Memorial University of Newfoundland
Faculty of Engineering and Applied Science

Engineering 2205

Chemistry and Physics
of Engineering Materials I

Laboratory Manual

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LABORATORY # 4: THERMAL CONDUCTIVITY

OBJECTIVE:

To study heat flow in a system consisting of an aluminum heater block with a heating element embedded in it at one end and a water-cooled aluminum block at the other end. These two opposing ends are separated by a system of hollow aluminum cylinders held together by a threaded rod or tool steel test specimen. Three thermisters are located in the mid-section to measure the temperature within the system.

In this laboratory you will be studying heat flow through metals and evaluating how this depends on the *thermal conductivity* of the materials and the dimensions of the object through which the heat flows. You should understand the concept of *heat flux* and *thermal gradient*. You will be measuring thermal conductivity as a materials parameter.

One approach to the concept of thermal conductivity is to appreciate the similarity, or analogy, between the flow of electricity and the flow of heat, hence the use of the term *conductivity* in both contexts. Similarly, one can think of a thermal resistance in much the same way as one treats an electrical resistance, and you will be studying thermal resistances in series and in parallel, with heat flowing between points of differing temperature as electricity flows between points of differing potential, i.e voltage. The concept of resistance applies to flow from one point to another in a solid; it also applies to the flow of heat across the interface (the contact) between two solids.

While the principles involved in the work are simple, the work is complicated by heat flow away from the equipment, along routes that we can think of as heat losses. These losses have to be estimated on the basis of a separate study, the results of which are provided to you.

THEORY:

Heat and Heat Flow

What is heat?

A simple answer is that heat is a form of energy. We cannot observe heat directly. We cannot even feel it. What we feel when we touch a hot or cold object is the *temperature*. Yet we usually think that the warmer an object is, the more heat it contains. Yet temperature alone is not a simple indication of heat content. For instance, liquid water at 0°C has more heat in it than the heat in the same mass of ice at the same temperature.

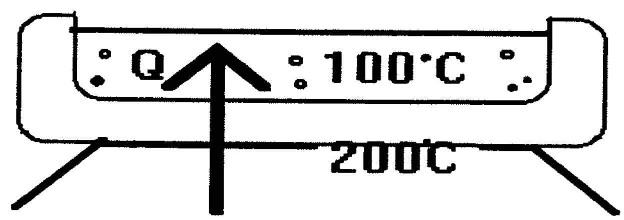
Heat flow

Heat flows naturally from a higher temperature to a lower temperature. This is happening all the time in our homes in winter. All the heat produced in an electrical baseboard heater, for instance, in a house, goes from the hot elements in the heater, which may have a surface temperature of 300°C or more, to the air in the room at about 20°C and then through the walls and windows of the house to the air outside the house which may be at -10°C. That is, heat can be at +300°C, or at -10°C, and it can be the same *amount* of heat at both temperatures.

Heat flows in solids by conduction. Conduction is also possible in liquids and gases. However, convection is likely in a liquids and gases, where the liquid or gas transports heat by movement of the liquid or gas. Heat can also be transported by radiation through transparent solids, liquids, or gases, as well as through a vacuum. The transport occurs in this case by electromagnetic waves.

Thermal Resistance

How fast does heat flow? The rate at which heat flows from a higher temperature to a lower temperature depends inversely on the *resistance* encountered by the heat. The greater the resistance, the slower the heat flow rate. This is what the >R-value= on insulation means. The higher the >R= number the bigger the resistance and the slower the rate at which heat will flow through it for a given temperature difference betwen the two sides of the insulation.



Consider a metal pan with some water in it, sitting on heating element on a stove, Figure 1. If the water is boiling, the inner surface of the pan in contact with the water must be at about 100°C. The outer bottom surface of the pan, on the heating element, may be at 200°C, say. Heat entering the pan at 200°C must exit into the water at 100°C. Though the temperature is different on the two surfaces, whatever heat enters at the higher temperature also leaves the metal at the lower temperature. *There is no heat lost* in the sense that any of this heat disappears. When we refer to a Aheat loss@ we usually mean that the heat goes somewhere we don=t intend or want it to go.

