CHAPTER 5
VEL. ANALYSIS

PROBLEM 1

1) A two-link manipulator with rotational joint is shown in fig. 1 Calculate the velocity of the tip of the arm using

$L_1 = 5$ cm, $L_2 = 10$ cm; $L_3 = 5$ cm;
$\theta_1 = 30^\circ$, $\theta_2 = 80^\circ$, $\theta_3 = 0^\circ$
$\dot{\theta}_1 = 5$ rad/s $\dot{\theta}_2 = 10$ rad/s $\dot{\theta}_3 = 7$ rad/s

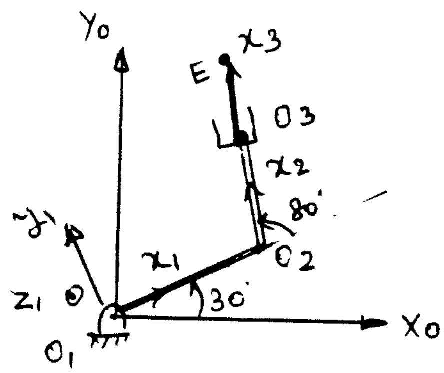


fig. 1

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	30
2	0	0.05	0	80
3	0	0.10	0	0

Table 1 D-H Parameters

SOLUTION

$$ {}^{0}_{1}[T] = \begin{bmatrix} 0.8659 & -0.5001 & 0.0 & 0.0 \\ 0.5001 & 0.8659 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$

→ Use 2: in Robotics Software

$$ {}^{1}_{2}[T] = \begin{bmatrix} 0.1730 & -0.9849 & 0.0 & 0.05 \\ 0.9849 & 0.1730 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \rightarrow $$ From 2: in Robotics Software

$$ {}^{2}_{3}[T] = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.10 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$

$$ {}^{0}_{3}[T] = {}^{0}_{1}[T] * {}^{1}_{2}[T] * {}^{2}_{3}[T] $$

$$ = \begin{bmatrix} -0.3427 & -0.9394 & 0.0 & 0.009 \\ 0.9394 & -0.3427 & 0.0 & 0.118 \\ 0.0 & 0.0 & 1.0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$

All the above four D-H matrices are got from Case no: 2 (D-H parameters) in ROBOTICS SOFTWARE

For calculations below, use 12 for matrix multiplication 3 for inverse & 8 for cross product of vectors;

Note: (i) [R] is got from [T] by removing the last column and row

(ii) $ {}^{1}_{0}[R] = {}^{0}_{1}[R]^{-1} $

in general $ {}^{i+1}_{i}[R] = {}^{i}_{i+1}[R]^{-1} $ } use 3 to find the inverse

$$ {}^1\{\omega_1\} = {}^1_0[R]\,{}^0\{\cancel{\omega_0}\}^{=0} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{Bmatrix} $$

$$ {}^1\{\omega_1\} = \begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix} $$

$$ {}^1\{v_{0_1}\} = {}^1_0[R]\left({}^0\{\cancel{v_0}\}^{=0} + {}^0\{\cancel{\omega_0}\}^{=0} \times \{\cancel{R_{a}}\}^{=0} \right) $$

$$ {}^1\{v_{0_1}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} $$

$$ {}^2\{\omega_2\} = \underbrace{{}^2_1[R]^{\circledast}\,{}^1\{\omega_1\}}_{\text{use } 12} + {}^2\begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{Bmatrix} \qquad \circledast\ {}^2_1[R] = {}^1_2[R]^{-1} $$

$$ = \begin{bmatrix} 0.173 & 0.9849 & 0.0 \\ -0.9849 & 0.173 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 10 \end{Bmatrix} $$

$$ = \begin{Bmatrix} 0 \\ 0 \\ 15 \end{Bmatrix} $$

$$ {}^2\{v_{0_2}\} = {}^2_1[R]\left({}^1\{\cancel{v_{0_1}}\}^{=0} + \underbrace{{}^1\{\omega_1\} \times {}^1\{R_{0_2 0_1}\}}_{\text{use } 8} \right) $$

