CHAPTER 2

WORKED OUT PROBLEMS

2.1 [15] A vector ${}^{A}P$ is rotated about Z_A by $\theta\,(30)$ degrees and is subsequently rotated about X_A by φ (20) degrees. Give the rotation matrix which accomplishes these rotations in the given order.

GIVEN $\theta = 30^\circ \; ; \; \phi = 20^\circ$

ONE CAN USE PROGS 12 OR 2

SOLUTION

$$[R] = \left[ROT(\hat{x}, 20^\circ)\right]\left[ROT(\hat{z}, 30^\circ)\right]$$

USING PROG 2

THETA VECTOR

$$\{\theta\} = \begin{Bmatrix} 20 \\ 30 \\ 0 \end{Bmatrix}$$

IDENTITY MATRIX (ALL DIAGONALS = 1)

AXIS VECTOR

$$\{I\}_{AXIS} = \begin{Bmatrix} 1 \\ 3 \\ 1 \end{Bmatrix} \begin{array}{l} \hat{x} \\ \hat{z} \\ \hat{x} \end{array}$$

ONE CAN USE ANY AXIS 1, 2, 3 AS LONG AS $\theta = 0$

POSITION VECTORS OF ALL AXES.

$$[0.0 \quad 0.0 \quad 0.0]^T \longrightarrow \begin{Bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

RESULT

$$\begin{bmatrix} 0.8659 & -0.5 & 0.0 & 0.0 \\ 0.4699 & 0.8136 & -0.34 & 0.0 \\ 0.1711 & 0.2962 & 0.9396 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

4 x4 MATRIX

2.2 [15] A vector ^{A}P is rotated about Y_A by 30 degrees and is subsequently rotated about X_A by 45 degrees. Give the rotation matrix which accomplishes these rotations in the given order.

<u>USE PROG 2</u>

$$[R] = \left[Rot(\hat{X}, 45^\circ)\right]\left[Rot(\hat{Y}, 30^\circ)\right]$$

$$\{\theta\} = \begin{Bmatrix} 45 \\ 30 \\ 0 \end{Bmatrix} \rightarrow \text{THETA VECTOR}$$

$$\{I\} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} \rightarrow \text{AXIS VECTOR}$$

POSITION VECTORS FOR EACH CO-ORDINATE SYSTEM

$$\begin{Bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}, \begin{Bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}, \begin{Bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix}.$$

<u>RESULT</u>

$$[T] = \begin{bmatrix} 0.8659 & 0.0 & 0.5 & 0.0 \\ 0.3537 & 0.7068 & -0.6124 & 0.0 \\ -0.3537 & 0.7073 & 0.6121 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

2.3 [16] A frame $\{B\}$ is located as follows: initially coincident with a frame $\{A\}$, we rotate $\{B\}$ about Z_B by θ (20) degrees and then we rotate the resulting frame about X_B by φ (25) degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}P$ to ${}^{A}P$.

USE $\theta = 20^\circ$ AND $\phi = 25^\circ$.

PART A

$$ {}^{A}_{B}[R] = [ROT(\hat{z}, 20)]\,[ROT(\hat{x}, 25)] $$

USE PROG 2 OR 12

BY FOLLOWING A PROCEDURE SIMILAR TO EARLIER TWO PROBLEMS THE RESULT IS

$$ {}^{A}_{B}[T] = \begin{bmatrix} 0.9396 & -0.31 & 0.1446 & 0.0 \\ 0.3421 & 0.8515 & -0.3972 & 0.0 \\ 0.0 & 0.4227 & 0.9062 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$

PART B

$$ {}^{B}_{A}[T] = {}^{A}_{B}[T]^{-1} = \begin{bmatrix} 0.9396 & 0.3421 & 0.0 & 0.0 \\ -0.31 & 0.8515 & 0.42 & 0.0 \\ 0.1446 & -0.3972 & 0.9 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$

$$ {}^{A}\{P\} = {}^{A}_{B}[T]\,{}^{B}\{P\} $$

2.4 [16] A frame $\{B\}$ is located as follows: initially coincident with a frame $\{A\}$, we rotate $\{B\}$ about Z_B by 30 degrees and then we rotate the resulting frame about X_B by 45 degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}P$ to ${}^{A}P$.

$$ {}^{A}_{B}[R] = [\mathrm{ROT}(\hat{z}, 30^\circ)]\,[\mathrm{ROT}(\hat{x}, 45)] $$

USE PROG. 2

$$ \{\theta\} = \begin{Bmatrix} 30 \\ 45 \\ 0 \end{Bmatrix};\quad \{I\} = \begin{Bmatrix} 3 \\ 1 \\ 1 \end{Bmatrix} $$

RESULT

$$ {}^{A}_{B}[T] = \begin{bmatrix} 0.8659 & -0.35355 & 0.3537 & 0 \\ 0.500 & 0.6121 & -0.6124 & 0 \\ 0.0 & 0.7073 & 0.7068 & 0 \\ 0.0 & 0.0 & 0.0 & 1 \end{bmatrix} $$

2.12 [14] A velocity vector is given by

$$^{B}V = \begin{bmatrix} 10.0 \\ 20.0 \\ 30.0 \end{bmatrix}$$

Given

$$^{A}_{B}T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 11.0 \\ 0.500 & 0.866 & 0.000 & -3.0 \\ 0.000 & 0.000 & 1.000 & 9.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

compute ^{A}V.

