Solar Resource Problems – Engi 9614

9-1) a and b

9-2)

9-3)

9-6)

9-7)

9-8)

The Solar Resource – Engi 9614
Problems.

9-1) Latitude, $L = 48°N$
Due south, $\gamma = 0°$
Tilt angle, $\beta = 30°$
Equinox = 21 March or 21 September.

a) Find the incident energy/m^2/day in KWh
use average value for each hour between
sunrise and sunset.

Direct normal clear sky insolation in W/m^2
values can be taken from Table 9-3, 6.

First: calculate hour angle, ω which is 15°
+ or − per hour after or before noon

Second: find polar altitude α_s using L, δ, and ω

Third: find polar azimuth, γ_s using α_s, ω and δ

Fourth: find angle of incidence, θ_i from
α_s, γ_s, β and surface azimuth γ

Consider times from 6:00 a.m. to 6:00 p.m.
as given in Table 9-3.

See table next page.

Declination angle, δ between the rays of the
sun and the plane of the equator
is 0° at the equinox.

9-1) a) continued.

Time	Hour angle ω	Solar altitude α_s	$\sin\omega$	$\cos\alpha_s$	solar azimuth γ_s	$\sin\alpha$	$\cos\gamma_s$	$\cos\theta_i$
6:00	−90	0	−1	1	−90	0	0	0
7:00	−75	9.97	−0.969	0.985	−79.66	0.173	0.179	0.238
8:00	−60	19.54	−0.866	0.942	−66.83	0.334	0.393	0.474
9:00	−45	28.23	−0.707	0.881	−60.79	0.473	0.488	0.625
10:00	−30	35.41	−0.5	0.815	−37.84	0.579	0.790	0.823
11:00	−15	40.26	−0.259	0.763	−19.84	0.646	0.941	0.918
noon	0	42.0	0	0.743	0	0.669	1	0.951
1:00	15	40.26	0.259	0.763	19.84	0.646	0.941	0.918
2:00	30	35.41	0.5	0.815	37.84	0.579	0.790	0.823
3:00	45	28.23	0.707	0.881	60.79	0.473	0.488	0.625
4:00	60	19.54	0.866	0.942	66.83	0.334	0.393	0.474
5:00	75	9.97	0.969	0.985	79.66	0.173	0.179	0.238
6:00	90	0	1	1	90	0	0	0
5:30	82.5	5.01	0.991	0.996	84.26	0.087	0.100	0.125

$$\begin{aligned}
\sin\alpha_s &= \sin\delta \sin L + \cos\delta \cos L \cos\omega \\
&= \sin 0^\circ \sin 48^\circ + \cos 0^\circ \cos 48^\circ \cos\omega \\
&= 0 + (1)(0.669)\cos\omega \\
&= 0.669 \cos\omega \\
\alpha_s &= \sin^{-1}[0.669\cos\omega]
\end{aligned}$$

$$\sin\gamma_s = \frac{\cos\delta \sin\omega}{\cos\alpha_s} = \frac{(1)\sin\omega}{\cos\alpha_s}$$

$$\gamma_s = \sin^{-1}\left[\frac{\sin\omega}{\cos\alpha_s}\right]$$

$-$ve γ_s is east of due south
$+$ve γ_s is west of due south
(p.299)

$$\cos\theta_i = \sin\alpha \cos\beta + \cos\alpha \sin\beta \cos(\gamma - \gamma_s)$$

and $I_{\beta b} = I_{nb} \cos\theta_i$

9-1) a) continued

$$\cos \theta_i = \sin\alpha \cos 30^\circ + \cos\alpha \sin 30^\circ \cos(-\gamma_s)$$
$$= 0.866 \sin\alpha + 0.5 \cos\alpha \cos(\gamma_s)$$

Time	$\cos\theta_i$	I_{nb}	$I_{\beta b}$
6:00	0	0	0
7:00	0.238	481	114
8:00	0.474	743	352
9:00	0.625	852	533
10:00	0.823	905	745
11:00	0.918	931	855
noon	0.951	939	893
1:00	0.918	931	855
2:00	0.823	905	745
3:00	0.625	852	533
4:00	0.474	743	352
5:00	0.238	481	114
6:00	0	0	0
5:30	0.125	241	30

note: The times of 6:30 a.m and 5:30 p.m. could have I_{nb} values slightly greater than 241 W/m^2.