The rate at which heat flows depends inversely on the thermal resistance encountered. Two analogies may help. To get water to flow through a pipe you have to have a higher pressure at one end than at the other end, and water will flow from the higher pressure to the lower pressure. The same water which flows into the pipe at the higher pressure flows out at the lower pressure, the only difference between the water at the two ends is the pressure in the water.

$$Q = \frac{\Delta P}{R}$$

where Q is the rate of flow of the water, ΔP is the difference in pressure, and R the resistance.. The longer and thinner the pipe, the bigger the resistance.

$$I = \frac{\Delta V}{R}$$

In electrical conduction

where I, V and R have the usual electrical meanings. Electrical charge enters the resistance at a higher potential and exits at the lower potential

In each of these cases, the quantity of heat, the mass of water, and the amount of electrical charge, stays the same, only the temperature, the pressure, or the potential, changes for the heat, water, or charge.

$$Q = \frac{\Delta T}{R}$$

The equivalent heat flow equation is.

where Q is the rate of flow of heat and ΔT is the temperature difference and R is the thermal resistance.

Steady State Heat Flow Equation

$$Q = -K.A.\frac{\Delta T}{\Delta x}$$

Another form of equation (3) is

where K is the thermal conductivity of the material, A is the area, or cross-sectional area, through which the heat flows and ΔT/Δx is the temperature gradient. The minus sign appears if we use a normal sign convention, because the gradient must be negative (decreasing temperature with increasing x) if the heat flow is to be in the direction of increasing x, i.e. Q is positive. This equation describes heat flow in a *steady state*, that is when the temperatures are constant (but not the same) at all points in the material. (There is a more complicated equation for non-steady-state heat flow). ΔT/Δx is the temperature gradient, which we can also write as dT/dx, getting the other standard form of equation

$$Q = -K.A.\frac{dT}{dx}$$

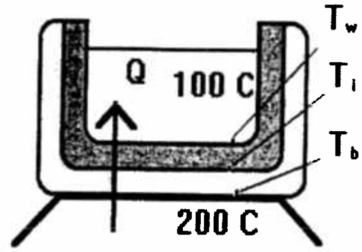
(4):

Ignoring signs, it follows from equations (3) and (4) that the thermal resistance, R, is given by

$$R = \frac{\Delta x}{A}\frac{1}{K}$$

where Δx is the thickness, or length, and A is the cross sectional area the area the heat flows through. In the case of a pan on a stove, considering just the bottom of the pan and ignoring any heat flow elsewhere, Δx is the thickness of the pan, A is the area of the bottom part which receives heat from the heating element, and K is, of course, the thermal conductivity of the pan material.

Some pans are made of two materials in a sandwich, e.g. an outside layer of copper on a layer of stainless steel, Figure 2. Heat will flow first through the copper and the *same* heat flows through the stainless steel layer, at the rate Q.



Each layer presents its own resistance to the heat flow. These resistances are in *series*, i.e. they follow each other in the path of the heat flow, so *the two resistances add together* to get the total resistance against the flow of

$$Q = \frac{\Delta T_{Cu}}{R_{Cu}}$$

heat. For the copper we can write

$$Q = \frac{\Delta T_{SS}}{R_{SS}}$$

For the stainless steel, we can write

$$R_{Cu} = \frac{(\Delta x)_{Cu}}{A}\frac{1}{K_{Cu}}$$

where

$$R_{SS} = \frac{(\Delta x)_{SS}}{A} \frac{1}{K_{SS}}$$

and
$\Delta T_{Cu} = T_b - T_i$ and $\Delta T_{SS} = T_i - T_w$ where T_b is the temperature at the bottom surface against the element, T_i is the temperature at the interface between the copper and the stainless steel layers, and T_w is the temperature of the inside surface against the water.

Determining Thermal Conductivity
some notes

Basic principle

The *principle* in this laboratory exercise is simple (but the details are not!). The simple part is that we make use of the standard equation for steady-state heat conduction: $Q = -k.A. (dT/dx)$. Q is the rate at which heat flows through a cross-sectional area, A, in a sample of the material where the temperature gradient is dT/dx or $\Delta T/\Delta x$. If we know Q, A, and $\Delta T/\Delta x$ we can easily calculate k, the *thermal conductivity*.

Source of heat and heat sink

In this experiment the heat is generated in an electrical heating element, where the power, W, dissipated is equal to the heat generated, and the standard electrical equation, $W=V.I$, applies.

