LABORATORY # 3: ELECTRICAL RESISTANCE AND RESISTIVITY

OBJECTIVE:

To experimentally measure and report on the electrical properties (resistance, resistivity and conductivity) of copper and Kanthal wire as a function of temperature.

In this laboratory you will be measuring the *electrical resistance* of two metallic materials as a function of temperature. From these measurements you will determine the *electrical resistivity* and *electrical conductivity* of the materials. You should understand how *resistivity* and *conductivity* are materials properties that depend on the composition of the material and the temperature, while *resistance* depends on the material, the temperature and on the dimensions of the object. You should come to this laboratory with a good idea of how resistivity is calculated from the resistivity, and what kind of relationship to expect between resistivity and temperature.

Calibration of the equipment used to measure the resistance is an important part of this exercise.

Calculations are needed in order to determine the resistivity of the materials in question from measurements of their resistance and other parameters. Here you will learn what is good practice in processing data and evaluating the uncertainties involved.

THEORY:

Ohm's law relates the applied voltage, V, to the current, I, and resistance, R, of the material through which the current is passing in the following equation:

$$V = IR$$

The units for voltage, V, are given in volts, (J/C), for current, I, are given in amperes (C/s), and for resistance, R, are given in ohms (V/A). It should be understood that the value of R is influenced by specimen geometry, and for many materials is independent of current.

The **resistivity**, ρ, is independent of specimen geometry but related to R through the following formulas

$$\rho = RA/L$$

$$\rho = VA/IL$$

where L is the distance over which the voltage is measured, and A is the cross-sectional area perpendicular to the direction of the current in the sample (specimen). The units for resistivity are given in ohm-meters (Ω-m).

Electrical **conductivity**, σ, is the reciprocal of the resistivity given below in the formula

$$\sigma = 1/\rho$$

and it is used to specify the electrical character of a material in terms of how well a material is capable of conducting an electric current. The units for conductivity are therefore given as the reciprocal of ohm-meters $[(\Omega\text{-m})^{-1}]$.

Electrical Resistivity of Metals

Metals are very good conductors of electricity and have high conductivities because of the large number of free electrons excited into empty states above the Fermi energy and into the conduction band.

Crystalline defects behave as scattering centers for conduction electrons in metals, and any increase in the numbers of these defects will increase the resistivity of a given metal. It is also known that the concentration of these imperfections is highly dependant on the temperature, composition, and the degree of cold work of a given metal sample (specimen). It has been observed experimentally that the total resistivity of a given metal or metal alloy can be the arithmetic sum of the contributions from the following: temperature (thermal vibrations), impurity concentration, and plastic deformation (degree of cold work) as reported in the equation below:

$$\rho_{total} = \rho_t + \rho_i + \rho_d$$

where the total resistivity is the sum of the resistivities with given subscripts for t, i, and d corresponding to thermal, impurity, and deformation resistivity contributions.

Influence of Temperature

In the case of pure metals, their resistivity has been shown to increase linearly with temperature above –200°C according to the formula

$$\rho_t = \rho_o + aT$$

where ρ_o and a are constants for each particular pure metal.

Influence of Impurities

For the case where impurities are added to a pure metal to form a solid solution alloy, the impurity resistivity can be related to the impurity concentration c_i in terms of the atom fraction (at%/100) as follows:

$$\rho_i = Ac_i(1-c_i)$$

where A is a composition-independent constant.

Influence of Plastic Deformation

Plastic deformation also increases the electrical resistivity of a metal since deformation involves the creation of line defects called dislocations that are known to scatter electrons and contribute to the overall scattering centers of conduction electrons.

EXPERIMENTAL:

1. Use the vernier caliper to find the diameter of the copper wire and Kanthal wire and also determine the length of the wires.

2. Determine the offset of the digital multimeter (DMM) by checking its resistance against two known resistor banks of 1.0 Ohm and 10.0 Ohm. The reading may be, for example, 1.1 Ohm and 10.1 Ohm so then the offset would be +0.1 Ohm for each resistor bank.

3. Determine the offset of the thermometer at room temperature against a known thermocouple. Here, the thermometer may read, for example, 20.7°C when the known thermocouple is 20.0°C for room temperature, giving an offset of +0.7°C.

4. Measure the resistance of both wires at room temperature with a digital multimeter (DMM).

5. Measure the resistance of the Kanthal wire at 100°C by placing the coiled Kanthal wire and tube assembly into a test tube of oil with a thermometer and then enclose it into a heater box. The resistance is measured using a digital multimeter (DMM).

