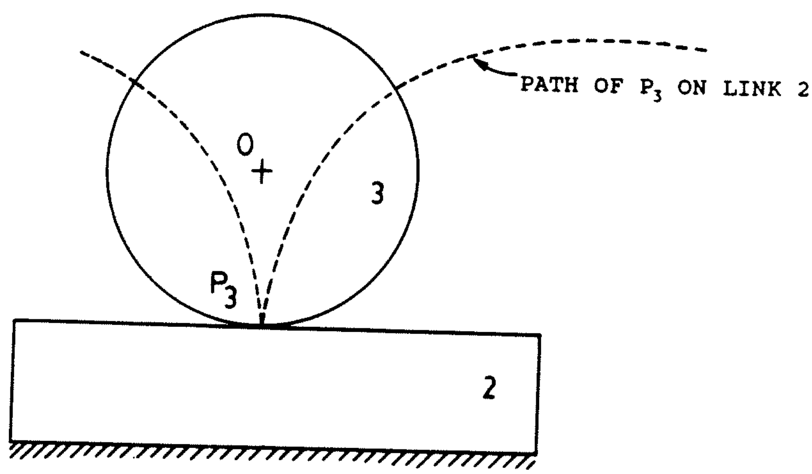

FIG. 4.5

Similarly, for the acceleration of P_3 we can write using Eq. (4.24)

$$\mathbf{A}_{p_3} = \mathbf{A}_{p_2} + A^r_{p_3/p_2} + 2\omega_2\, V_{p_3/p_2} \angle(\theta_{V_{p_3/p_2}} + 90^0)$$

$$\mathbf{A}_{p_3} = \mathbf{A}^r_{p_3/p_2} + \mathbf{A}_{p_2}$$

$$= (\mathbf{A}^n_{p_3/p_2})^r + (\mathbf{A}^t_{p_3/p_2})^r$$

$$= \frac{\left(V_{p_3/p_2}\right)^2}{\rho} \angle(\theta_n + 180) + (\mathbf{A}^t_{p_3/p_2})^r \angle\, \theta_n + 90 + \mathbf{A}_{p_2}$$

$$= 0 \angle(\theta_n + 180) + (\mathbf{A}^t_{p_3/p_2})\ (\theta_n + 90) + \underline{0}$$

Example 4.1: For the mechanism shown in Fig. 4.6a, the following data are given:

$\mathbf{R}_{AO_2} = 0.5 \angle 30^0$, $R_{CA} = 0.5 \angle 330^0$,

$\mathbf{R}_{BA} = 0.5 \angle 150^0$, $\alpha_2 = 150$ rad/sec^2 (cw)

and $\omega_2 = 30$ rad/sec (cω).

Find $\mathbf{V}_B$, $\mathbf{A}_B$ and α_3.

Solution

Velocity Analysis

Link 2

$$\mathbf{V}_A = \omega_2\, R_{AO_2} \angle(30 - 90) = 30 \times 0.5) \angle -60^0$$
$$= 15 \angle 300^0$$

Link 3

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

$$V_C \angle 0 = 15 \angle 300^0 + (\omega_3 R_{CA}) \angle (330^0 + 90^0)$$

$$15 \angle 0 = 15 \angle 300^0 + 15 \angle 60^0 \qquad \text{(Using program 11)}$$

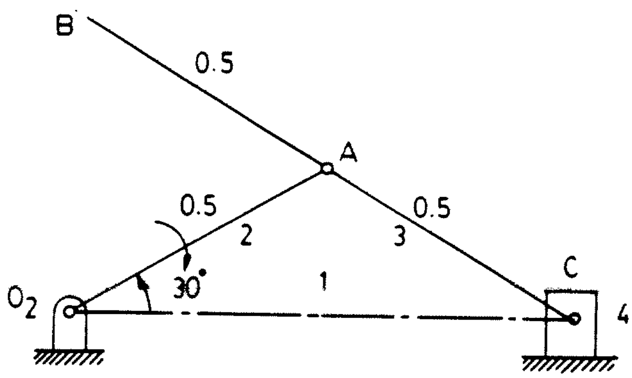

FIG. 4.6(a)

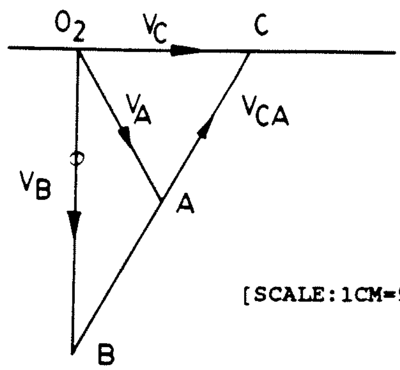

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FIG. 4.6(b)

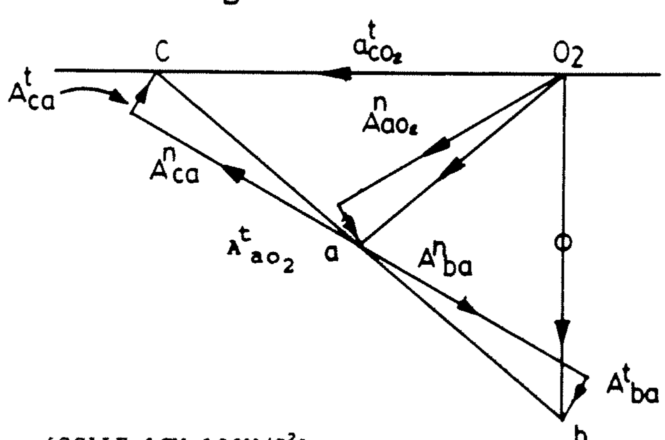

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FIG. 4.6(c)

