Chapter 9

LINEAR CONTROL OF MANIPULATORS

9.1 INTRODUCTION

We would want to control the wrist motions to take the tool through a specified space curve. To achieve this, we would represent the dynamics of the system by a linear differential equation whose solutions are simpler and known. A linear differential equation has constant coefficients. Since these coefficients are a function of joint variables, the solution will be valid in the neighborhood of a point in space. Since the space curve is made up of large number of points, one has to recompute each of the coefficients and the forces involved, point by point.

$$\tau = M(\theta_d)\ddot{\theta}_d + V(\theta_d \dot{\theta}_d) + G(\theta_d) \qquad \text{Equation (9.1)}$$

STEP 1

In robotic control problems, one carries out the inverse kinematic analysis to calculate

$\theta_d \dot{\theta}_d \ddot{\theta}_d$ from the specification of the task.

STEP 2

Compute the (τ) by substituting kinematic parameters θ etc. on the right hand side of Equation (9.1).

If our dynamic model was accurate then the end effector would move along the specified path. However, this would not happen. Actually, there would be errors. In an open loop control, one applies the (τ)) vector and no effort is made to correct for the errors.

On the other hand, to achieve greater accuracy, one modifies the applied torque based on the error at the previous instant of time. These errors are used to calculate additional torques, besides the theoretical torque vector. This process of correction is called the feed-back which is dependent upon the actual position, actual velocity of the end effector which is sensed by the sensors, and the corresponding desired position, or desired velocity etc, one can define the errors as:

$$
\begin{aligned}
E &= \theta_d - \theta_d \\
\dot{E} &= \dot{\theta}_d - \dot{\theta}_a
\end{aligned}
\qquad (9.2)
$$

Such a system of torque based control is called closed - loop system. A stable system is one where the errors build up with time.

9.4 CONTROL OF SECOND-ORDER SYSTEMS

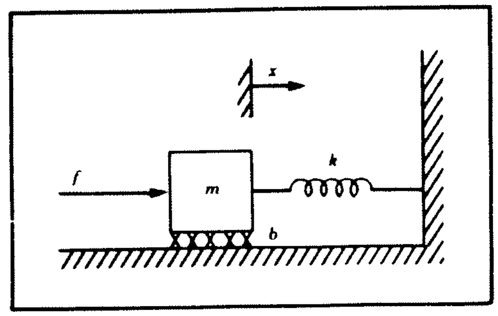

FIGURE 9.6 A damped spring-mass system with an actuator.

A second order system with a forcing function is written as:

$$m\ddot{x} + b\dot{x} + kx = f \qquad (9.34)$$

Suppose we have sensors which can measure and report $x\,,\ \dot{x}$ etc.

We propose a control law which computes the force which should be applied by the actuator (motor for rotary motion or solenoid for linear motion). This force is proportional to the displacement and velocity errors (by applying in opposite direction). Here the proportionality constaints are k_p and k_v. The minus sign is to compensate for the positive or negative values of x *and* $\dot{x}$

In this case, the resulting differential equation will be

$$m\ddot{x}+bx+kx=-k_p x-k_v x \qquad \text{Equation 9.36}$$

$$m\ddot{x}+(b+k_v)x+(k+k_p)x=0 \qquad \text{Equation 9.37}$$

$$m\ddot{x}+b^1x+k^1x=0 \qquad \text{Equation 9.38}$$

Here we have

$$b^l = b + k_v\,;\; k^l = k + k_p$$

Here k_p, k_v are control gains. Often these k_p and k_v are chosen such that we have a critically damped system. These values should not make b^l or k^l negative; in which case if negative the system will become unstable.