The heater element is embedded in a cylindrical block of aluminum, and the heat generated in the *heater block* is transmitted from it to the test specimen. One end of the test specimen is clamped to the heater block, the other end to an aluminum box which is cooled by cold water running through it. Therefore, in principle, the heat generated in the heater element travels through the heater block, through the test specimen to the cooled box, which we can call a *heat sink*.

Heat lost or "diverted"

Unfortunately, the situation is not as simple as the above would seem to indicate. Some of the heat generated in the heater is "lost", i.e. conducted from the sides of the heater not in contact with the test specimen. Also, some heat passes down a threaded rod used to clamp the heater block, the specimen, and the heat sink together. Furthermore, some heat is lost from the outer exposed surface of the test specimen, and corrections for all these have to be estimated.

The rate at which heat is lost, i.e. conducted away, from the surfaces of the heater block and the test specimen is kept to a minimum by surrounding these items with thermal insulation, in this case vermiculite, which consists of expanded mica chips. Nevertheless this heat loss cannot be ignored.

The rate at which heat is transferred away from the surface of the heater block, is a function of the temperature of the block. This has been studied, and the results of this investigation described in a report available to you. You will see there that the rate of heat loss is more or less a linear function of the surface temperature of the block. Note that this result is stated in terms of *heat flux*, the rate of heat transfer per unit (surface) area. If you know, or can estimate, the surface temperature, and know the size of the area in question, you can estimate the rate at which heat is transferred from that area into the insulation next to it. The same relationship between surface temperature and heat flux is used for estimates of heat loss from the specimen surface.

In this experiment you will have measurements of the temperature in at three points in the test specimen, two of them close to, but not at, the ends of the specimen, and you can use these measurements to interpolate anywhere between the measurements and extrapolate to estimate the temperature beyond the three thermistors, including the end surfaces of the specimen. You can also estimate the temperature at points along the length of the heater block.. Note that you have to have a temperature gradient to have heat conduction, and that heat flows down the gradient. Since heat flows in the heater block to the test specimen it is reasonable as a first approximation to assume that there is a temperature gradient in the heater block, which is similar to that in the specimen. We can, therefore use the temperature gradient in the heater block to extrapolate back to get an estimate for the temperature at the mid point in the block, and use this as an average of the temperature of the heater block surface in estimating the rate of heat loss from the heater block surface.

As the surface temperature of the test specimen varies, it is appropriate to consider separate sections of the test piece, and estimate the average temperature of each section and its surface area and use that to estimate the heat flow from that section into the insulation.

The specimen can be considered to be divided in four sections: sections II and III between the t thermistors, sections, I and IV, between the outer thermistors and the ends of the specimen.

Thermistor calibration

The temperature at the thermistors is determined using the calibration provided. Note that the equation relating resistance to temperature is what you would expect from a semiconductor.

$$\sigma = \sigma_O \exp(-E_g/(2kT))$$

which we can write as $\sigma = \sigma_O \exp(-C/T)$, or $\rho = \rho_O \exp(C/T)$. Using $R = \rho\ (l/A)$, we have

$$R = R_O \exp(C/T)$$

or

$$ln\ R = ln\ R_O + (C/T)$$

It follows from the derivation of this equation that T has to be on the absolute, Kelvin, scale.

Taking heat losses and diversions into account

Treat the surface of the heater block as section H, and estimate the heat loss from each, as $L_H, L_I, L_{II}, L_{III}$. For the surface temperature of the heater block extrapolate the temperature from the specimen to the middle of the heater block. For the test specimen estimate the average temperature at the middle of each section.

Estimate the rate of heat loss from the heater block surface, and from each portion of the outer surface of the specimen, using an estimate of the temperature at the mid point of the area in question to obtain the rate of heat transfer from that area. As the data available provides us with heat flux as a function of temperature, multiply the heat flux value chosen (for the mean temperature of the area) by the area in question to obtain the rate of heat loss.

There is heat lost from the heater block surface which does not enter the specimen, but there is also heat going down the threaded rod in the centre of the test specimen. The rod is made of tool steel, i.e. a strong steel. The thermal conductivity of the tool steel is 46 W/(m.s) (± 20%).