Therefore,

$$\omega_3 = 15/0.5 = 30 \text{ rad/s} \quad \text{ccw}$$

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

$$= 15 \angle 300^0 + (30 \times 0.5) \angle (150^0 + 90^0) = 25.98 \angle 240^0$$

Acceleration Analysis

Link 2

$$\mathbf{A}^n_{AO_2} = \omega_2^2 R_{AO_2} \angle (\theta_{AO_2} + 180^0) = (30^2 \times 0.5)\angle(30^0 + 180^0)$$

$$= 450 \angle 210^0$$

$$\mathbf{A}^t_{AO_2} = \alpha \quad R_{AO_2} \angle (\theta_{AO_2} - 90^0) = (150 \times 0.5)\angle(30^0 - 90^0)$$

$$= 75 \angle -60^0 = 75 \angle 300^0$$

$$\mathbf{A}_A = 450 \angle 210^0 + 75 \angle 300^0 = 456.207 \angle 219.462^0$$

Link 3

$$\mathbf{A}_C = \mathbf{A}_A + \mathbf{A}^n_{CA} + \mathbf{A}^t_{CA}$$

$$A_C\angle 0 = 456.207 \angle 219.462 + \omega_3^2 R_{CA} \angle (330^0 + 180^0)$$
$$+ \alpha_3 R_{CA}\angle(330^0 + 90^0)$$

$$A_C\angle 0^0 = 456.207 \angle 219.462^0 + 30^2 x \ 0.5 \angle 150^0$$
$$+ \alpha_3 R_{CA}\angle(60^0)$$

Using program 11 we get

$$A_C = 704.426 \angle 180^0$$

$$\alpha_3 = 74.997/0.5 = 149.994 \text{ rad/s} \quad \text{ccw}$$

$$\mathbf{A}_B = \mathbf{A}_A + \omega_3^2 R_{BA} \angle (\theta_{BA} + 180) + \alpha_3 R_{BA}\angle (\theta_{BA} + 90)$$

$$= 456.207 \angle 219.462^0 + (30^2 x \ 0.5) \angle 330^0$$
$$+ (149.994 \times 0.5)\angle (150^0 + 90^0)$$

$$= 456.207 \angle 219.462^0 + 450 \angle 330^0 + 74.997 \angle 240^0$$

$$= 579.899 \angle 270^0$$ (Using program 15)

Graphical Solution:

Velocity Analysis: The velocity of the point A is calculated first and it will be equal to $\omega_2 \times R_{AO_2} = 30 \times 0.5 = 15$ m/s. Next we write the equations for links 3 and 4 for the point C as

$$\mathbf{V}_C = \underbrace{\overset{\surd\surd}{\mathbf{V}_A} + \overset{*\surd}{\mathbf{V}_{CA}}}_{\text{LINK 3}} = \underbrace{\overset{*\surd}{\mathbf{V}_{C4}}}_{\text{LINK 4}} \qquad \text{(case 2a)}$$

We draw lines parallel to $\overset{*\surd}{\mathbf{V}_C}$ from the tail of $\mathbf{V}_A$ and another line from the tip of $\mathbf{V}_A$ and another line from the tip of $\mathbf{V}_A$ (Perpendicular to R_{CA}). In this way obtain the point C. TO get the point B, we draw the image of the link BAC by extending the line CA in the velocity diagram in the proportion :

$$\mathbf{V}_{BC} \ / \ \mathbf{V}_{AC} = R_{BC} \ / \ R_{AC}$$

After locating the point B, we join it with the origin to get the absolute velocity of this point.

Acceleration Analysis: In the acceleration analysis, we start with the calculation of the normal and tangential components of the acceleration of A given by

$$A^n_{AO_2} = \left(\frac{V^2_{AO_2}}{AO}\right) \angle\ (\theta_{AO} + 180) = \frac{15^2}{0.5} \ \angle(30 + 180) \qquad \text{(b)}$$

$$A^t_{AO_2} = (\alpha_2 \ R_{AO_2}) \ \angle(\theta_{AO} \ - 90) = (150 \times 0.5) \ \angle(30\text{-}90) \qquad \text{(c)}$$

To calculate the acceleration of the point C we write equations for links 3 and 4 again which are

$$\mathbf{A}_C = \underbrace{\overset{\surd\surd}{\mathbf{A}^n_C} + \overset{*\surd}{\mathbf{A}^t_C}}_{\text{LINK 4}} = \underbrace{\overset{\surd\surd}{\mathbf{A}_A} + \overset{\surd\surd}{\mathbf{A}^n_C} + \overset{*\surd}{\mathbf{A}^t_C}}_{\text{LINK 3}} \qquad \text{(d)}$$