For 11 h there are 6,091 $\frac{Wh}{m^2}$

For 12 h 6,121 $\frac{Wh}{m^2}$

I_{nb} is taken table 9-3 for March 21st.

The above only accounts for 11 hours of the day. At the two ends of the day, the 2 half hours are not accounted for. Consider times of 6:30 a.m. and 5:30 p.m. by interpolation.

Sum last column: 6,121 W/m^2

or 6.12 kWh/m^2

Since each value is considered to occur over a period of one hour.

9-1) b) The time of 2:00 p.m. represents the hour from 1:30 p.m. to 2:30 p.m.

$$30(0.5) + 114 + 352 + 533 + 745 + 855 + 893 + 855 + 745$$

$$= 5{,}107 \text{ Wh/m}^2$$

$$\frac{5107}{6121} = 83.4\%$$

The loss of energy is $100\% - 83.4\% = 16.6\%$.

9-2) $L = 42° N$
$\gamma = 0°$ (due south)
$\beta = 23°$ tilt angle
At 10:00 a.m. on May 5th find θ_i

At 10:00 a.m., hour angle $\omega = -30$

On May 5, $N = 125$.

Declination $\delta = 23.45 \sin\left[\frac{360\,(284+125)}{365}\right] = 16.1°$

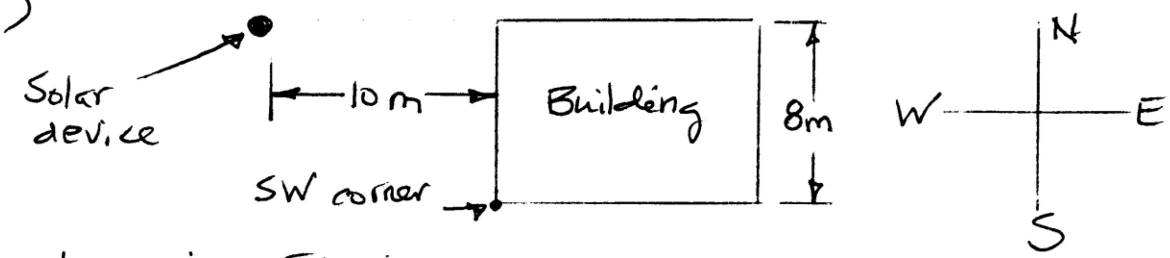
$$\begin{aligned}
\sin\alpha_s &= \sin\delta \sin L + \cos\delta \cos L \cos\omega \\
&= \sin 16.1° \sin 42° + \cos 16.1° \cos 42° \cos(-30) \\
&= (0.277)(0.669) + (0.961)(0.743)(0.866) \\
&= 0.1853 + 0.6183 \\
\alpha_s &= 53.5°
\end{aligned}$$

$$\sin\gamma_s = \frac{\cos\delta \sin\omega}{\cos\alpha_s} = \frac{(0.961)(-0.5)}{0.595} = -0.808$$

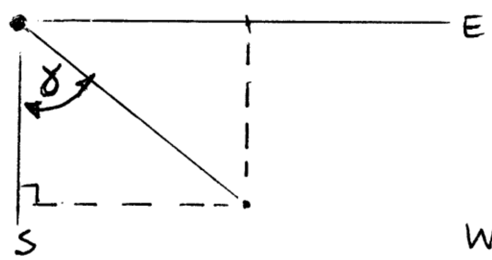
$\gamma_s = -53.9°$ —ve sign means east of due south

$$\begin{aligned}
\cos\theta_i &= \sin\alpha \cos\beta + \cos\alpha \sin\beta \cos(\gamma - \gamma_s) \\
&= (0.804)(0.921) + (0.595)(0.391)(0.589) \\
&= 0.740 + 0.137 \\
\theta_i &= 28.7°
\end{aligned}$$