Since this rod goes from the heater block to the heat sink it carries heat *in parallel* to the heat flowing down the specimen. It can be assumed to have the same temperature gradient as the specimen, and that is determined from the temperatures registered by the two thermistors and the distance between them. So knowing the temperature gradient and estimating the effective cross-sectional area of the rod we get the rate of heat flow down the rod, Q_R, from

$$Q_R = -k_R.A_R.\ \Delta T/\Delta x$$

where k_R is the thermal conductivity of the rod material and A_R is the cross-sectional area of the *rod.*

Work through the heat flow step by step, starting with the heater block. That is, if Q_H is rate at which heat is developed by the heater, the rate of heat entering the specimen, Q_S,, is given by

$$Q_S = Q_H - L_H - Q_R$$

We can also call this Q_I, the rate at which heat enters section I of the specimen, i.e.

$$Q_I = Q_S.$$

The rate at which heat leaves section I and enters section II is Q_{II}:

$$Q_{II} = Q_I - L_I$$

Likewise,

$$Q_{III} = Q_{II} - L_{II}$$

and

$$Q_{IV} = Q_{III} - L_{III}$$

Sections II and III are the sections of the specimen for which we have a direct measurement of a temperature gradient from the two thermistors at each end of the section, i.e a value of ΔT along with Δx, but there is a small difference between the heat entering and the heat leaving the section. The best approach is the use the *average* of the rates of heat flow within the specimen in and out of the section, e.g. for section II the average of Q_I and Q_{II} as the heat flow rate in the equation $Q = -k.A. \Delta T/\Delta x$ from which to determine k, *and likewise the average of* Q_{II} and Q_{III} for section III.

Uncertainties

There are many steps in the calculations before you arrive at the value of the thermal conductivity of the test specimen material and each step will involve some uncertainty!

You will have uncertainties in the dimensions, i.e. the diameter and hence the cross sectional areas, the distance between the thermistors, the length of the specimen and each portion of it, the area of the heater block. The thermistor resistance measurements will be uncertains and hence also the temperature values. There is also an uncertainty in the values of voltage and current use to calculate the rate of heat generation in the heater, the uncertainty (given) in the thermal conductivity of the rod material, and the uncertainty (given) in the value of heat flux associated with a particular temperature on the heater block and specimen surface.

Use these uncertainties to estimate the uncertainty in *each* of the heat flow rates (Q and L values).

As you calculate W and each Q or L value you may be *multiplying* or *dividing* parameters, and so you can add the uncertainty in each parameter, calculated as a *percentage* of the value of the parameter. However, as you will also be successively *subtracting* a Q or L value from the previous Q value in the chain of heat flows, you must calculate the uncertainty of each Q and L value in W (Watts), not in %. All these uncertainties in Q and L, *expressed in Watts,* must be *added* to get a total uncertainty, in Watts, in the Q value used to calculate k for the specimen.

You will also estimate the uncertainty of the value of the temperature gradient, by estimating the uncertainly in the distance used as will as estimating how far off your measurements of temperature may be, using the worst case (one thermistor registering too high and one too low), and estimate the % effect that may have on your estimate of the temperature gradient.

At this point you should express your uncertainties in the temperature gradient and the cross-sectional area, and express your uncertainty in the derived Q value in %, and *add* that percentage to the % uncertainties of the other parameters in the steady state heat flow equation, since here you will be back to multiplying and adding parameters.

Be systematic and tidy in going through all this, step by step.

Note that the uncertainty in some parameters may be large as a percentage of that parameter value, but contribute relatively little to the uncertainty in the value of thermal conductivity of the specimen. The uncertainty in other parameters may me more significant. While you may carry a number of digits in the various intermediate values you calculate, make sure the final answer in stated in the appropriate number of digits consistent with the estimated total uncertainty. Note which parameters contribute the most to the uncertainty in the final answer.

EXPERIMENTAL:

1. Measure the basic dimensions of the aluminum cylinder and heater block.

2. Slide the threaded steel rod through the aluminum cylinder and screw it into the connections at each end while making sure the heat sink holes 1, 2, 3 are facing upwards.

3. Connect the four springs which holds the box together.

4. Insert the three thermisters into each corresponding hole in the aluminum cylinder. The three holes must be filled with conducting paste by the laboratory supervisor. The thermisters are then attached to a channel box which is connected to the multimeter. Make note of the numbers tagged to each precalibrated thermister.