6. Measure the resistance of the Kanthal wire at 0°C by first removing it from the heater and allowing it to cool to room temperature, then placing it into a bucket of ice water until it stabilizes at 0°C. Measure the resistance of the Kanthal wire using a digital multimeter (DMM).

7. Summarize the results for length, diameter, and resistance at three temperatures from 1, 4, 5 and 6 in a table. Summarize the offsets in another table. Include the uncertainties in your calculations.

8. Measure the resistance of the Copper wire at 100°C and 0°C using the same procedure as was used for the Kanthal wire.

9. Summarize the results of temperature, resistance, resistivity, and uncertainties in a table for each Copper and Kanthal wire.

10. Graph the results of resistivity versus temperature on the computer in the laboratory.

11. Add a discussion of the results and uncertainties.

12. Conclude the main findings of the behavior of the resistivity and temperature for each wire and the uncertainty.

UNCERTAINTIES:

The following is some theory on how to derive uncertainties for this laboratory.

Uncertainties for Laboratory 3

Conductivity of copper and Kanthal ™ as a function of temperature
An introduction
particularly regarding uncertainties in measurements and derived quantities

Uncertainties in measurements

One possible source of uncertainty will be the resolution of the instrument, i.e. the smallest division will likely determine the lowest digit in our readings. With an "analog" instrument, such as the thermometer, while the manufacturer is unlikely to guarantee the accuracy to better than the smallest division on the scale provided we may be able to estimate readings between the smallest divisions of the scale. However, such an estimate is not likely to be justified unless we can make a calibration check of that accuracy. The same applies to the vernier calipers, where the resolution of the vernier scale provides the lower limit to the possible accuracy of the instrument. However, the accuracy is probably really determined by the consistency with which we can apply a standard low force between the jaws of the calipers during measurements and the care with which we conduct calibration checks.

With instruments with digital readouts the reading is clear and we may assume that the resolution is half the smallest digit of the display. However, a digital display does not guarantee the accuracy implied by the smallest digit of the display. The electronics or mechanical components involved will have some error, however small it is. With an electronic meter calibration checks must be performed for the ranges actually used in the tests.

We can determine the calibration error of an instrument by comparing its reading with that of a more reliable instrument, or by measuring a known standard for the parameter in question. The difference between the instrument reading and the standard, or between the instrument and that of a better instrument, is called a calibration error or correction. In this exercise we provide some means of determining calibration corrections for the thermometer, the vernier calipers, and the resistance meter.

It is important to realize that not only will our measurements with any instrument be uncertain to some extent, each calibration check will also be uncertain to a certain amount, depending on the uncertainty in the value of the standard and on how you perform the calibration check.

Thus the uncertainties in your measurements with each instrument will depend on the way you use the instrument and also on uncertainties in the calibration check. This is true of measurements with a thermometer, the vernier calipers, and the resistance meter.

To repeat, the uncertainty in using any instrument includes the uncertainty in the calibration as well as the uncertainty in the use of the instrument for the measurement you are interested in.

Uncertainty considerations for derived quantities

Obviously the fractional (or percent) uncertainty in a determination of resistivity is a combination of the uncertainties in R, d^2, and L, quantities that are multiplied together in calculating resistivity.

Uncertainties in products and quotients

A general rule in estimating the combined uncertainty in a quantity derived from a product or quotient of two or more parameters (i.e. multiplied or divided by each other) is that you add the uncertainties in each parameter, expressed as fractions. The result is a also a fraction. For instance the uncertainty in the area of a rectangle ab is $u_a \times u_b$, where u_a and $u_b$are the uncertainties in a and b, respectively. Suppose the uncertainty in each length measurement is 1mm and a =10mm, while b = 30mm. Then u_a = 1/10= 0.1 and u_b = 1/30= 0.03, and the uncertainty in ab is 0.1+0.03 = 0.13, or 13%. 13% of 300 mm^2 is 39 mm^2.

Uncertainties in sums and differences

On the other hand if quantities are added or subtracted the general rule is to add the uncertainties as quantities. Thus if we add two lengths 10 mm and 30 mm, and the uncertainty in the measurement of each length is 1 mm, the uncertainty of the sum 30 mm + 10 mm = 40 mm is 1mm+1mm=2 mm, 5% uncertainty. The uncertainty in the difference 30 mm-10 mm=20 mm would also be 2 mm, a 10% uncertainty.

Uncertainties in measured resistivities

The principal result in Laboratory 4 is the resistivity, ρ, of each of the two wires provided as a function of temperature. It corresponds to the Greek letter "rho" - pronounced "row". The materials tested are copper and Kanthal. The copper wire is coated with insulation as it is normally used for transformer or inductance coils. Kanthal is produced primarily for heater elements. As it is not insulated it is wound on a plastic tube to keep the turns from touching each other.

