CHAPTER 6

MANIPULATOR DYNAMICS

In this chapter we study the forces required to cause the motion. There are two types of problems.

1) Given the vectors $\{ \theta \}$, $\{ \dot{\theta} \}$, and $\{ \ddot{\theta} \}$ find $\{\tau \}$

2) Given $\{ \tau \}$ find $\{ \theta \}$, $\{ \dot{\theta} \}$ and $\{\ddot{\theta} \}$

The first type of problem is useful in controlling the manipulator. The second type is useful in simulation.

6.3 MASS DISTRIBUTION

We identify an inertia tensor $^A[I]$ as

$$^A[I] = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

where

$$I_{xx} = \iiint_V (y^2+z^2)\rho \ dv$$

$$I_{yy} = \iiint_V (x^2+z^2)\rho \ dv$$

$$I_{xy} = \iiint_V \rho xy \ dv$$

The elements I_{xx}, I_{yy}, I_{zz} are mass moment of inertia and the other terms are mass products of inertia. ${}^{A}[I]$ is a symmetric matrix, so it has six independent elements. If the reference frame is the principal axes then the product of inertia terms are zero and the matrix is diagonal.

6.4 NEWTON'S EQUATION, EULER'S EQUATION

We will consider each link of a manipulator as a rigid body. If the location of the centre of mass and inertia tensor are given then the mass distribution is completely characterized. Newton's equations and their rotational analogies, Euler's equation, describe how forces, inertias and accelerations relate.

$$\{F\} = m\,\{\dot{v}_c\} \qquad (6.29)$$

Where $\{\dot{v}_c\}$ is the acceleration of the mass centre. Equation (6.29) is the Newton's equation of motion and

$$\{N\} = {}^{C}[I]\{\dot{\omega}\} + \{\omega\}\times {}^{C}[I]\{\omega\} \qquad (6.30)$$

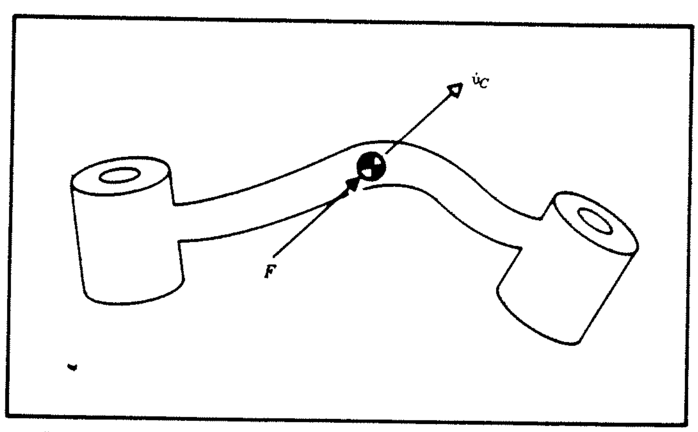

FIGURE 6.3 A force F acting at the center of mass of a body causes the body to accelerate at $\dot{v}_C$.

is the Euler's Equation for this link in frame {C} where the origin is located at the centre of the mass.

6.5 ITERATIVE NEWTON-EULER DYNAMIC FORMULATION

We now consider the problem of computing the torques that correspond to a given trajectory of a manipulator. We assume that we know the position, velocity and acceleration of the joints i.e.,

$$\{\theta\},\ \{\dot{\theta}\},\ \text{and}\ \{\ddot{\theta}\}$$

are known. With this knowledge, and the knowledge of the kinematics and inertia tensors and mass of the links, we can calculate the joint torques required to cause the motion. To use the following algorithm it is also know that ${}^{0}_{0}\{\omega\} = {}^{0}_{0}\{\dot{\omega}\} = \{0\}$, i.e. the base is fixed.

The angular velocity and angular acceleration for a <u>rotational</u> joint are given by

$$^{i+1}\{\omega\}_{i+1} = {}^{i+1}_{\ \ i}[R]\,^{i}\{\omega\}_{i} + \dot{\theta}_{i+1}\,^{i+1}\{\hat{Z}_{i+1}\} \tag{6.31}$$

$$^{i+1}\{\dot{\omega}\}_{i+1} = {}^{i+1}_{\ \ i}[R]\,^{i}\{\dot{\omega}\}_{i} + {}^{i+1}_{\ \ i}[R]\,^{i}\{\omega\}_{i} \times \dot{\theta}_{i+1}\,^{i+1}\{\hat{Z}_{i+1}\} + \ddot{\theta}_{i+1}\,^{i+1}\{\hat{Z}_{i+1}\} \tag{6.32}$$

The linear equation for each link frame origin in <u>such cases</u> is

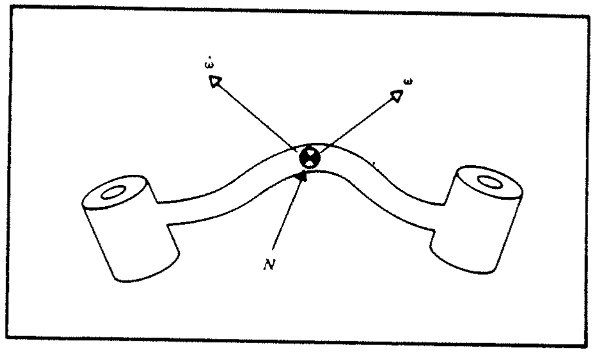

FIGURE 6.4 A moment N is acting on a body, and the body is rotating with velocity ω and accelerating at $\dot{\omega}$.