METHOD 1

$$^{A}\begin{Bmatrix} V \end{Bmatrix} = \underset{4\times 4}{^{A}_{B}\begin{bmatrix} T \end{bmatrix}} \; \underset{\text{4×1 VECTOR}}{^{B}\begin{Bmatrix} 10.0 \\ 20.0 \\ 30.0 \\ 0.0 \end{Bmatrix}}$$

USE PROG 12

$$^{A}\begin{Bmatrix} V \end{Bmatrix} = \begin{Bmatrix} -1.34 \\ 22.32 \\ 30.0 \\ 0.0 \end{Bmatrix}$$

METHOD 2

USE

FIND θ USING PROG 11

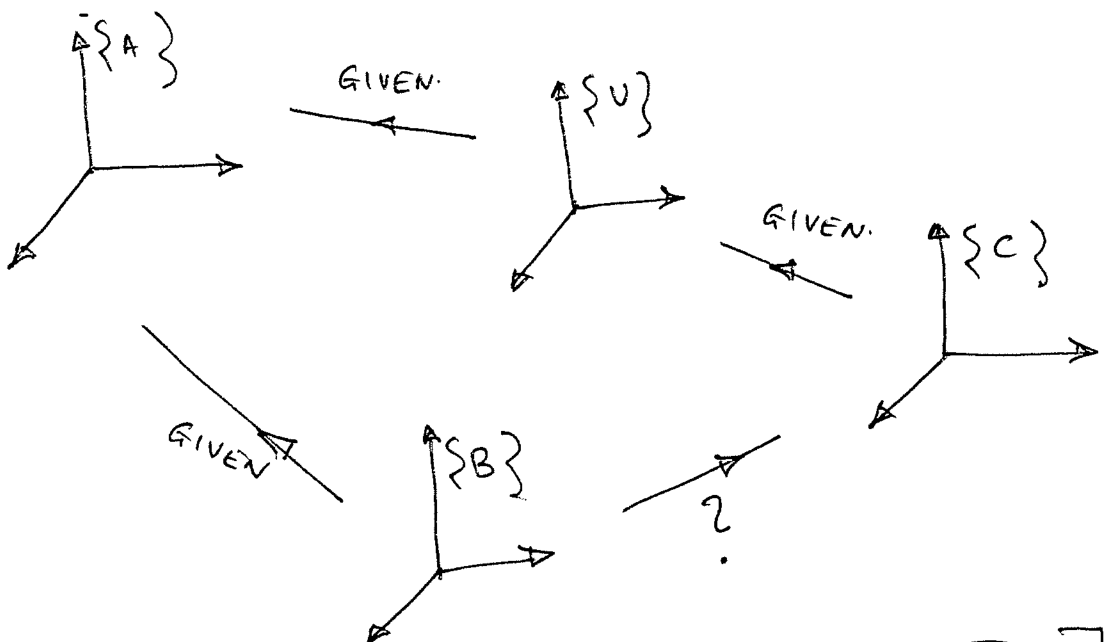
$$^{A}\begin{Bmatrix} V \end{Bmatrix} = \underbrace{\underset{3\times 3}{^{A}_{B}\begin{bmatrix} R \end{bmatrix}} \; \underset{3\times 1}{\begin{Bmatrix} 10.0 \\ 20.0 \\ 30.0 \end{Bmatrix}}}_{\text{PROG 12}}$$

2.13 [21] The following frame definitions are given as known. Draw a frame diagram (like that of Fig. 2.15) which qualitatively shows their arrangement. Solve for ${}^B_C T$.

$$
{}^U_A T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 11.0 \\ 0.500 & 0.866 & 0.000 & -1.0 \\ 0.000 & 0.000 & 1.000 & 8.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^B_A T = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.0 \\ 0.000 & 0.866 & -0.500 & 10.0 \\ 0.000 & 0.500 & 0.866 & -20.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^C_U T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & -3.0 \\ 0.433 & 0.750 & -0.500 & -3.0 \\ 0.250 & 0.433 & 0.866 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



$$
{}^B_C[T] = {}^B_A[T]\ {}^A_U[T]\ {}^U_C[\cancel{T}]
$$

$$
= {}^B_A[A]\ {}^U_A[T]^{-1}\ {}^C_U[T]^{-1}
$$

STEP 1

CALCULATE

$${}^{U}_{A}\left[T\right]^{-1} \quad \text{USING PROG 3}$$

$${}^{C}_{U}\left[T\right]^{-1} \quad \text{USING PROG 3}$$

STEP 2

MULTIPLY TWO MATRICES AT A TIME USING PROG 12

STEP 1

RESULT

$${}^{U}_{A}\left[T\right]^{-1} = \begin{bmatrix} 0.866 & 0.5 & 0.0 & -9.026 \\ -0.5 & 0.866 & 0.0 & 6.366 \\ 0.0 & 0.0 & 1.0 & -8.00 \\ 0.0 & 0.0 & 0.0 & 1.00 \end{bmatrix}$$

$${}^{C}_{U}\left[T\right] = \begin{bmatrix} 0.866 & 0.433 & 0.250 & 3.147 \\ -0.5 & 0.75 & 0.433 & -0.549 \\ 0.0 & -0.5 & 0.866 & -4.098 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

STEP 2

MULTIPLY TWO MATRICES AT A TIME USING PROG 12

FINAL RESULT

$${}^{B}_{C}\left[T\right] = \begin{bmatrix} 0.499 & 0.749 & 0.433 & -6.575 \\ -0.749 & 0.625 & -0.217 & 14.783 \\ -0.433 & -0.216 & 0.874 & -28.311 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$