SOLAR PHOTOVOLTAIC PROBLEMS - Engi 9614

10-1)

10-2)

10-3)

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10-1) $\eta_{col} = 97\%$ for photon energy from

1.5×10^{-19} J to 6.0×10^{-19} J.

$\alpha(E,W) = 82\%$, photon frequency as in Fig. 10-5.

$S = 2.5 \times 10^{21}$ particles / s·m², Weibull ($\alpha = 3$, $\beta = 3$).

Find $I_L = e \int_{E_G}^{\infty} \eta_{col}(E) S(E) \alpha(E,W)\, dE$

$$I_L = e\, \eta_{col}(E)\, \alpha(EW)\, S \int_{1.5}^{6.0} F(E) \quad \text{where } F(E) = 1 - \exp\left[-\left(\frac{E}{\beta}\right)^{\alpha}\right]$$

F = cumulative distribution function of Weibull equation

$$P(1.5 < x\ 6.0) = F(6) - F(1.5)$$
$$F(6) = 1 - \exp\left[-(6/3)^3\right] = 0.9997$$
$$F(1.5) = 1 - \exp\left[-(1.5/3)^3\right] = 0.1175$$

$$P(1.5 < x < 6) = 0.9997 - 0.1175 = 0.8822$$

$$I_L = e(0.97)(0.82)(2.5 \times 10^{21} \text{ particles/s·m}^2)(0.8822)$$

$$= 1.754 \times 10^{21}\ e/\text{s·m}^2$$

1 A current flow = 6.241×10^{18} electrons / s.

$$I_L = \frac{1.754 \times 10^{21}\ e}{\text{s·m}^2} \cdot \frac{\text{s·A}}{6.241 \times 10^{18}\ e}$$

$$= 281 \frac{A}{m^2} \quad \text{or} \quad 0.0281 \frac{A}{cm^2}$$

10-2) $I_L = 0.035\ A/cm^2$ light current
$I_0 = 1.5 \times 10^{-10}\ A/cm^2$ saturation current
$T = 32^\circ C = 32 + 273.15 = 305.15$ kelvins
$m = 1.$

Find maximum voltage (V_{oc}), maximum current (I_{sc}) and fill factor. (FF)

Plot I and V as a function of voltage to show that V_{oc} and I_{sc} values are correct.

$I_L = I_{sc}$ because $I_D = 0$ when $V=0$.

$$V_{oc} = \frac{mkT}{e} \ln\left(\frac{I_L}{I_0} + 1\right)$$
$$= \frac{(1)(1.38 \times 10^{-23}\ J/K)(305.15\ K)}{1.602 \times 10^{-19}\ J/V} \ln\left(\frac{0.035\ A/cm^2}{1.5 \times 10^{-10}\ A/cm^2} + 1\right)$$
$$= (0.02629\ V)(19.27)$$
$$= 0.5066\ V$$

$$a = 1 + \ln\left(\frac{I_L}{I_0}\right) = 1 + \ln\left(\frac{0.035\ A/cm^2}{1.5 \times 10^{-10}\ A/cm^2}\right) = 20.27$$
$$b = \frac{a}{a+1} = \frac{20.27}{20.27 + 1} = 0.953$$

$$FF = (1 - a^{-b})\left(1 - \frac{\ln a}{a}\right) = (1 - 20.27^{-0.953})\left(1 - \frac{\ln 20.27}{20.27}\right)$$
$$= (0.9432)(0.8515) = \underline{\underline{0.803 = FF}}$$

$I_{sc} = \underline{\underline{0.035\ A/cm^2}}$
$V_{oc} = \underline{\underline{0.5066\ V}}$

I_m and V_m will maximize power output.

$$I_m = I_L(1 - a^{-b}) = 0.035\ A/cm^2\ (1 - 20.27^{-0.953})$$
$$= 0.0330\ A/cm^2$$

$$V_m = V_{oc}\left(1 - \frac{\ln a}{a}\right) = 0.5066\ V\left(1 - \overbrace{\frac{\ln 20.27}{20.27}}^{0.8515}\right)$$
$$= 0.4314\ V$$