To draw Eq.(d), we draw a line of magnitude $\left(\frac{V_{CA}^2}{R_{CA}}\right)$ from the point 'a' (acceleration diagram) in the direction of $\mathbf{R}_{AC}$. Since the tangential component's ($\mathbf{A}_{CA}^t$) magnitude is not known, we just draw a line perpendicular to R_{AC} direction from the tip of the $\mathbf{A}_{CA}^n$ vector. Since the normal acceleration $\mathbf{A}_C^n = 0$, we simply draw a line parallel to the vector $\mathbf{A}_C^t$. The point of intersection gives the point C. If we measure the vector $\mathbf{A}_{CA}^t$ then we can obtain

$$\alpha_3 = \frac{A_{CA}^t}{R_{CA}} = \frac{85}{0.5} \text{ (scaled)} = 170 \text{ rad/s}^2 \qquad \text{(e)}$$

The acceleration of B can be calculated by two methods. In the first method we write

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t \qquad \text{(f)}$$

We know A_A from above and the other two terms will be

$$\mathbf{A}_{BA}^n = \omega_3^2 \ R_{BA} \angle(\theta_{BA} + 180) \qquad \text{(g)}$$

$$= \left(\frac{V_{BA}^2}{R_{BA}}\right) \angle(\theta_{BA} + 180)$$

$$= \left(\frac{15^2}{0.5}\right) \angle(150 + 180)$$

$$= 450 \angle 330^0$$

$$\mathbf{A}_{BA}^t = (\alpha_3 \ R_{BA}) \angle(\theta_{BA} + 90) \qquad \text{(h)}$$

$$= (170 \times 0.5) \angle(150 + 90) = 85 \angle 240^0$$

In this way all the three vectors in Eq. (f) can be added to locate the position of point B in the acceleration diagram.

In the second method we use the scaling factor in the acceleration image concept. We can see in Eqs. (g) and (h) above that $\mathbf{A}_{BA}^{n}$ and $\mathbf{A}_{BA}^{t}$ are at right angles to each other.

Thus we can write

$$|\mathbf{A}_{BA}| = \sqrt{|\mathbf{A}_{BA}^{n}|^2 + |\mathbf{A}_{BA}^{t}|^2}$$

$$= \sqrt{(\omega_3^2 R_{BA})^2 + \alpha_3^2 R_{BA}^2}$$

$$= R_{BA} \left(\sqrt{\omega_3^4 + \alpha_3^2}\right)$$

$$= R_{BA} \times SF \qquad \text{(i)}$$

Where the Sealing Factor (SF) in the case of the acceleration difference of two points on link i can be given as

$$(SF)_i = \sqrt{\omega_i^4 + \alpha_i^2} \qquad \text{(j)}$$

In this problem we can produce the line 'ca' in acceleration diagram such that

$$|\underline{A}_{BA}| = \left(\sqrt{30^4 + 170^2}\right) 0.5$$

$$= 457.957$$

to locate the point b. The acceleration $\mathbf{A}_B$ can be measured to be equal to 580 m/s^2 $\angle 271^0$.

Example 4.2

For the mechanism shown in Fig 4.7a the following data are given

$\mathbf{R}_{AO_2} = 0.040 \angle 120$, $\mathbf{R}_{BA} = 0.08 \angle 18.704$,

$\mathbf{R}_{O_4B} = 0.07 \angle 239.214$, $\mathbf{R}_{CA} = 0.044 \angle 352.13$

$\underline{\omega}_2 = 50$ rad/s (cw) $\underline{\omega}_3 = 33.75$ rad/s (cw)

$\underline{\omega}_4 = 44.286$ rad/s (cw)

Find $\mathbf{A}_C, \underline{\alpha}_3$ and $\underline{\alpha}_4$

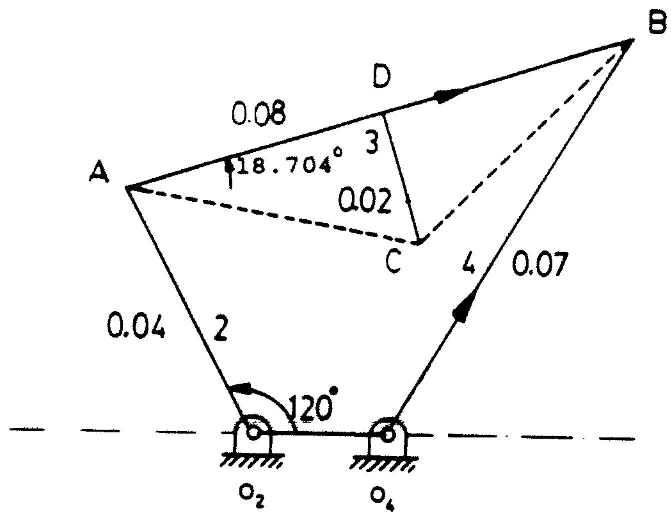
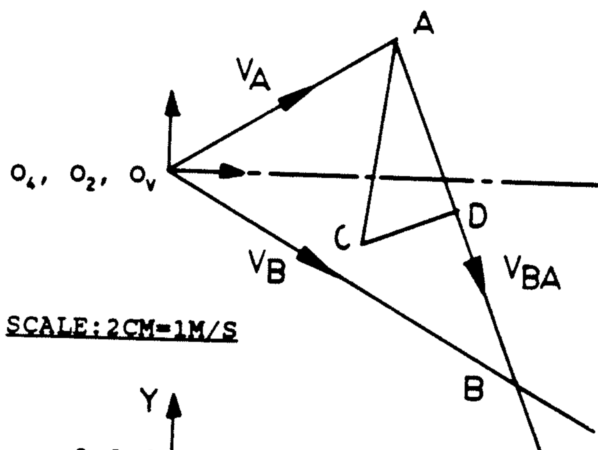
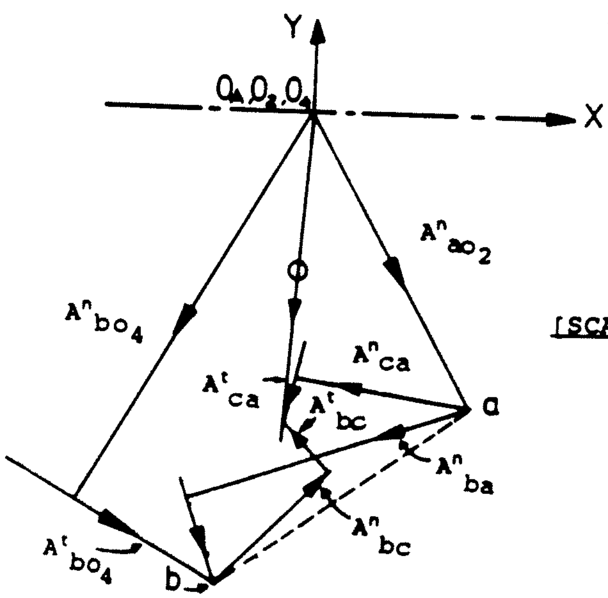