$$ = \begin{bmatrix} 0.173 & 0.9849 & 0.0 \\ -0.9849 & 0.173 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \left(\begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix} \times \begin{Bmatrix} 0.05 \\ 0 \\ 0 \end{Bmatrix} \right) $$

$$= \underbrace{\begin{bmatrix} 0.173 & 0.9849 & 0.0 \\ -0.9849 & 0.173 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.25 \\ 0 \end{Bmatrix}}_{\text{use } \underline{12}}$$

$$= \begin{Bmatrix} 0.246 \\ 0.04325 \\ 0.0 \end{Bmatrix}$$

$${}^3\{\omega_3\} = {}^3_2[R]\,{}^2\{\omega_2\} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{Bmatrix}$$

$$= \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 15.0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 22.0 \end{Bmatrix}$$

$${}^3\{v_{03}\} = {}^3_2[R]\left({}^2\{v_{02}\} + {}^2\{\omega_2\} \times \begin{Bmatrix} 0.10 \\ 0 \\ 0 \end{Bmatrix}\right)$$

$$= \underbrace{\begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \left(\begin{Bmatrix} 0.246 \\ 0.04325 \\ 0.0 \end{Bmatrix} + \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 15 \end{Bmatrix} \times \overbrace{\begin{Bmatrix} 0.10 \\ 0 \\ 0 \end{Bmatrix}}^{\text{use } 8}}_{\text{use } 4} \right)}_{\text{use } 12}$$

$$= \begin{Bmatrix} 0.246 \\ 1.54325 \\ 0.0 \end{Bmatrix}$$

$${}^3\{v_{E3}\} = {}^3\{v_{03}\} + {}^3\{\omega_3\} \times {}^3\begin{Bmatrix} O_3E \\ 0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.246 \\ 1.54325 \\ 0.0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 22.0 \end{Bmatrix} \times \begin{Bmatrix} 0.05 \\ 0.0 \\ 0.0 \end{Bmatrix} = \begin{Bmatrix} 0.246 \\ 1.543 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1.10 \\ 0 \end{Bmatrix}$$

$$^{3}\{V_{E_3}\} = \begin{Bmatrix} 0.246 \\ 2.643 \\ 0 \end{Bmatrix}$$

$$^{0}\{V_{E_3}\} = {}^{0}_{3}[R] \begin{Bmatrix} 0.246 \\ 2.643 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2.567 \\ -0.674 \\ 0 \end{Bmatrix}$$

PLANAR ANALYSIS

$$\underline{V}_{O_1} = \underline{0}$$

$$\underline{V}_{O_2} = \cancel{\underline{V}_{O_1}}^{\,0} + \underline{V}_{O_2O_1}$$

$$= \omega_1 \times \underline{R}_{O_2O_1}$$

$$= 5 \times 0.05 \angle(30+90)$$

$$= 0.25 \angle 120^\circ$$

PLANAR ANALYSIS

$$\underline{V}_{B_3} = \underline{V}_{B_2} + \underline{\omega}_2 \times \underline{R}_{B_3 O_2}$$

$$= 0.25\angle 120 + \underbrace{1\cancel{5}}_{\omega_2} \times \underbrace{0.1}_{R_{O_3 O_2}} \angle(110+90)$$

$$= 0.25\angle 120 + 1.5\angle 200^0$$

$$= 1.563\angle 190.94 = \begin{Bmatrix} -1.534 \\ -0.296 \\ 0 \end{Bmatrix}$$

$$\underline{V}_{E_3} = \underline{V}_{O_3} + \underline{\omega}_3 \times R_{E_3 O_3}$$

$$= 1.563\angle 190.94 + 22 \times .05\angle 200$$

$$= 1.563\angle 190.94 + 1.1\angle 200$$

$$= 2.654\angle 194.68 = \begin{Bmatrix} -2.567 \\ -0.672 \\ 0 \end{Bmatrix}$$

PROBLEM 2

I A three-link manipulator is shown in fig. 2:

$L_1 = 5$ cm, $L2 = 10$ cm, $L3 = 5$ cm
$\theta_1 = 30^\circ$; $\theta_2 = ?$ $\theta_3 = ?$
$\dot{\theta}_1 = 5$ rad/s $\dot{\theta}_2 = 10$ rad/s $\dot{d}_3 = 4$ cm/s

Calculate ?