5. Connect up the water cooling hose connections and when all connections are done, pour vermiculite insulation into the box and cover with a Plexiglas lid.

6. Adjust power source to 70 volts.

7. Measure the resistance of each thermister at 10 minute intervals until they stabilize. Then continue to measure the resistance of each thermister at 2 minute intervals until they stabilize. Place these data into a table of Time (min), Thermister Hole#1(kΩ) , Thermister Hole #2(kΩ), Thermister Hole#3(kΩ).

8. Make a table for the offset of the digital voltmeter: Theory (ohm) Measured (ohm) Offset

9. Make a table for the results: Time (min) Thermister1 (kOhm) Thermister 2 (kOhm) Thermister 3 (kOhm)

10. Determine the value of constant resistance at each thermister (i.e. average of last 3 readings).

11. Calculate the thermister temperature for these constant resistance values using the thermister equation and the unique constant, C, for each thermister.

12. Create a table of results: Section, Thermister, Length of Section, Resistance, Temperature, Temperature Gradient, Surface Area, Mean Temperature.

13. Graph the results of Temperature versus Distance on the computers in the laboratory.

14. Calculate the heat loss in the heater block, rod, sections I to IV,

15. Calculate the thermal conductivity, k, of the steel rod in sections II and III and the average, k, for the steel rod in sections II and III.

16. Account for the uncertainties in all calculations.

17. Add a discussion of results and uncertainties.

18. Conclude the main results of thermal conductivity determined for the steel rod and its uncertainty.

UNCERTAINTIES:

The following is some additional theory on how to handle uncertainties in this laboratory.

Laboratory 4 Uncertainties
Thermal Conductivity and Heat Flow

The rules:

If parameters are multiplied or divided: add % uncertainties to get % uncertainty of result.
(If uncertainty in A is 1%, and uncertainty in B is 5%, uncertainty in AB is 1%+5%=6%)

If parameters are added or subtracted add magnitudes of uncertainties. (If uncertainty in A is 3mm, and uncertainty in B is 2mm, the uncertainty in A-B is 5mm)

The application:

To estimate k we use Q = -k.A. dT/dx or k = (Q/A)/(-dT/dx)

Uncertainty in Q, rate of heat generated, results from uncertainties in V and I. Consider how to combine the uncertainties of V and I to find the uncertainty in Q. For the purposes of the following example this combined uncertainty is assumed to be 5%.

Uncertainty in Q_I, heat entering specimen, includes uncertainty in Q and also uncertainties in Q_R, rate of heat flow down rod, and L_H, rate of heat loss from heater surface.

Q_R is estimated using $Q_R = -k_R.A_R.\ dT/dx$

The % uncertainty in k_R is 20%. The % uncertainty in A_R can be estimated from the % uncertainty in the rod diameter. The % uncertainty in dT/dx is the sum of the % uncertainties of the enumerator, dT, and denominator, dx, used to calculate it.