With each wire you will be measuring the resistance at different temperatures so that you can determine the temperature coefficient describing the dependence of resistivity on temperature.

Copper wire:

The equation for resistivity, ρ, of a wire with length, L, and diameter, d, and resistance, R, is:

$$\rho = R\,(A/L) = R\,(\pi d^2/4L)\ \text{Ohm.m}$$

Values for L and d are given, R is measured, so resistivity is easily calculated. Although the length and diameter of the wire are given, these values like all other measurements are uncertain. Assume that the uncertainty is ±1 in the last digit of each value given.

From the values at about 0°C, room temperature, and about 100°C, we can get the temperature coefficient, a, for resistivity, ρ, from

$$\rho = \rho_0 + at$$

by plotting resistivity against temperature (Celsius).

ρ_o is the resistivity at 0°C, while a is the slope, or temperature coefficient.

Each result will be uncertain due to uncertainties in each of the measurements from which we calculate resistivity, as well as from the uncertainties in our measurements of temperature.
If we were not able to check the calibration of the digital resistance meter we might assume that the uncertainty is ±1 in the last digit of the display in the range chosen. We accept the calibration certificates provided by the supplier of the standard resistances. These certificates show uncertainties which are negligible compared with our ohmmeter reading uncertainty. However, you can check the calibration in two ways. In both cases make sure you are making the calibration check in the same range on the meter as the range you will use for the measurements in question.

Short the two leads from the meter and it should read zero. If it is not zero the reading, R_0, with shorted leads will be a correction to subtract from measurements. Measure the resistance of a standard resistor appropriate for the range of the meter chosen. The difference, if any, between the reading, R, and the standard value R_S is also a correction, $R\text{-}R_S$, to subtract from measurements. If these two checks give different corrections, consider what is the most sensible way to handle the correction to apply to other measurements.

We can put an absolute value to the uncertainty in R based on the calibration. For example, it might be 0.1 Ohm in a reading of a few Ohms. This is the uncertainty in our correction. However, there is a similar uncertainty in both the calibration reading and our resistance readings, so the total uncertainty will be the sum of both uncertainties that i.e.

0.2 Ohm in this example. If, for instance, the resistance reading is 3.4 Ohm, the uncertainty in this is 0.2/3.4 =0.058 or 6%.

Note that the diameter of the wire is squared in the formula for resistivity. If, for example the diameter is 0.27 mm and the uncertainty is 0.01 mm i.e.0.01/0.27=0.037 or 3.7% in the diameter, the uncertainty in the cross-sectional area is 2 x 0.037 = 0.074 or 7.4%, because the diameter is multiplied by the diameter in the calculation.

The uncertainty in L has to be assumed to be one unit (1 cm) in the last digit in the stated value. Even though the quantity error appears to be large it could be the smallest fractional uncertainty in this case.

Adding all these fractional uncertainties gives us the limiting uncertainties for the resistivity values. This uncertainty is also part of the uncertainty in r_0 provided the zero temperature used is not much outside the range of temperature range for the measurements used to calculate this constant. This would be the case if we use the Celsius scale. However, there is the separate issue of the range of intercepts and slopes for lines which provide a reasonable fit to the data points in a plot of resistivity against temperature.

With each of the three points having an uncertainty which can be represented by error bars we can chose a variety of lines which will more or less fit the error bars. Our first thought could be that there will be extremes in terms of the intercepts with T = 0, and we should use these extremes for our range of possible values of r_0 and use the combination of all the parameter uncertainties to calculate the total uncertainty to use for error bars. However, while any one value for resistivity could be uncertain due to a combination of uncertainties in diameter, length, and resistance, only the error in resistance can be a different value in each of the three (or however many) resistance measurements taken, as we use the same length and diameter measurements in all the resistivity calculations for that wire. Therefore only the uncertainty in the resistance measurements should be used for error bars when determining the range of possible values for intercept and slope determined from that.

The uncertainty in slope, a, should be that determined this way. However, you should combine the uncertainty in the intercept with the uncertainties in cross-sectional area and length to get the uncertainty in ρ_0.

Kanthal:

The situation with the Kanthal wire is similar to that for the copper wire, except that in this case you have to estimate the total length of the wire from measurements you make on the coil with all the uncertainties involved in that.

It is important to have a good sketch of the coil and a clear record, using the sketch, of the measurements made on it. There are both more or less straight portions as well as a portion coiled in the form of a helix. Estimate the length of each portion and add them together.