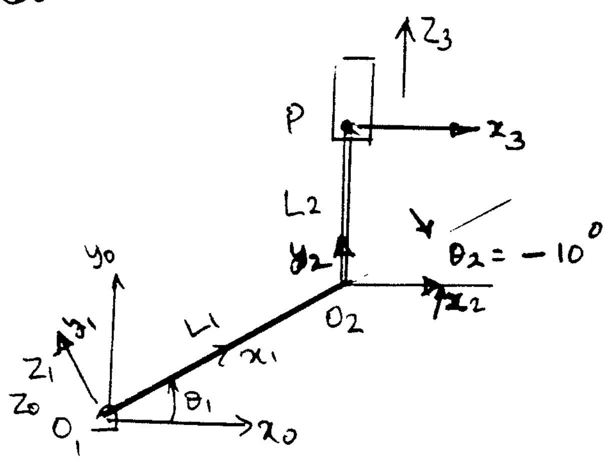


fig. 2

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	30°
2	0	0.05	0	−10°
3	−90°	0	0.10	0

Table 2. D-H Parameters.

SOLUTION

$$ {}^0_1[T] = \begin{bmatrix} 0.8659 & -0.5001 & 0.0 & 0.0 \\ 0.5001 & 0.8659 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} $$ → Use 2: in Robotics Software

$$
{}^{1}_{2}[T] = \begin{bmatrix} 0.98479 & 0.17371 & 0.0 & 0.05 \\ -0.17371 & 0.98479 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \rightarrow \text{from 2 in Robotics software}
$$

$$
{}^{2}_{3}[T] = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -6.32e-4 & 0.9999 & 0.099 \\ 0 & -0.999 & -6.32e-4 & -6.32e^{-5} \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \rightarrow \text{"}
$$

$$
{}^{0}_{3}[T] = {}^{0}_{1}[T] * {}^{1}_{2}[T] * {}^{2}_{3}[T] \quad \rightarrow \text{USING PROG 2}
$$

$$
= \begin{bmatrix} 0.93964 & 2.1635E-4 & -3.4215E-1 & 9.0807E-3 \\ 3.4212E-1 & -5.94165E-4 & 9.3964E-1 & 1.18973E-1 \\ 0 & -9.999E-1 & -6.3232E-4 & -6.3232E-5 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}
$$

All the above four D-H matrices are got from case no 2 (D-H parameters) in ROBOTICS SOFTWARE

For calculations given below: use $\underline{12}$ for matrix multiplication, $\underline{\underline{3}}$ for inverse & $\underline{\underline{8}}$ for cross product of vectors.

<u>Note</u>: (i) [R] is got from [T] by removing the last column and row

(ii) $${}^{i+1}_{i}[R] = {}^{i}_{i+1}[R]^{-1} = {}^{i}_{i+1}[R]^{T}$$

Use $\underline{\underline{3}}$ to find inverse

$$ {}^{1}\{\omega_1\} = {}^{1}_{0}[R]\,\cancel{{}^{0}\{\omega_0\}}^{=0} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{Bmatrix} \qquad \text{EQ (5-45)} $$

$$ = \begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix} $$

$$ {}^{1}\{v_{01}\} = {}^{1}_{0}[R]\left(\cancel{{}^{0}\{v_0\}}^{=0} + \cancel{{}^{0}\{\omega_0\}}^{=0} \times \cancel{{}^{0}\{R_{01}\}}^{=0} \right) \qquad \text{EQ (5-47)} $$

$$ = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} $$

$$ {}^{2}\{\omega_2\} = {}^{2}_{1}[R]\,{}^{1}\{\omega_1\} + {}^{2}\begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{Bmatrix} $$

$$ = \begin{bmatrix} 0.98479 & -0.17371 & 0.0 \\ 0.17371 & 0.98479 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 10 \end{Bmatrix} $$

