CHAPTER 3

MANIPULATOR KINEMATICS

3.1 INTRODUCTION

In this chapter, we will study the location and orientation of the end effector relative to the base. There are no motions involved in this chapter. This will be achieved by first defining a stationary frame at the base and a moving frame attached to each of the links. By knowing the displacement of the origins of each of the moving frames with respect to each other and also the orientations of the axes, it will be possible to calculate the position and the orientation of the axes at the end effector with respect to the base coordinate system.

3.2 LINK DESCRIPTION

fig

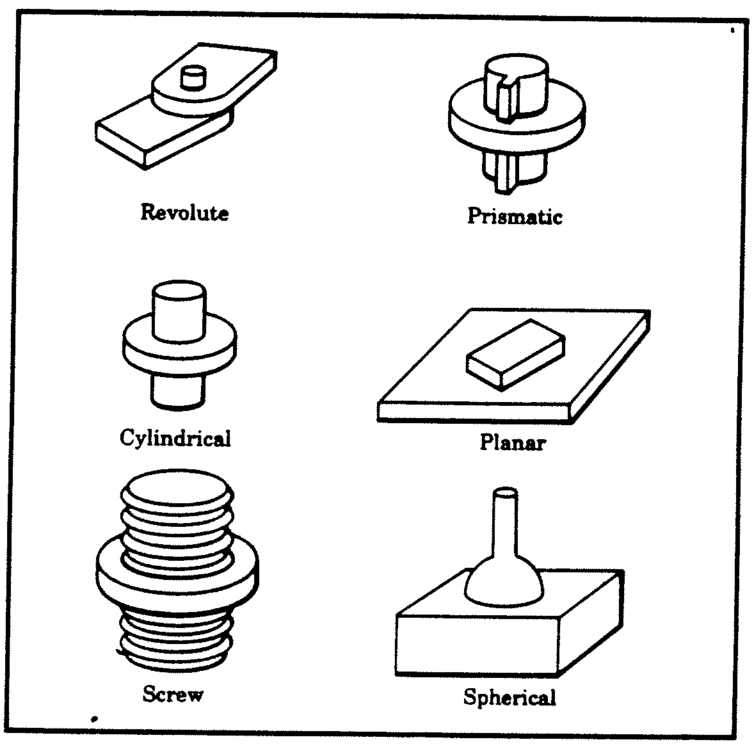

FIGURE 3.1 The six possible lower pair joints.

The manipulator may be thought of as a series of links connected to each other through the joints which are either revolute or sliding, in this course. In other words, these joints have only single degree of freedom.

A typical manipulator has six degrees of freedom. In the field of Robotics, the transformation matrices between two coordinate systems are written in accordance with the work of Denevit and Hartenberg. These matrices are also 4 x 4 but they require only 4 independent parameters, instead of 6 as seen in the last chapter. In this convention, the coordinate system i-1 is fixed on the link i-1 such that z_{i-1} axis points in the direction of the rotation vector, if it is a rotating or pinned connection, or along the sliding direction, in the case of the prismatic joint. The x_{i-1} axis is located or aligned along the link length. By fixing these two axes, one automatically fixes the y_{i-1} axis.

In this convention, there are two screw axes involved. At first one rotates about x_{i-1} axis such that z_{i-1} axis coincides with the z_i axis and this rotational angle is called α_i. Next, the second rotation is given about this z_{i-1} axis which is now coincident with the z_i axis until x_{i-1} axis coincides with the x_i axis. In the first rotation i.e., when screw rotates by α degrees, it also advances or translates by a_i meters. Similarly, in the second rotation, the translation involved is d_i meters and the rotation of the z_{i-1} axis is by θ_i degrees. This is clearly shown in fig 3.5. In actual robotic manipulators the design is such that only θ_i is a variable and the other parameters are fixed for a given manipulator. It