If you think of the helix as unwound and flat, it is clear that a right angled triangle will give the length of one turn, L_t , where p is the pitch, i.e. the separation between the turns, and c the circumference of the cylinder traced out by the helix.

$$L_t = (C^2 + p^2)^{1/2} = C(1 + (p/c)^2)^{1/2} = \pi d(1 + (p/\pi)^2)^{1/2}$$

Note that the diameter, d, in this equation should be d_c, the diameter of the cylinder described by the helix, as discussed below. For the total length, the length of a turn has to be multiplied by the number of turns, which is not necessarily an integer. You should make a clear note of the number of turns plus the extra fraction, if any. You should also have measurements which will give you the pitch of the helix.

Assuming that the jaws of the vernier calipers were placed in grooves on top of the wire, while measuring the diameter, the helix diameter should first be corrected by subtracting one diameter of the wire to get the helix diameter, d_h, from centre to centre of the wire. However, the helix diameter, measured this way, say, over half a turn, is not perpendicular to the cylinder axis. This has to be corrected using $d_c = (d_h^2 - p^2/4)^{1/2}$ to get d_c, the diameter of the cylinder described by the helix. Use the corrected helix diameter to get the circumference. Then measure the two more or less straight lengths in the inside of the tube from the ends of the helix to the screw terminals.

With the total length of the Kanthal wire calculated, the procedure is the same as for the copper wire. There are similar considerations of uncertainty, except that in this case the uncertainty in the length estimate is made up of the sum of uncertainties in all the separate estimates. Clearly the uncertainty in each of the two short lengths will be at least 1 mm and probably 2 mm, for a total of 4 mm.

The uncertainty in the estimate of the length of each turn in the helix. In the expression for the length of one turn, $p/(\pi.d)$ is small compared to 1, and the uncertainty in $p/(\pi.d$ will have little effect on the square root term in the expression. The uncertainty in L_t will, therefore be mainly due to uncertainty in the first $\pi.d$ in the expression. Therefore the fractional, or percentage, uncertainly in L_t will be the same as the uncertainty in d, the diameter of the helix. For example, with a helix diameter of about 25 mm, and an uncertainty in that measurement of, say, 0.1 mm, the fractional uncertainty is 0.1/25 =0.004 or 0.4%. With each turn of the helix about 80 mm long and 10 whole turns, say, for a total length of the whole number of turns equal to 800 mm, a 0.4% uncertainty in this is about 3 mm

The estimate of the fractional portion of the helix will be pretty rough. With a helix diameter of about 25 mm or a circumference of about 80 mm, an uncertainty of one-tenth of a turn is 8 mm.

The three uncertainties just discussed are all for lengths that are added to get the total length, so we add the quantity values of the uncertainties, not the fractions. With the numbers chosen for this example the total is 4 mm+3 mm+8 mm for a total of 15 mm. Divide this by the total length of the wire to get a fractional uncertainty in the value of l in the expression for resistivity.

From this stage on the procedures to obtain the estimates of ρ_0 and a for Kanthal are as for the copper wire, and so are the procedures for estimates of the uncertainties involved in these derived quantities.

Uncertainties of uncertainties:

The rules given here for the cumulative effect of uncertainties in separate parameters used to derive a parameter are only simple rules of thumb which can be used if only a few parameters are involved, and the estimates of the individual parameter uncertainties are only crude estimates. The theory of probability and statistics provides more sophisticated rules, which, however, require more sophisticated estimates of uncertainties in the individual parameters to apply properly. In particular each measurement should be repeated many times to obtain a good estimate of the "variance" of each measurement. This is not feasible in your exercise.

However, it is worth noting that your estimates of uncertainties are also subject to uncertainties, so there is no point in quoting any uncertainty estimate with more than one or two significant digits. For instance, if the result of your calculations of an uncertainty is 3.516 % it makes no sense to quote it other than 3.5%. Maybe even 4% will be more sensible.

Likewise if this is the uncertainty in a value of 105.156, it makes no sense to quote that value as anything other than 105, as the true value could be anywhere between 101 and 109.

In practice you are likely to encounter uncertainties much greater than these. That is life!

References:

1. W.D. Callister, Materials Science and Engineering: An Introduction, 6th Edition, John Wiley & Sons, 2003. (Chapter 18: Electrical Properties)
2. J.F. Shakelford, Introduction to Materials Science for Engineers, 6th Edition, Pearson Education, Prentice Hall, 2004. (Chapter 15: Electrical Behavior)