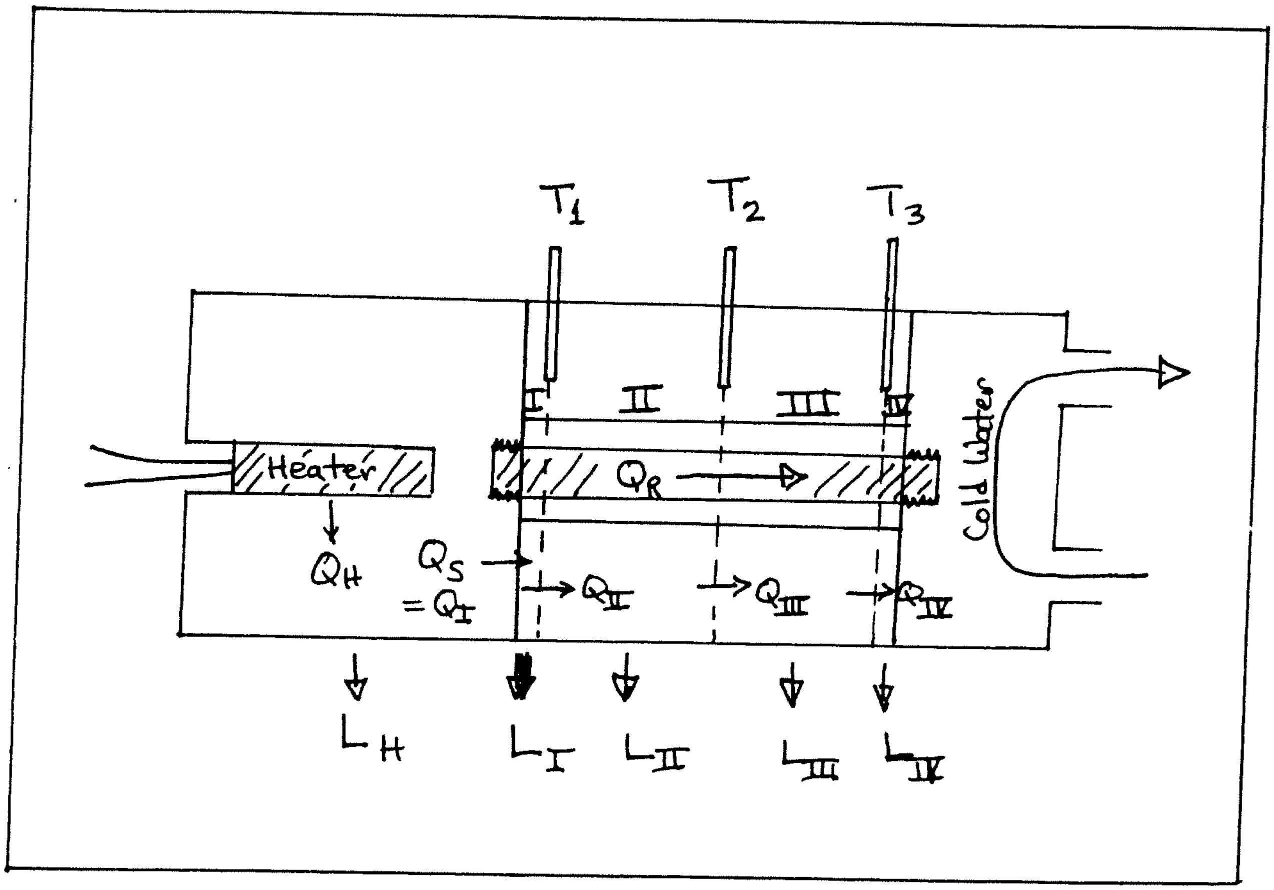
The % uncertainty of Q_R is the sum of all these % uncertainties. This % uncertainty must then be converted into a quantity uncertainty, as Q_R is one of two heat losses SUBTRACTED from Q. *For example*, if Q_R = 1W and the uncertainty in it is 30%, the quantity uncertainty in Q_R is 0.3W, which is contributed to the uncertainty in Q_I.

L_H will be uncertain due to the 20% uncertainty in the equation Q=7.35(MT-300). L_H will also be uncertain due to uncertainties in the area used, and in (T-300). If, for example, if L_H is 20W and these contribute another 20% to the uncertainty of L_H, for a total uncertainty in 40%, L_H is uncertain by 40% of 20W or 8W. This uncertainty is added to the uncertainty in Q to get the uncertainty in Q_I. For instance, if Q is 200W and $Q_I = Q-Q_R-L_H$ = 200W-1W-20W=179W, the uncertainty in Q_I is 10W+0.3W+8W=18.3W, rounding off to 18W (or about 10%)

Q_{II} will be a little more uncertain than Q_I, due to the uncertainty in L_{II}, but as L_{II} is small the effect of its uncertainty on that of Q_I will also be small, so (in this example) the uncertainty in the rate of heat flow in the first part of the specimen is not far from 10% or 11%. Consider also the uncertainties in A and especially dT/dx, which could provide an additional 9% or 10%, for a *total uncertainty of about 20% in k using the example values.* Similarly for Q_{III} and Q_{IV}.

References:

1. W.D. Callister, Materials Science and Engineering: An Introduction, 6th Edition, John Wiley & Sons, 2003. (Chapter 19: Thermal Properties)
2. J.F. Shakelford, Introduction to Materials Science for Engineers, 6th Edition, Pearson Education, Prentice Hall, 2004. (Chapter 7: Thermal Behavior)


T1
T2
T3
I
II
III
IV
Heater
QR
Cold Water
QH
QS
= QI
QII
QIII
QIV
LH
LI
LII
LIII
LIV