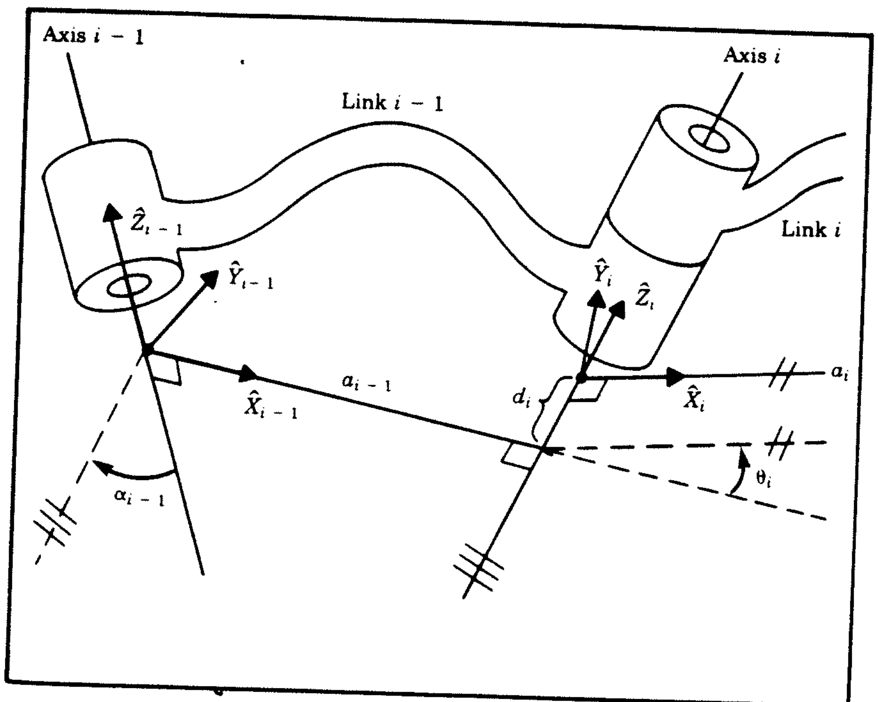

FIGURE 3.5 Link frames are attached so that frame $\{i\}$ is attached rigidly to link i.

will be the convention in this course to use

1) $$a_o = a_n = 0.0$$

2) $$\alpha_o = \alpha_n = 0.0$$

3) The origin or the coordinate system will be located at the beginning of the link.

4) If the joint 1 is a revolute then the zero position is selected at θ_1 degrees from the base coordinate system.

The Denevit-Hartenberg Transformation matrices can be written as a product of four individual matrices and the expression will be

$$ {}^{i-1}_{i}[\,T\,] = [R_x(\alpha_{i-1})]\ [D_x(a_{i-1})]\ [R_z(\theta_i)]\ [D_z(d_i)] $$

$$ = [Screw_x(a_{i-1},\alpha_{i-1})][Screw_z(d_i,\theta_i)] $$

$$ {}^{i-1}_{i}[T] = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

THREE LINK MANIPULATOR (PLANAR)

The D-H parameters for this manipulator are given in fig 3.8. It is a good example where one can verify our concepts from planar kinematics. Suppose we use a numerical value for $\theta_1 = 30°$, $\theta_2 = 40°$ $\theta_3 = 30°$ and $L_1 = L_2 = 10$ cm. If $L_3 = 5$ cm then

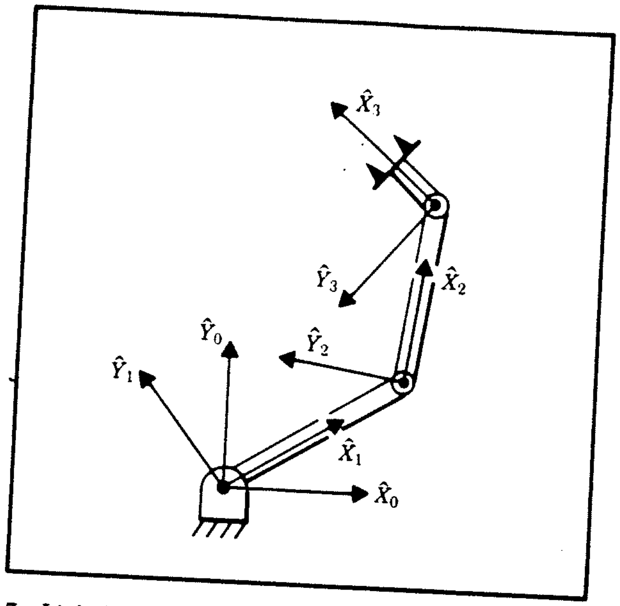

FIGURE 3.7 Link frame assignments.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