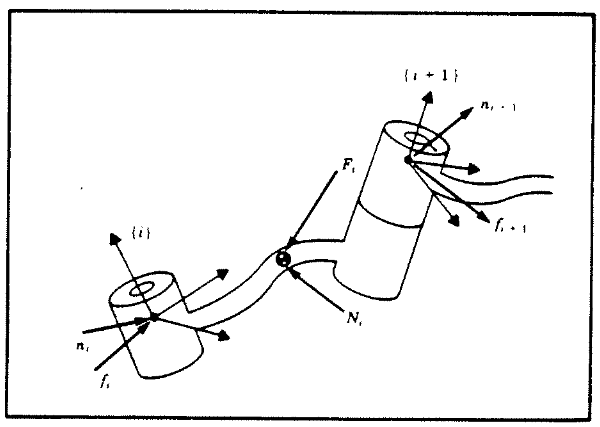

FIGURE 6.5 The force balance, including inertial forces, for a single manipulator link.

given by

$$^{i+1}\{\dot{v}\}_{i+1} = {}^{i+1}{}_{i}[R]({}^{i}\{\dot{\omega}\}_{i} \times {}^{i}\{P_{i+1}\} + {}^{i}\{\omega\}_{i} \times ({}^{i}\{\omega\}_{i} \times {}^{i}\{P_{i+1}\}) + {}^{i}\{\dot{v}\}_{i}) \tag{6.34}$$

on the other hand, if the joint is prismatic, then one has to use

$$^{i+1}\{\dot{\omega}\}_{i+1} = {}^{i+1}{}_{i}[R]\,{}^{i}\{\dot{\omega}\}_{i} \tag{6.33}$$

$$^{i+1}\{\dot{v}\}_{i+1} = {}^{i+1}{}_{i}[R]({}^{i}\{\dot{\omega}\}_{i} \times {}^{i}\{P_{i+1}\}+{}^{i}\{\omega\}_{i} \times ({}^{i}\{\omega\}_{i} \times {}^{i}\{P_{i+1}\})+{}^{i}\{\dot{v}\}_{i}) + 2\,{}^{i+1}\{\omega\}_{i+1} \times \dot{d}_{i+1}{}^{i+1}\{\hat{Z}_{i+1}\} + \ddot{d}_{i+1}{}^{i+1}\{\hat{Z}_{i+1}\} \tag{6.35}$$

The acceleration of the centre of the link in either of the two cases will be

$$^{i}\{\dot{v}\}_{Ci} = {}^{i}\{\dot{\omega}\}_{i} \times {}^{i}\{P_{Ci}\} + {}^{i}\{\omega\}_{i} \times ({}^{i}\{\omega\}_{i} \times {}^{i}\{P_{Ci}\}) + {}^{i}\{\dot{v}\}_{i} \tag{6.36}$$

The inertial force $\{\underline{F}_i\}$ and torque $\{N_i\}$ will be given by

$$^{i}{}_{i}\{F\} = m\,{}^{i}{}_{i}\{\dot{v}_{Ci}\} \tag{6.37}$$

$$^{i}\{N\} = {}^{Ci}[I]\,{}^{i}{}_{i}\{\dot{\omega}\} + ({}^{i}{}_{i}\{\omega\} \times {}^{Ci}[I]\,{}^{i}{}_{i}\{\omega\})$$

INCLUSION OF GRAVITY FORCES IN THE DYNAMICS ALGORITHM

The effect of gravity on the links can be included quite easily by setting ${}^{0}{}_{0}\{\dot{v}\} = {}^{0}{}_{0}\{g\}$ where g = 9.81. This fictitious upward acceleration causes exactly the same effect on the links as gravity would. This does not introduce additional computational expense.

6.8. THE STRUCTURE OF THE MANIPULATOR DYNAMIC EQUATIONS

THE STATE SPACE EQUATION

It is quite often convenient to express the dynamic equations in the general form as

$$\underset{n\times1}{\{\tau\}} = \underset{n\times n}{[M(\underline{\theta})]}\{\ddot{\theta}\} + \underset{n\times1}{\{V(\underline{\theta},\underline{\dot{\theta}})\}} + \underset{n\times1}{\{G(\underline{\theta})\}} \qquad (6.59)$$

Where

$[M(\underline{\theta})]$ is the n×n mass matrix

$\{V(\underline{\theta},\underline{\dot{\theta}})\}$ is an n×1 vector having coriolis and centrifugal terms

and $\{G(\theta)\}$ is an n×1 vector of gravity terms.