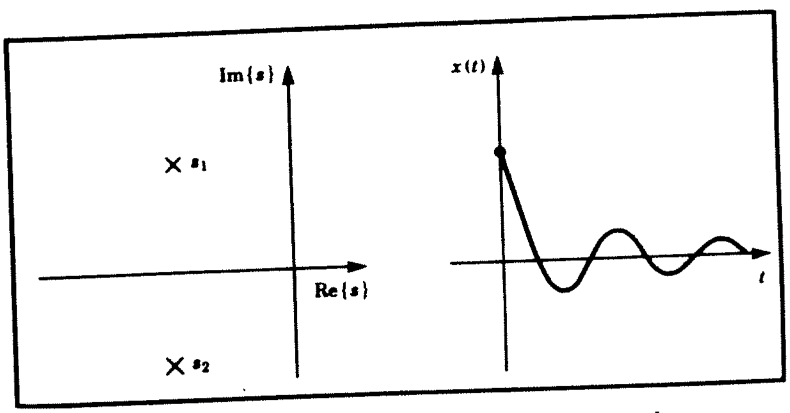

FIGURE 9.4 Root location and response to initial conditions for an underdamped system.

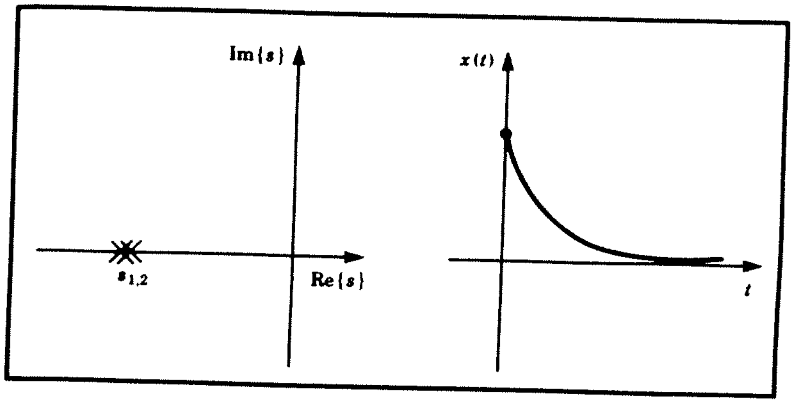

FIGURE 9.5 Root location and response to initial conditions for a critically damped system.

9.6 TRAJECTORY FOLLOWING CONTROL

A trajectory is specified where the subscript d represents the desired value. Defining error as $e = x_d - x$, a servo control law which will cause trajectory following is

$$f^{1} = x_d + k_v \dot{e} + k_p e \qquad (9.50)$$

If we combine this equation with the equation of motion of a unit mass equation (9.44), which is

$$\ddot{x} = f^{1}$$

we get

$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e \qquad (9.52)$$

This is a second order differential equation where, we can choose the coefficients k_v, and k_p to make the error e, stable.

9.7 DISTURBANCE REJECTION

We have the equation

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

and , we want to maintain good performance (small errors) even in the presence of some external disturbance or noise.

The error equation in this case is

$$\ddot{e} + k_v \dot{e} + k_p e = f_{dist} \qquad (9.53)$$

STEADY STATE ERROR

Let us consider the simplest case when f_{dist} = constant (like a static force).

The steady state response will also be a constant value

$$k_p e = f_{dist} \qquad (9.55)$$

$$e = f_{dist} / k_p \qquad (9.56)$$

The higher the k_p value, the smaller will be the error.

ADDITION OF AN INTEGRAL TERM

Suppose we have an additional integral term

$$f^1 = \ddot{x}_c + k_v \dot{e} + k_p e + k_i \int e\, dt \qquad (9.57)$$

This results in the error equation as

$$e + k_v e + k_p e + k_i \int e\, dt = f_{dist} \qquad (9.58)$$

This term has been added to make the steady state error = 0

If e(t) < 0 for t < 0. We can write for t > 0 by differentiating Equation (9.58) which in the case of constant distanbance becomes {here $f_{dist} = 0$}.

$$k_i e = 0$$

$$or \qquad e = 0$$

Here one can solve Equation (9.59) to calculate the error e as a function of time.

The form of Equation (9.57) is called PID (Proportional Integral Derivative Control Law).