FIG. 4.7(a)



SCALE:2CM=1M/S

FIG. 4.7(b)



[SCALE:1CM=20M²/S]

FIG. 4.7(c)

Solution

Link 2

$$\mathbf{A}_A = \omega_2^2 R_{AO_2} = (50^2 \times 0.04)\angle 300^0 = 100\angle 300^0$$

Link 3

$$\mathbf{A}_B = \mathbf{A}_A + \omega_3^2 R_{BA}\angle 198.704 + \alpha_3 R_{BA}\angle 108.704$$

$$= 100 \angle 300^0 + 91.125 \angle 198.704^0 + (\alpha_3 R_{BA}) \angle 108.704^0$$

$$= 121.383 \angle 252.593^0 + (\alpha_3 R_{BA}) \angle 108.704^0 \quad .....(a)$$

Link 4

$$A_B = A_{0_2} + \omega_4^2 R_{BO_4} \angle 239.214^0 + \alpha_4 R_{BO_4} \angle 149.214^0$$

$$A_B = 0 + 137.287\angle 239.214^0 + \alpha_4 R_{BO_4} \angle 149.214^0 \quad .....(b)$$

Equating Eqs (a) and (b) we get

$$121.383 \angle 252.593 + (\alpha_3 R_{BA}) \angle 108.704^0 = 137.287 \angle 239.214^0 + (\alpha_4 R_{BO_4}) \angle 149.214^0$$

or,

$$34.021 \angle 3.568^0 + (\alpha_3 R_{BA}) \angle 108.704^0 = (\alpha_4 R_{BO_4}) \angle 149.214^0$$

Using program 11 we get

$$34.021 \angle 3.568^0 + (-29.555) \angle 108.704^0 = (-50.557)\angle 149.214^0$$

Therefore, we can calculate

$$\alpha_3 = \frac{-29.555}{0.08} = -369.428 \text{ rad/s}^2 \quad \text{cw},$$

$$\alpha_4 = \frac{-50.557}{0.07} = -722.243 \text{ rad/s}^2 \quad \text{cw},$$

we can write

$$\mathbf{A}_C = \mathbf{A}_A + \omega_3^2 R_{CA}\angle 172.139 + \alpha_3 R_{CA}\angle (352.139^0 - 90^0)$$

$$= 100 \angle 300^0 + 33.75^2 \text{x } 0.044 \angle 172.139^0 + (369.438 \times 0.044) \angle 262.139^0$$

$= 100 \angle 300^0 + 50.119 \angle 172.139^0 + 16.255 \angle 262.139^0$

$= 95.868 \angle 268.882^0$ (Using program 15)

Graphical Solution:

Acceleration of A:

$$\mathbf{A}_A = \mathbf{A}_{O_2} + \mathbf{A}^n_{AO_2}$$

$$\mathbf{A}^n_{AO_2} = \frac{V_A^2}{AO_2} = 4 / 0.4 = 100 \text{ m/s}^2$$

Acceleration of B:

We can write equations for links 3 and 4 as

$$\overset{\surd\surd}{\mathbf{A}_B} = \overset{\surd\surd}{\mathbf{A}_A} + \overset{\surd\surd}{\mathbf{A}^n_{BA}} + \overset{*\surd}{\mathbf{A}^t_{BA}}$$

$$= \overset{\surd\surd}{\mathbf{A}^n_{BO_4}} + \overset{*\surd}{\mathbf{A}^t_{BO_4}} \qquad \text{(a)}$$

$$\mathbf{A}^n_{BA} = \frac{V^2_{BA}}{BA} = (2.7)^2/0.08 = 91.1 \text{ m/s}^2 \angle 198^0$$

$$\mathbf{A}^n_{BO_4} = \frac{V^2_B}{BO_4} = (3.1)^2/0.07 = 137.2 \text{ m/s}^2 \angle 240^0$$