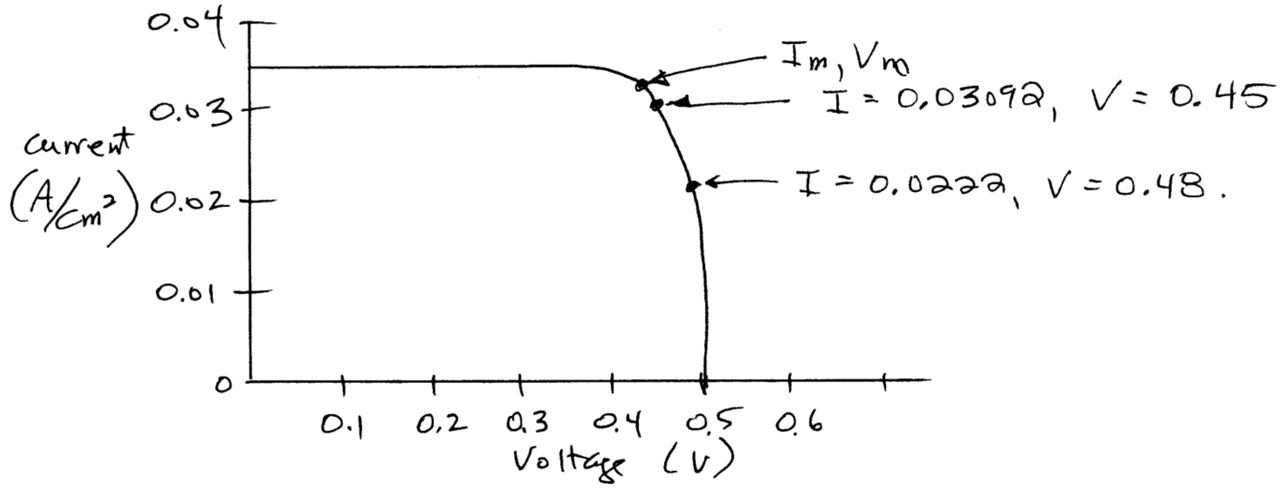
0.4312

10-2) continued.

max. power output $= I_m \times V_m$
$= 0.0330\ A/cm^2 \times 0.4314\ V$
$= 0.01424\ W/cm^2$
$= 14.24\ mW/cm^2.$

As a double check of calculations:

$V_{oc} \times I_{sc} \times FF = 0.5066\ V \times 0.035\ A/cm^2 \times 0.803$
$= 0.01424\ W/cm^2$
$= 14.24\ mW/cm^2.$



$I_D = 0$ at I_{sc}, but then increases exponentially with voltage.

At $V = 0.45\ V$.

$$I_D = I_0 \left[\exp\left(\frac{eV}{mKT}\right) - 1\right]$$

$$= 1.5\times10^{-10}\frac{A}{cm^2}\left[\exp\left(\frac{1.602\times10^{-19}\ J/V \times 0.45\ V}{1 \times 1.38\times10^{-23}\ \frac{J}{K} \times 305.15\ K}\right)\right]$$

$$= 1.5\times10^{-10}\left[\exp(17.119)\right] = 4.082\times10^{-3} = 0.004082\ \frac{A}{cm^2}$$

$$I = I_L - I_D = 0.635\ \frac{A}{cm^2} - 0.004082\ \frac{A}{cm^2} = 0.03092\ \frac{A}{cm^2}$$

10-2) continued.

At $V = 0.4314 \quad (= V_m)$

$$I_D = I_0 \left[\exp\left(\frac{eV}{mKT}\right) - 1\right]$$

$$= 1.5\times10^{-10}\left[\exp\left(\frac{1.602\times10^{-19}\times 0.4314}{1\times 1.38\times10^{-23}\times 305.15}\right)\right]$$