$$ = \begin{Bmatrix} 0 \\ 0 \\ 15 \end{Bmatrix} $$

$$ {}^{2}\{v_{02}\} = \underbrace{{}^{2}_{1}[R]}_{\text{use 12 — OPER 3}}\left(\cancel{{}^{1}\{v_{01}\}}^{=0} + \underbrace{\underbrace{\{\omega_1\} \times {}^{1}\{R_{0_2 0_1}\}}_{\text{use 8 — OPER 1}}}_{\text{use 4 — OPER 2}} \right) $$

$$ = \underbrace{\begin{bmatrix} 0.98479 & -0.17371 & 0 \\ 0.17371 & 0.98479 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^{2}_{1}[R]} \left(\underbrace{\begin{Bmatrix} 0 \\ 0 \\ 5 \end{Bmatrix}}_{{}^{1}_{1}\{\omega\}} \times \underbrace{\begin{Bmatrix} 0.05 \\ 0 \\ 0 \end{Bmatrix}}_{{}^{1}\{R_{0_2 0_1}\}} \right) $$

$$= \begin{bmatrix} 0.98479 & -0.17371 & 0 \\ 0.17371 & 0.98479 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.25 \\ 0 \end{Bmatrix}$$

$$^2\{v_{O_2}\} = \begin{Bmatrix} -0.043427 \\ 0.24619 \\ 0.0 \end{Bmatrix}$$

$$^3\{\omega_3\} = {}^3_2[R]\,{}^2\{\omega_2\} + \begin{Bmatrix} 0 \\ 0 \\ \cancel{\dot{\theta}_3} \end{Bmatrix} = 0$$

SLIDING CONNECTION

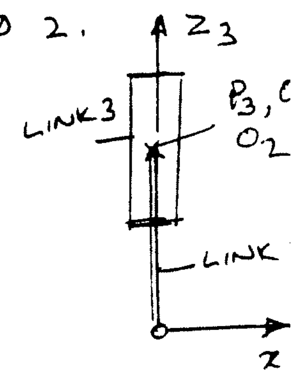
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 15 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ -15 \\ 0 \end{Bmatrix}$$

ROTATING CW FROM LOCAL Y_3 DIRECTION

NOTE:

P_3 AND O_3 ARE SAME POI[NT]
P_3 AND P_2 ARE COINCIDEN[T] POINTS BUT ON LINKS 3 AND 2.



$$^3\{v_{P_2}\} = {}^3_2[R]\left({}^2\{v_{O_2}\} + {}^2\{\omega_2\} \times \begin{Bmatrix} 0 \\ 0.10 \\ 0 \end{Bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \left(\begin{Bmatrix} -0.043427 \\ 0.24619 \\ 0.0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 15 \end{Bmatrix} \times \begin{Bmatrix} 0 \\ 0.10 \\ 0 \end{Bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} -1.543427 \\ 0.24619 \\ 0.0 \end{Bmatrix} = \begin{Bmatrix} -1.543427 \\ 0 \\ 0.24619 \end{Bmatrix}$$

$$^3\{v_{P_3}\} = {}^3\{v_{O_3}\} = {}^3\{v_{P_2}\} + {}^3\begin{Bmatrix} 0 \\ 0 \\ \dot{d}_3 \end{Bmatrix}$$

$$= \begin{Bmatrix} -1.543427 \\ 0.0 \\ 0.24619 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0.04 \end{Bmatrix} \quad \dot{d}_3 = 4\,cm/s$$

$$= \begin{Bmatrix} -1.543427 \\ 0 \\ 0.28619 \end{Bmatrix}$$

$${}^0\{v_{03}\} = {}^0_3[R]\,{}^3\{v_{03}\}$$

$$= \begin{bmatrix} 0.9396 & 0. & -0.34215 \\ 0.34215 & 0. & 0.9396 \\ 0 & -1.0 & 0.0 \end{bmatrix} \begin{Bmatrix} -1.543427 \\ 0 \\ 0.28619 \end{Bmatrix}$$

$$= \begin{Bmatrix} -1.548124 \\ -0.26 \\ 0 \end{Bmatrix}$$