We plot the vectors which are known in Eq. (a) and the unknown tangential components we can scale. The results are:

$$| A^t_{BA} | = 28$$

$$\alpha_3 = \frac{| A^t_{BA} |}{R_{BA}} = \frac{28}{0.08} = -350 \text{ rad/s}^2 \text{ cw}$$

$$| A^t_{BO_4} | = 48$$

$$\alpha_4 = \frac{A^t_{BO_4}}{R_{BO_4}} = \frac{48}{0.07} = -685.7 \text{ rad/s}^2 \text{ cw}$$

Acceleration of C:

It is given with respect to A and B and the equations will be

$$\mathbf{A}_C = \overset{\surd\surd}{\mathbf{A}_A} + \overset{\surd\surd}{\mathbf{A}^n_{CA}} + \dot{\mathbf{A}}^t_{CA} = \mathbf{A}_B + \mathbf{A}^n_{CB} + \dot{\mathbf{A}}^t_{CB} \qquad \text{(b)}$$

We know the following:

$$A^n_{CA} = \frac{V^2_{CA}}{CA} = (1.5)^2/0.045 = 50.0 \text{ m/s}^2$$

$$A^n_{CB} = \frac{V^2_{CB}}{CB} = (1.5)^2/0.045 = 50.0 \text{ m/s}^2$$

Similarly we can add the tangential acceleration also because α_3 and α_4 are known from above. Acceleration of C can also be traced according to its relative position in the displacement diagram by plotting the acceleration image of the link 3. It is easy because we know the locations of two points A and B. If we do this, we will find

$$\mathbf{A}_C = 100 \text{ m/s}^2 \angle 266^0$$

Example 4.3

For the velocity analysis of the mechanism shown in Fig 3.24 was performed in Example 3.3. Find α_4, α_3, $\mathbf{A}_{B_3}$ and $\mathbf{A}_{B_4}$ when α_2 = 10 rad/s^2(ccw). Use the other results obtained in the example.

Solution

Link 2

$$\mathbf{A}_{A_2} = \mathbf{A}_{O_2} + \omega_2^2 R_{A_2O_2} \angle (45^0 + 180^0) + \alpha_2 R_{AO_2} \angle (45^0 + 90^0)$$

$$= 40^2 x\ 5 \angle 225^0 + (10 \times 5) \angle 135^0$$

$$= 8000 \angle 225^0 + 50 \angle 135^0$$

$$= 8000.156 \angle 224.642^0$$

Link 3

$$\mathbf{A}_{B_3} = \mathbf{A}_{A_3} + \omega_3^2 R_{BA} \angle (80.881^0 + 180^0) + \alpha_3 R_{BA} \angle (80.881^0 + 90^0)$$

$$= 8000.156 \angle 224.642^0 + (-7.060)^2 7 \angle 260.881^0$$

$$+ (\alpha_3\ 7) \angle 170.881^0$$

$$= 8283.980 \angle 226.069^0 + (7\ \alpha_3) \angle 170.881^0 \qquad \text{---(a)}$$

Link 4*

*It is better to locate the moving coordinate system on the guiding link (link 4) and let the guided link (slider) move on the guiding link. In this way the error in the Coriolis term is avoided. The apparent path is quite clear.

$$\mathbf{A}_{B_3} = \mathbf{A}_{B_4} + 2\ \omega_4 V_{B3/B4} \angle (\theta_{v_{B3/B4}} + 90^0) + \mathbf{A}_{B3/B4}$$

$$= (-7.060)^2 (18.382) \angle (145.470 + 2(-7.060)(168.315) \angle 170.881 + 90)^*$$

$$+ A^t_{B3/B4} \angle 350.881 + (\alpha_4 R_{B4O_1}) \angle (145.470 + 90)$$

$$\mathbf{A}_{B3} = 916.225 \angle 145.470 + 2376.608 \angle 80.881 + A^t_{B3/B4} \angle 350.881$$

$$+ (18.382\ \alpha_3) \angle 235.47$$

$$= 2890.763 \angle 97.517 + A^r_{B3/B4} \angle 350.881$$

$$+ (18.382\ \alpha_3) \angle 235.47 \qquad \text{(b)}$$

Equating Eqs. (a) and (b) we obtain

$$8283.980 \angle 226.069 + (7\ \alpha_3) \angle 170.881 = 2890.763 \angle 97.517 + A^t_{B3/B4} \angle 350.881 + (18.382\ \alpha_3) \angle 235.47$$

or

$$10335.84 \angle 238.703^0 + \alpha_3 (7 \angle 170.881^0 + 18.382 \angle 235.47^0) = A^t_{B3/B4} \angle 350.881$$

or,

$$10335.84 \angle 238.703 + \alpha_3\ 22.301 \angle 219.00 = A^t_{B3/B4} \angle 350.881$$

If use program 11 we will get

$10335.84 \angle 238.703^0 + (-12855.24) \angle 219.00^0 = 4680.346 \angle 350.881^0$

Therefore we can write

$$A^t_{B3/B4} = 4680.346 \angle 350.881$$

$$\alpha_4 = \alpha_3 = -12835.24 / 22.301 = -576.442 \text{ rad/s cw}$$

Example 4.4

The velocity analysis of the mechanism in Fig 2.28a was performed in Example 3.4. Given that