FIGURE 3.8 Link parameters of the three-link planar manipulator.

$${}^{0}_{3}[T] = {}^{0}_{1}[T]\,{}^{1}_{2}[T]\,{}^{2}_{3}[T]$$

$$= {}^{0}_{3}\begin{bmatrix} -0.174 & -.0984 & 0 & 1.207 \times 10^{-1} \\ 0.984 & -0.174 & 0 & 0.144 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [R_z\,(\theta-108)]$$

From the fig 3.7, we can see that

$${}^{0}\{E.E.\} = {}^{0}_{3}[T]\begin{Bmatrix} 0.05 \\ 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0.11 \\ 0.193 \\ 0 \\ 1 \end{Bmatrix}$$

In the polar coordinates the location will be

$E\ E = 10\ \angle 30 + 10\ \angle 70 + 5\ \angle 100$

$= 22.339\ \angle 59.873 = 11.212i + 19.321j + 0k$ cm

$= 0.112i + 0.193j + 0k$ m

PUMA 560

The D-H Parameters for PUMA 560 manipulator are given in the Table (P 91). The Position vector of the origin at the end effector can be calculated using the formula

$$ {}^{0}_{6}[T] - {}^{0}_{1}[A] \ldots {}^{5}_{6}[A] \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} $$

where the symbol ${}^{i-1}{}_{i}[A]$ has been used instead of ${}^{i-1}{}_{i}[T]$ to indicate the transformation between the coordinate systems attached to the moving links.

fig

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

FIGURE 3.21 Link parameters of the PUMA 560.

If we do not substitute the numerical values for α_i, a_i, etc. and keep it in the symbolic form then we can obtain the expression

$$
{}^{0}_{6}[T] = \begin{bmatrix} r_{11} & \cdot & \cdot & \cdot & r_{13} & p_x \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ r_{31} & \cdot & \cdot & \cdot & r_{33} & p_z \\ 0 & \cdot & \cdot & \cdot & 0 & 1 \end{bmatrix}
$$

where

Eq(3.14)

$$
\begin{aligned}
r_{11} &= c_1\left[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6\right] + s_1(s_4c_5c_6 + c_4s_6),\\
r_{21} &= s_1\left[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6\right] - c_1(s_4c_5c_6 + c_4s_6),\\
r_{31} &= -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,\\
\\
r_{12} &= c_1\left[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6\right] + s_1(c_4c_6 - s_4c_5s_6),\\
r_{22} &= s_1\left[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6\right] - c_1(c_4c_6 - s_4c_5s_6),\\
r_{32} &= -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,\\
\\
r_{13} &= -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,\\
r_{23} &= -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,\\
r_{33} &= s_{23}c_4s_5 - c_{23}c_5,\\
\\
p_x &= c_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] - d_3s_1,\\
p_y &= s_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] + d_3c_1,\\
p_z &= -a_3s_{23} - a_2s_2 - d_4c_{23}.
\end{aligned}
\tag{3.14}
$$

This equation constitutes the kinematics of the PUMA 560. It specifies how to compute the position and orientation of the end effector.

3.8 FRAMES WITH STANDARD NAMES

In the task performance or planning, several "Standard" frames are necessary. All robot motions are described in terms of these frames.

(a) The Base Frame {B}

It is located at the base of the manipulator. It is merely another name for frame $^{0}\{\ \}$.

(b) The Station Frame {S}

The {S} is located at a task relevant location. It serves as the standard frame or the reference frame for the task. As far as the robot system is concerned, {S} serves as the universal frame and all actions of the robot are made relative to it. It is also called the WORLD FRAME or the TASK FRAME.

(c) The Wrist Frame {W}

{W} is affixed to the last link of the manipulator. It is another name for frame {N}, the link frame attached to the last link of the robot.

(d) The Tool Frame {T}

It is affixed to the end of the tool, the robot happens to be holding. The tool frame is specified with respect to the wrist frame.