$$= 1.5\times10^{-10}\left[\exp(16.41)\right] \approx 0.00201 \ A/cm^2$$

$$I = I_L - I_D = 0.035 - 0.002 = 0.033 \quad (= I_m)$$

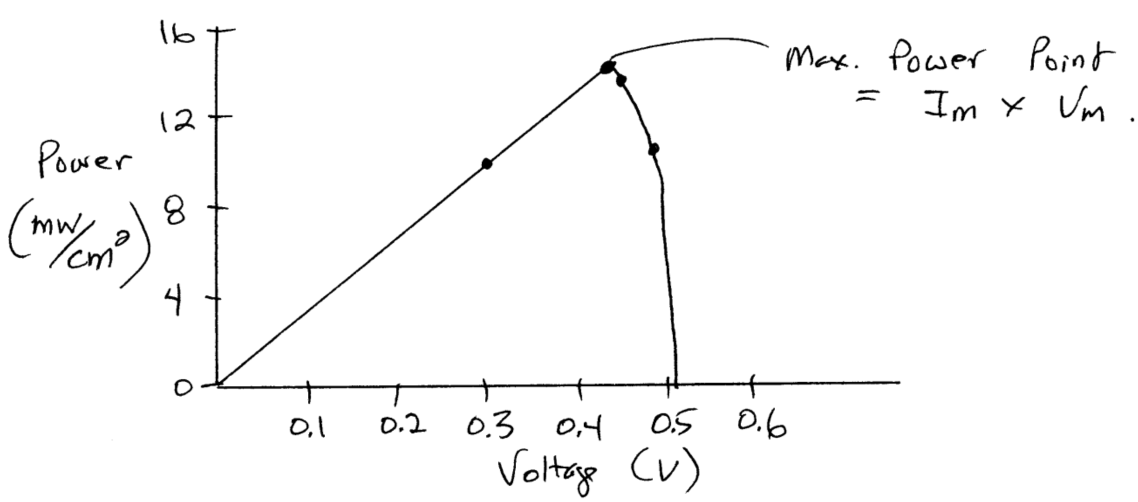
At $V = 0.48$

$$I_D = 1.5\times10^{-10}\left[\exp\left(\frac{1.602\times10^{-19}\times 0.48}{1\times 1.38\times10^{-23}\times 305.15}\right)\right] = 0.0128$$

$$I = 0.035 - 0.0128 = 0.0222 \ A/cm^2$$

Also plot current and power as a function of voltage.

$P = I \times V$; $\quad 0.03092 \times 0.45 = 0.013914 \ W/cm^2 = 13.91 \ mW/cm^2$
$0.0222 \times 0.48 = 0.010656 \ W/cm^2 = 10.66 \ mW/cm^2$
$0.0330 \times 0.4314 = \qquad = 14.24 \ mW/cm^2$



10-2) continued.

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At $V = 0.30$ V

$$I_D = 1.5\times10^{-10}\left[\exp\left(\frac{1.602\times10^{-19}\times0.30}{1\times1.38\times10^{-23}\times305.15}\right)\right]$$

$$= 1.357\times10^{-5} = 0.00001357\ A/cm^2$$

$$I = I_L - I_D = \simeq 0.035$$

$$P = 0.035\times0.30 = 0.0105\ \frac{W}{cm^2} = 10.5\ mW/cm^2$$

1.23
1.35
3.74

10-3) For $1\,m^2$ of PV panel at 39.4° North
Temp. coeff. for cell η, $\beta = 0.0055\,°C^{-1}$
Permanent tilt, 40° due south.
Efficiency at rated conditions, $\eta_r = 0.16$, $T_r = 20$
$\alpha\tau = 0.85$
Direct solar use through inverter at 93% efficiency.
Assume $U_L = 20\,W/m^2{\cdot}K = 0.02\,kW/m^2{\cdot}K$

In Jan. $\bar{H}_d = 6.15\,MJ/m^2$ avg. monthly insolation
$\bar{K}_T = 0.40$ atmospheric clearness index
$\bar{t}_{air} = -1.0°C$ avg. monthly air temperature.