$\omega_2 = 30$ rad/s ccw, $\omega_3 = 0.297$ rad/s ccw,

$\omega_4 = -0.639$ rad/s cw, $\omega_5 = -0.682$ rad/s cw,

$\mathbf{R}_{AO_1} = 2 \angle 100$, $\mathbf{R}_{AC} = 20.099 \angle 15.709$

$\mathbf{R}_{GC} = 39.916 \angle 5.660$, $\mathbf{R}_{DC} = 41.284 \angle 25.419$,

$\mathbf{R}_{ED} = 20 \angle 330$, $\mathbf{R}_{FG} = 20 \angle 330$

$\alpha_2 = 10$ rad/s (ccw) , and $V_{A2/A3} = -59.702 \angle 15.709$

$= 59.702 \angle 105.709^0$

$= 59.702 \angle 195.709$

Find α_3 , α_4 , $\mathbf{A}_D$ and $\mathbf{A}_E$

Solution:

Link 2

$$\mathbf{A}_{A2} = \omega_2^2 R_{AO_1} \angle (100^0 + 180^0) = (30^2 x2) \angle 280^0 = 1800 \angle 280^0 \quad \text{(a)}$$

Link 3

$$\mathbf{A}_{A_2} = \mathbf{A}_{A_3} + 2\,\omega_3 V_{A2/A3} \angle (\theta_{v_{A2/A3}} + 90^0) + \mathbf{A}^t_{A_2/A_3}$$

$$= \omega_3^2 R_{A_3C} \angle 195.709^0 + \alpha_3 R_{A_3C} \angle 105.709^0$$

$$+ (2 \times \omega_3 \times 59.702) \angle (195.709^0 + 90^0) + \mathbf{A}^t_{A_2/A_3}$$

$$= (0.297^2 \times 20.099) \angle 195.709^0 + (\alpha_3 \times 20.099^0)) \angle 105.709^0$$

$$+ (2 \times 0.297 \times 59.702) \angle 285.719^0 + \mathbf{A}^t_{A_2/A_3}$$

$$= 1.773 \angle 195.709^0 + (20.099\ \alpha_3) \angle 105.709^0$$

$$+35.463 \angle 285.719^0 + \mathbf{A}^t_{A_2/A_3} \quad \text{----(b)}$$

Equating Eqs. (a) and (b)

$$1800 \angle 280^0 = = 1.773 \angle 195.709^0 + (20.099\ \alpha_3) \angle 105.709^0$$

$$+35.463 \angle 285.719^0 + \mathbf{A}^t_{A2/A3}$$

or,

$$1764.537 \angle 279.943 = (20.099\ \alpha_3) \angle 105.709 + A^t_{A2/A3} \angle 15.709$$

Using program 2 we get

$$1764.537 \angle (279.943) = -1755.609 \angle 105.709 + (-177.276) \angle 15.709$$

Therefore, we can write

$$\alpha_3 = -1755.609 / 20.099 = -87.348 \text{ rad/s cw}$$

$$= -87.348 \text{ k}$$

$$A^t_{A2/A3} = 177.276 \angle 195.709$$

$$\mathbf{A}_D = \omega_3{}^2 R_{DC} \angle (25.419^0 + 180^0) + \alpha_3 R_{DC} \angle (25.419^0 + 90^0)$$

$$= (0.297)^2 (41.284) \angle 205.419^0 + (-87.384)(41.284) \angle 115.419^0$$

$$= 3.642 \angle 205.419^0 + 3607.571 \angle 295.419^0$$

$$= 3607.573 \angle 295.361^0$$

$$\mathbf{A}_E = \mathbf{A}_D + \mathbf{A}^n_{ED} + \mathbf{A}^t_{ED}$$

$$\mathbf{A}_E = \underline{A}_D + \omega_4^2 R_{ED} \angle (\theta_{ED} - 180^0) + \alpha_4 R_{ED} \angle (\theta_{ED} + 90^0)$$

$$A_E \angle 0 = 3607.573 \angle 295.361^0 + \{(-0.639)^2 \times 20\} \angle (330^0 - 180^0)$$

$$+ (\alpha_4\ 20^0) \angle (330^0 + 90^0)$$

$$= 3607.573 \angle 295.361^0 + 8.166 \angle 150^0 + (20\ \alpha_4) \angle 60^0$$

$$= 3600.857 \angle 295.287^0 + (20\ \alpha_4) \angle 60^0$$

If we use program 11 we will get

$$3417.863 \angle 0^0 = 3600.857 \angle 295.287^0 + 3759.499 \angle 60^0$$

Therefore, we obtain from above

$$\mathbf{A}_E = 3417.863 \angle 0^0$$

$$\alpha_4 = (3759.499)/ 20 = 187.975 \text{ rad/s ccw}$$

Example 4.5:

The velocity analysis of the mechanism shown in Fig.2.29(a) was performed in Example 3.8. For this mechanism it is given that

$\alpha_2 = -7\ k$,	$\omega_4 = -3.544\ k$,
$\omega_2 = 5\ k$,	$\omega_5 = 4.392\ k$,
$\omega_3 = -3.393\ k$,	$\omega_6 = -2.878\ k$

Find α_3, α_4, α_5, α_6, $\mathbf{A}_D$ and $\mathbf{A}_H$.