(e) The Goal Frame {G}

{G} is a description of the location to which the robot has to move the tool to. In other words, at the end of the motion the tool frame {T} should be identical to the Goal Frame {G}. It requires

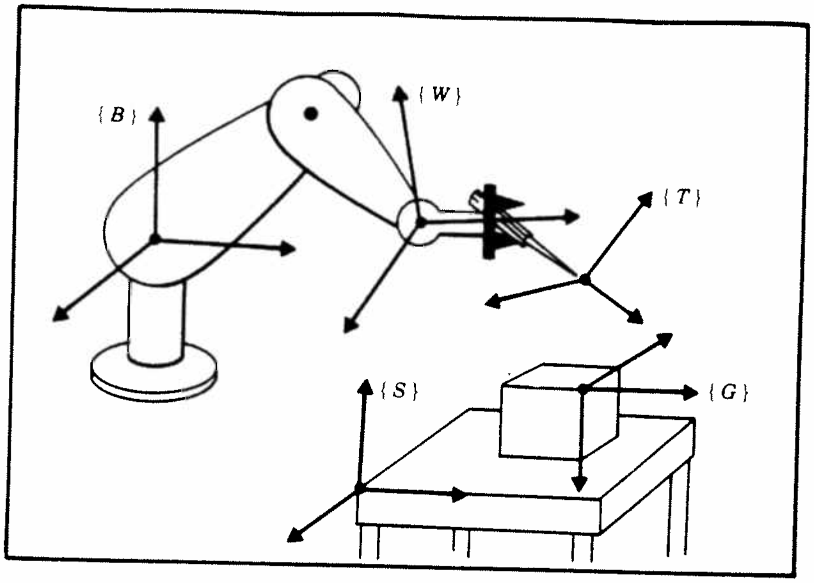

FIGURE 3.27 The standard frames.

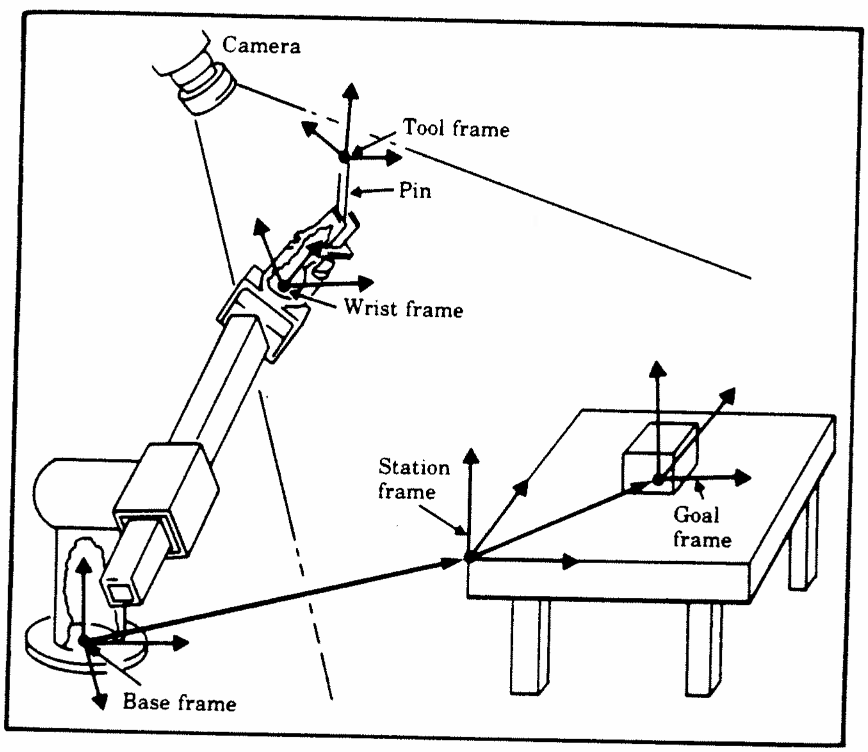

FIGURE 3.28 Example of the assignment of standard frames.

identity in both the location as well as the orientation.

3.9 WHERE IS THE TOOL ?

One of the first capabilities a robot must have is to be able to calculate the position and orientation of the tool it is holding with respect to {S}. This can be done using the relationship

$$ {}^{S}_{T}[T] = {}^{B}_{S}[T]^{-1}\ {}^{B}_{W}[T]\ {}^{W}_{T}[T] \qquad (3.18) $$