9-3)



Location is Edinburgh, Scotland
$L = 56°$
Summer solstice ≈ 21 June, Declination, $\delta = 23.45°$
Time is 9:00 a.m. solar time



, $\tan \gamma = \dfrac{10\ m}{8\ m} = 1.25$

$\gamma = 51.3°$

When $\gamma_s < 51.3°$ the sun will reach the device.

At 9:00 a.m., hour angle $\omega = -45°$

$$\alpha_s = \sin^{-1}\left[\sin\delta \sin L + \cos\delta \cos L \cos\omega\right]$$
$$= \sin^{-1}\left[(0.398)(0.829) + (0.917)(0.559)(0.707)\right]$$
$$= \sin^{-1}\left[0.330 + 0.362\right]$$
$$= 43.8°$$

$$\gamma_s = \sin^{-1}\left[\frac{\cos\delta \sin\omega}{\cos\alpha_s}\right] = \sin^{-1}\left[\frac{(0.917)(-0.707)}{0.722}\right] = -63.9$$

$-ve$ γ_s means sun is east of due south.

$\gamma_s > 51.3°$ so sun will not reach the device by 9:00 a.m.

4-6) At 76° W long. and $L = 37°$ N, sun rise is at 05:10:20 hours local time

At this local time system, when is sun rise at 87° W long. and $L = 49°$ N ?

$$\text{Solar time} = \text{std. time} + 0{:}04\,(\text{long std} - \text{long loc}) + ET$$
$$= 05{:}10{:}20\,h + 0{:}04\,(90 - 76) + ET$$

Time zones at multiples of 15° gives 90°
Neglect ET which would be the same in each case

$$\text{Solar time} = 05{:}10{:}20\,h + 0{:}56{:}00 = 06{:}06{:}20$$
$$= 6.11\,h$$

$$\text{Hour angle} = -88.35°$$

At sun rise $\alpha_s = 0$ and $\sin \alpha_s = 0$

$$\sin \alpha = \sin \delta \sin L + \cos \delta \cos L \cos \omega$$
$$0 = \sin \delta\,(0.602) + \cos \delta\,(0.799)(0.0288)$$
$$0 = 0.602 \sin \delta + 0.0230 \cos \delta$$

$$0 = \frac{0.602 \sin \delta}{0.0230 \cos \delta} + 1 = 26.17 \tan \delta + 1$$

$$-1 = 26.17 \tan \delta$$
$$-0.0382 = \tan \delta$$
$$\tan^{-1}(-0.0382) = \delta = -2.19°$$

$$0 = \sin(-2.19) \sin(49°) + \cos(-2.19) \cos(49) \cos \omega$$
$$0 = (-0.0382)(0.755) + (0.999)(0.656) \cos \omega$$
$$0 = -0.0242 + 0.655 \cos \omega$$
$$\frac{0.0242}{0.655} = \cos \omega$$

$$\cos^{-1} 0.0369 = \omega = 87.89 = \text{hour angle}$$
$$= 6.211\ \text{hours.}$$

9-6) continued

$\omega = -87.89^\circ$ or 2.11° after 6:00 a.m.