Solution:

Link 2

$$\mathbf{A}_{A2} = (\omega_2^2\ R_{A_2O_1}) \angle(\theta_{AO_1} + 180^0) + \alpha_2 R_{A_2O_1} \angle(\theta_{AO_1} + 90^0)$$

$$= (5^2 x\ 2.5) \angle (150^0 + 180^0) + (-7\ x\ 2.5) \angle (150^0 + 90^0)$$

$$= 62.5 \angle 330^0 + 17.5 \angle 60^0$$

$$= 64.904 \angle 345.642^0$$

Link 3

$$\mathbf{A}_B = \mathbf{A}_A + \omega_3^2\ R_{BA} \angle(\theta_{BA} + 180^0) + \alpha_3 R_{BA} \angle(\theta_{BA} + 90^0)$$

$$\mathbf{A}_B = 64.904 \angle 345.642^0 + (-3.393)^2 10 \angle(214.658^0 + 180^0)$$

$$+ (10\ \alpha_3) \angle (214.658^0 + 90^0) \qquad \text{(a)}$$

Link 4

$$\mathbf{A}_B = \omega_4^2 R_{BO_2} \angle(\theta_{BO_2} + 180) + \alpha_4 R_{BO_2} \angle(\theta_{BO_2} + 90)$$

$$= (-3.544)^2 7 \angle(97.713 + 180) + 7\,\alpha_4 \angle(97.713 + 90) \qquad \text{(b)}$$

Equating Eqs (a) and (b) we get

$$64.904 \angle 345.642 + 115.125 \angle 34.658 + 10\,\alpha_3 \angle 304.658$$

$$= 87.919 \angle 277.713 + 7\,\alpha_4 \angle 187.713$$

Using program 15 we get

$$(199.704) \angle 43.118 + 10\,\alpha_3 \angle 304.658 = 7\,\alpha_4 \angle 187.713$$

Using program 11 we obtain

$$(199.704) \angle 43.118 + (129.789) \angle 304.658 = (-221.586) \angle 187.713$$

Now we can write

$$\alpha_3 = \frac{129.789}{10} = 12.979 \text{ rad/sec} \quad \text{ccw.}$$

$$\alpha_4 = \frac{-221.586}{7} = 31.655 \text{ rad/sec} \quad \text{cw.}$$

Links 3 and 5

$$\mathbf{A}_{C_5} = \mathbf{A}_{C_3} = \mathbf{A}_A + \omega_3^2 R_{CA} \angle (\theta_{CA} + 180) + \alpha_3 R_{CA} \angle(\theta_{CA} + 90)$$

$$\mathbf{A}_{C_3} = 64.704 \angle 345.642^0 + (3.393)^2 (7.496) \angle (79.826^0 + 180^0)$$

$$+ \; 12.979 \times 7.496 \angle (79.826^0 + 90^0)$$

$$= 64.704 \angle 345.642^0 + (86.297) \angle 259.286^0 + 97.291 \angle 169.826^0$$

$$= 96.953 \angle 239.692^0 \qquad \text{(Using program 15)}$$

Therefore we have

$$\mathbf{A}_{C_5} = \mathbf{A}_{C_3} = 96.953 \angle 239.692^0 \qquad \text{(c)}$$

Links 5 and 6

We locate the moving coordinate system on the link 6, the guiding link, and write the equation

$$A_{C_5} = \mathbf{A}_{C_6} + \mathbf{A}^C_{C5/C6} + \mathbf{A}^t_{C5/C6}$$

$$= \omega_6^2 R_{CF} \angle (14.032^0 + 180^0) + \alpha_6 R_{CF} \angle (14.032^0 + 90^0)$$

$$+ 2\omega_6 V_{C5/C6} \angle (\theta_{V_{C_5/C_6}} + 90\ ) + A^t_{C_5/C_6} \angle 10.032^0$$

Substituting the value for $\mathbf{A}_{C_5}$ from above, we get

$$96.953 \angle 239.692^0 = \{(-2.878)^2 \times 6.422\} \angle 194.032^0$$

$$+ (\alpha_6\ 6.422) \angle 104.032^0$$

$$+ 2\ (-2.878)(14.599) \angle (90^0 + 10.032^0) + A^t_{C5/C6} \angle 10.032^0$$

$$96.953 \angle 239.692^0 = 53.193 \angle 194.032^0 +$$

$$(\alpha_6\ 6.422) \angle 104.032^0 +$$

$$84.032 \angle (280.032^0) + A^t_{C5/C6} \angle 10.032^0$$

or,

$$16.901 \angle 135.041^0 = (6.422\ \alpha_6) \angle 104.032^0 + A^t_{C5/C6} \angle 10.032^0$$

Using program 2 we get

$$16.901 \angle 135.041 = (13.877) \angle 104.032 + (\ -8.728) \angle 10.032$$

Therefore we obtain,

$$\alpha_6 = \frac{13.877}{6.422} = 2.161 \text{ rad/s}^2 \text{ (ccw)}.$$

$$\mathbf{A}^t_{C5/C6} = -8.728 \angle 10.032 = 8.728 \angle 190.032$$

$$\mathbf{A}_{D_6} = (\omega_6^2\ R_{DF}) \angle (30.945^0 + 180^0) + (\alpha_6\ R_{DF}) \angle (30.945^0 + 90^0)$$

$$= \{(-2.878)^2 (6.858)\} \angle 210.945^0 + (2.161)(6.858) \angle 120.945^0$$

$$= 54.568 \angle 210.945^0 + 14.820 \angle 120.945^0$$

$$= 56.545 \angle 195.751^0 \qquad \text{(d)}$$

$$\mathbf{A}_{D_5} = \mathbf{A}_{D_6} + \mathbf{A}^r_{D_5/D_6} + (2\ V_{D_5/D_6}\ \omega_6) \angle (\theta_{V_{D_5/D_6}} + 90^0)$$

$$A_{D_5} = 56.545 \angle 195.751^0 + A^t_{D_5/D_6}\ (\ 100.032^0 + 180^0) \qquad \text{(e)}$$

It is given that $\underline{V}_{D_5/D_6} = 0$ because of the rolling contact.