For a two link manipulator shown in fig.6.6, these vectors and matrices are

$$[M(\underline{\theta})] = \begin{bmatrix} l_2^2m_2+2l_1l_2m_2c_2+l_1^2(m_1+m_2) & l_2^2m_2+l_1l_2m_2c_2 \\ l_2^2m_2+l_1l_2m_2c_2 & l_2^2m_2 \end{bmatrix} \qquad (6.60)$$

$$\{V(\underline{\theta},\underline{\dot{\theta}})\} = \underset{2\times1}{\begin{Bmatrix} -m_2l_1l_2s_2\dot{\theta}_2^2 - 2m_2l_1l_2s_2\dot{\theta}_1\dot{\theta}_2 \\ m_2l_1l_2s_2\dot{\theta}_1^2 \end{Bmatrix}} \qquad (6.61)$$

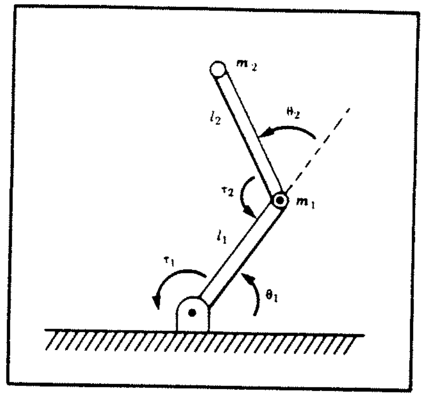

FIGURE 6.6 Two-link with point masses at distal end of links.

$$\{G(\underline{\theta})\} = \begin{Bmatrix} m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1 \\ \\ m_2 l_2 g c_{12} \end{Bmatrix}_{2\times 1} \qquad (6.62)$$

THE CONFIGURATION SPACE EQUATION

One can rewrite Eq. (6.59) in the form

$$\{\tau\} = \underset{n\times n}{[M(\underline{\theta})]}\underset{n\times 1}{\{\ddot{\theta}\}} + \underset{n\times(n-1)/2}{[B(\underline{\theta})]}\{\dot{\underline{\theta}}\,\dot{\underline{\theta}}\} + [C(\underline{\theta})]\{\dot{\underline{\theta}}^2\} + \{G(\underline{\theta})\} \qquad (6.63)$$

where $[B(\underline{\theta})]$ – coriolis coefficients

$$\{\dot{\theta}\;\dot{\theta}\} = [\dot{\theta}_1\dot{\theta}_2 \ldots \dot{\theta}_{n-1}\dot{\theta}_n]^T \qquad (6.64)$$

$[C(\underline{\theta})]$- centrifugal coefficients

$$\{\dot{\underline{\theta}}^2\} = [\dot{\theta}_1^2\dot{\theta}_2^2 \ldots \dot{\theta}_n^2]^T \qquad (6.65)$$

Eq.(6.63) gives a form in which parameters are only a function of joint position. It shows that the dynamics is computation intensive.

6.12. DYNAMIC SIMULATION

Problem → Given $\{\tau\}$, find $\{\theta\}$, $\{\dot{\theta}\}$, $\{\ddot{\theta}\}$.

Given the dynamics equations written in the closed form as in Eq. (6.59), simulation requires solving the dynamic equation for acceleration

$$\{\ddot{\theta}\} = [M]^{-1}\{\tau - V(\underline{\theta},\underline{\dot{\theta}}) - G(\underline{\theta}) - F(\theta,\dot{\theta})\} \qquad (6.115)$$

We may then apply any of the several known Numerical Integration Schemes to integrate acceleration to compute future position and velocities.

Given initial conditions of the manipulator, usually in the form

$$\{\theta(0)\} = {}_0\{\theta\}$$

and

$$\{\dot{\theta}(0)\} = \{0.0\}.$$

We numerically integrate Eq. (6.115) forward in time steps of size Δt.

EULER INTEGRATION SCHEME

At first calculate

$$\{\dot{\theta}(t+\Delta t)\} = \{\dot{\theta}(t)\} + \{\ddot{\theta}(t)\}\Delta t$$

$$\{\theta(t+\Delta t)\} = \{\theta(t)\} + \{\dot{\theta}(t)\}\Delta t + \tfrac{1}{2}\Delta t^2\{\ddot{\theta}(t)\} \qquad (6.117)$$

and then substitute these two $\{\dot{\theta}(t+\triangle t)\}$ and $\{\theta(t+\triangle t)\}$ in Eq.(6.115) to calculate $\{\ddot{\theta}(t+\triangle t)\}$. In this way, the position, velocity, and acceleration caused by a certain torque function can be calculated.