$(2.11 \div 15)(60 \text{ mins}) = 8.44$ mins
$= 8$ mins 26 secs.

solar time = 06:08:26

std. time = 06:08:26 − 0:04 (90 − 87)
= 06:08:26 − 0:12:00
= 05:56:26

9-7) 94° W Long.
July 14 means $N = 195$
Daylight savings time
Time is 13:55:23 local time
Mountain Time Zone is 105° (p. 298)
Find the solar time.

$$\text{Solar time} = \text{Std. time} + 0{:}04\,(\text{Long}_{std} - \text{Long}_{loc}) + ET$$
$$= 13{:}55{:}23 + 0{:}04\,(105 - 94) + ET$$
$$= 13{:}55{:}23 + 0{:}44 + ET$$
$$= 14{:}39{:}23 + ET.$$

$$\tau = \frac{360\,N}{365}\ \text{degrees} = \frac{360 \times 195}{365} = 192.3°, \text{ say } 192°$$

$$ET = -7.3412 \sin 192° + 0.4944 \cos 192°$$
$$- 9.3795 \sin(2 \times 192°) - 3.2568 \cos(2 \times 192°)$$
$$- 0.3179 \sin(3 \times 192°) - 0.0774 \cos(3 \times 192°)$$
$$- 0.1739 \sin(4 \times 192°) - 0.1283 \cos(4 \times 192°)$$

$$= 1.526 - 0.484 - 3.815 - 2.975$$
$$+ 0.1869 + 0.262 - 0.129 - 0.086$$

$$= -5.71 \text{ minutes} = -5 \text{ minutes } 43 \text{ seconds.} = 0{:}05{:}43$$

Daylight savings means that in summer the clocks have been moved ahead so that without daylight savings, the local time is 13:39:23.

$$\text{Solar time} = 13{:}39{:}23 - 0{:}05{:}43$$
$$= 13{:}33{:}40$$

9-8) July 17 with daylight savings
Find ET and if sun is "fast" or "slow"
Find clock time with sun due south
Nov. 17 without daylight savings also
Longitude = 90° W.

On July 17, N = 198

$$z = \frac{360\ N}{365} \text{ degrees} = 195°$$

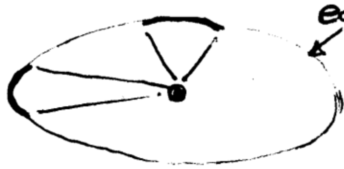
$$ET = -7.3412 \sin 195° + 0.4944 \cos 195° - 9.3795 \sin(2\times195°) - 3.2568 \cos(2\times195°) - 0.3179 \sin(3\times195°) - 0.0774 \cos(3\times195°) - 0.1739 \sin(4\times195°) - 0.1283 \cos(4\times195°)$$

$$= +1.900 - 0.4776 - 4.6898 - 2.820 + 0.2248 + 0.0547 - 0.1506 - 0.0642$$

$$= -6.0227$$

Solar time = std. time + 0:04 (90 − 90) + ET
= std. time + 0 − 0:06:01

Solar time is behind local time, sun may be farther from the earth, causing sun swept angle to be small and sun to appear slow.



Due south is solar noon

12:00:00 = clock time − 0:06:01
clock time = 12:06:01 when sun is due south

but with daylight savings

clock time = 13:06:01

9-8) continued.

On Nov. 17, $N = 321$

$$\tau = \frac{360\,N}{365} \text{ degrees} = \frac{360\,(321)}{365} = 317^\circ$$

$$\begin{aligned} ET = & -7.3412 \sin 317^\circ + 0.4944 \cos 317^\circ \\ & -9.3795 \sin(2 \times 317^\circ) - 3.2568 \cos(2 \times 317^\circ) \\ & -0.3179 \sin(3 \times 317^\circ) - 0.0774 \cos(3 \times 317^\circ) \\ & -0.1739 \sin(4 \times 317^\circ) - 0.1283 \cos(4 \times 317^\circ) \end{aligned}$$

$$\begin{aligned} = & +5.007 + 0.3616 + 9.357 - 0.2271 \\ & + 0.2471 + 0.0487 + 0.0242 + 0.1271 \\ = & \ 14.944 \text{ minutes} \end{aligned}$$

12:00:00 = clock time + 0:14:57

clock time = 11:45:03

so sun appears "fast"