We can also write an equation on link 5 as

$$A_{D_5} = A_{C_5} + \omega_5^2 R_{D_5C_5} \angle (100.032^0 + 180^0) + \alpha_5 R_{D_5C_5} \angle (100.032^0 + 90^0) \quad (f)$$

Substituting for A_{C_5} from Eq. (c) and equating Eqs. (e) and (f) we get

$$8.728 \angle 190.032 + \{ (4.392)^2 x\ 2 \} \angle 280.032 + 2\ \alpha_5 \angle 190.032$$
$$= 56.545 \angle 195.751 + A^t_{D_5/D_6} \angle 280.032$$

$$8.728 \angle 190.032 + 38.579 \angle 280.032 + 2\ \alpha_5 \angle 190.032 = 56.545 \angle 195.751$$
$$+ A^t_{D5/D6} \angle 280.032$$

$$57.836 \angle 335.308 + 2\ \alpha_5 \angle 190.032 = A^t_{D5/D6} \angle 280.032$$

Using program 11 we get

$$57.836 \angle 335.308 + 47.536 \angle 190.032 = 32.945 \angle 280.032$$

$$A^t_{D5/D6} = 32.945$$

$$\alpha_5 = \frac{47.264}{2} = 23.632 \text{ rad/s}^2 \quad (ccw)$$

Link 6

$$A_H = \omega_6^2 R_{HF} \angle (\theta_{HF} + 180) + \alpha_6 R_{DF} \angle (\theta_{HF} + 90)$$
$$= (-2.878)^2 x\ (10.308) \angle (24.068 + 180) +$$
$$(2.161 \times 10.308) \angle (24.068 + 90)$$

$$A_H = 85.380 \angle 204.068 + 22.756 \angle 114.086$$
$$= 88.361 \angle 189.144$$

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PAGE NUMBER	LINE NUMBER	ERROR	CORRECTION	
3	24	repeated words		
12	10	n=4	n2=4	
18	3	First Rpo	Rpo cap	
21	10	vector	position	
21	10	vector	position	
28	7	Matrix [2 6]	Matrix [2 3]	
28	8	6	3	
34	1	from the tail	from the head	
35	Fig 2.12 b	Incorrect labelling		
35	Fig 2.12 b	Incorrect labelling		
39	2	= (A COS..)	= A (COS..)	
39	2	= (A COS..)	= A (COS..)	
39	2	A should be outside the bracket		
39	2	A should be outside the bracket		
39	15	CCOS(.)+jSIN(.	C(COS(..)+jSIN(..))	
39	15	CCOS(.)+jSIN(.	C(COS(..)+jSIN(..))	
39	Eqn 2.24	C should be outside the bracket		
39	Eqn 2.24	C should be outside the bracket		
47	12	O.4	O x 4	
47	12	O.4	O x 4	
47	Line 12 Ex.2.4	D in place of C		
47	Line 12 Ex.2.4	D in place of C		
61	23		Equ (2.72)= Equ (2.75)	
61	23		Equ (2.72)= Equ (2.75)	
61	24		Equ (2.73)= Equ (2.76)	
61	24		Equ (2.73)= Equ (2.76)	
69	1	labor	labour	
69	1	labor	labour	
69	6	enigineers	engineers	
69	6	enigineers	engineers	
76	Fig 2.29a	AO1=2	AO1=2.5	
76	Fig 2.29a	AO1=2	AO1=2.5	
80	13	Rpc	Rdc	
80	13	Rpc	Rdc	
80	15	Rpc	Rdc	
80	15	Rpc	Rdc	
82	2	p/1	p/01	
82	5	p/1	p/01	
94	22	p/p	p/p1	
94	5 from bottom	Vp/p	Vp/p1	
94	5 from bottom	Vp/p	Vp/p1	
99	12	Fig 3.8	Fig 3.7	
99	12	Fig 3.8	Fig 3.7	
99	27	3.9	2.29	
99	Line 2 from be	Fig 3.9	Fig 2.29(a)	
102	8	2	20	
104	2	Eq.3.12	Eq.3.11	
104	2	Eq.3.12	Eq.3.11	
105	16	245.75	124.75	
105	Eq. 3.20	245.75	124.75	
113	6	link 3	link 4	
127	Last line	Minor mistake in eqn.		
131	7 from bottom	opposite	same	
138	Eq. d	8.382	18.382	
140	5	95.66	115.419	

147	8	angle 50-90	angle 59-90	
151	4	angle(+10	angle(+90)	
160	Eq. 4.16	Y1 j	Y B1 j	
172	8	sealing factor	scaling factor	