For optimum device tilt:

$$\frac{U_L\,(T_c - T_a)}{\alpha\tau} = 0.219 + 0.832\,\bar{K}_T.$$

for this equations units for U_L are $kW/m^2{\cdot}K$

$$\underset{\text{at optimum tilt.}}{(T_c - T_a)} = \frac{[0.219 + 0.832(0.4)](0.85)}{0.02} = 23.45°C.$$

Tilt correction $C_f = 1.0 - 0.000117(S_m - S)^2$

$$= 1.0 - 0.000117\,(39.4 + 29 - 40)^2$$
$$= 0.906$$

$$\underset{\text{at actual tilt}}{(T_c - T_a)} = (23.45)(0.906) = 21.24°C$$

Let $(T_a - T_m) = 3°C.$

$$(T_m - T_r) = -1.0°C - 20°C = -21°C.$$

$$\eta = \eta_r\left[1 - \beta\{(T_c - T_a) + (T_a - T_m) + (T_m - T_r)\}\right]$$
$$= 0.16\left[1 - \frac{0.0055}{°C}\{(21.2 + 3 - 21)°C\}\right]$$
$$= 0.16\,(0.9824) = 0.157$$

$$E_{Jan} = 0.157\,\bar{H}_d\left(\frac{31\,d}{Jan.}\right)$$

10-3) continued

$\bar{H}$ in Jan. $= 6.15 \frac{MJ}{m^2}$ or $\frac{6,150,000 \text{ J}}{m^2} \cdot x \frac{Wh}{3600 \text{ J}} = 1708 \frac{Wh}{m^2}$

$= \frac{1.708 \text{ kWh}}{m^2} \simeq \frac{1.71 \text{ kWh}}{m^2}$

Class example 10-5) had $\bar{H}_d = \frac{4.97 \text{ kWh}}{m^2 \cdot d}$ in May.

In this example May data for $\bar{H} \simeq 3 \times$ data in Jan.
Therefore $\bar{H}$ in this problem appears to be

$\bar{H}$ in Jan. $= \frac{1.71 \text{ kWh}}{m^2 \cdot d}$ or in other words insolation in one day.

$E_{Jan.} = 0.157 \bar{H}_d \,(31 \text{ d/month}) = (0.157)\left(\frac{1.71 \text{ kWh}}{m^2 \cdot d}\right)(31 d) = 8.32 \frac{kWh}{m^2}$

Considering inverter efficiency: $E_{Jan} = (0.93)\, 8.32 \frac{kWh}{m^2} = 7.74 \frac{kWh}{m^2}$

Finally include 11 derating factors $(0.71)\left(7.74 \frac{kWh}{m^2}\right) = 5.50 \frac{kWh}{m^2}$

In July: $\bar{H}_d = 22.67 \text{ MJ}/m^2 \cdot d$
$\bar{K}_T = 0.56$
$\bar{t}_{air} = 24° C$

Optimum tilt: $(T_c - T_a) = \frac{[0.219 + 0.832(0.56)](0.85)}{0.02} = 29.11°C$

$$C_f = 1.0 - 0.000117\,(S_m - S)^2$$
$$= 1.0 - 0.000117\,(L - 24 - 40)^2$$
$$= 1.0 - 0.000117\,(39.4 - 24 - 40)^2 = 0.929.$$

Actual tilt: $(T_c - T_a) = 27.0°C$.

Let $(T_a - T_m) = 3°C$.

10-3) continued

$$(T_m - T_r) = (24°C - 20°C) = 4°C.$$

$$\eta = \eta_r \left[1 - \beta\left\{(T_c - T_a) + (T_a - T_m) + (T_m - T_r)\right\}\right]$$

$$= 0.16\left[1 - 0.0055\left\{27.0 + 3° + 4°\right\}\right]$$

$$= 0.16(0.813) = 0.130$$

$$E_{July} = 0.130\ \bar{H}_d\ (31\,d/mo) = 0.130\,(6.30\ kWh/m^2 \cdot d)(31\,d) = 25.4\ \frac{kWh}{m^2}$$

$$\bar{H}_d = 22.67\ \frac{mJ}{m^2 \cdot d} \times \frac{Wh}{3600 J} = 6{,}297\ \frac{Wh}{m^2 \cdot d} = 6.30\ \frac{kWh}{m^2 \cdot d}$$

With inverter efficiency $(0.93)(25.4\ kWh/m^2 \cdot) = 23.62\ \frac{kWh}{m^2}$

Derating factors $(0.71)(23.62) = 16.8\ \frac{kWh}{m^2}$