Wind Energy Problems. — 9614

13–1) a.
b. first part only
c.
d.

13–2) a.
b. only answer part.

13–4)

13–5) a.
b.
c.

13–6)

13-1) a.

Bin	Wind speed (m/s) Min.	Max.	h/y	Bin. avg. m/s.	Frequency (%)	Power Watts.	Energy KWh/m²	
1	0.0	0.0	80	0	0.913	0.0	0	0
2	0.0	1.0	200	0.5	2.28	0.0719	0.0144	0.0114
3	1.0	2.0	501	1.5	5.72	1.94	0.972	0.0858
4	2.0	3.1	850	2.55	9.70	9.53	8.10	0.247
5	3.1	4.1	1300	3.6	14.8	26.8	34.8	0.533
6	4.1	5.2	1407	4.65	16.1	57.8	81.3	0.749
7	5.2	6.2	1351	5.7	15.4	106	143	0.878
8	6.2	7.2	990	6.7	11.3	173	171	0.757
9	7.2	8.2	641	7.7	7.32	263	169	0.564
10	8.2	9.3	480	8.75	5.48	385	185	0.480
11	9.3	10.3	375	9.8	4.28	541	203	0.419
12	10.3	11.3	291	10.8	3.32	724	211	0.359
13	11.3	12.3	180	11.8	2.05	945	170	0.242
14	12.3	13.4	78	12.85	0.890	1220	95.2	0.114
15	13.4	14.4	32	13.9	0.365	1544	49.4	0.0507
16	14.4	and above	4	16	0.0457	2355	9.42	0.00731
①	②	③	④	⑤	⑥	⑦	⑧	⑨

Column ⑨ ← $p_i \times V_{i,average}$

Column ⑤ = average columns ② and ③.

column ⑥ = column ④ ÷ 8760 h/y.

column ⑥ adds up to 99.96 % or ≈ 100 %

air density $\rho = 1.15 \text{ kg/m}^3$

$P = 0.5 \rho U^3 A$ use unit area of 1 m².

Bin 2: $P = (0.5)(1.15 \frac{kg}{m^3})(0.5 \frac{m}{s})^3 (1 m^2)$

$= 0.0719 \frac{kg}{m^3} \frac{m^3}{s^3} m^2 = 0.0719$ Watts.

Other Bins: $P = 0.575 (U_{avg})^3$ watts.

column ⑦ is actually Watts/m²

13-1) a. continued $AEO = \sum_i P_i \times t_i$

Column (8) = column (4) × column (7) $\left(\frac{h}{y} \text{ watts}\right) \div 1000 \frac{\text{watts}}{\text{KW}}$

and is estimated energy in one year.

sum of column (8) = 1,531 KWh/m^2 within one year

13-1) b. first part.

$$U_{average} = \sum_{i=1}^{n} p_i \times U_{i, average}.$$

$U_{i, average}$ = column (5) × column (6)

sum of column (9) = 5.50 m/s = average U

$P = 0.5 \rho U^3 A$ use unit area of $1\ m^2$

$$P = 0.5 \left(1.15 \frac{Kg}{m^3}\right)\left(5.50 \frac{m}{s}\right)^3 1\ m^2$$

$$= 95.7 \frac{Kg}{m^3} \frac{m^3}{s^3} m^2 = 95.7 \text{ watts}$$

Energy in one year = 95.7 Watts × 8760 h ÷ $\frac{1000\ W}{KW}$
= 838 KWh/m^2

13-1) c.
Equation 13-10 explains that if the average wind speed is adjusted by $6/\pi$, a more accurate estimate of power and energy is obtained.

Energy in one year = 838 KWh/m^2 × $(6/\pi)$
= 1600 KWh/m^2.

13-1) d. actual energy / year / m^2 = 1531 kWh / m^2

using $U_{average}$ only = 838 kWh / m^2

using $U_{average}$ + Eq. 13-10 = 1600 kWh / m^2.

$$\frac{1531 - 838}{1531} = 45.3\% \text{ error.}$$

$$\frac{1531 - 1600}{1531} = 4.51\% \text{ error.}$$

13-2) a. $2R = 80m$, $R = 40$ m
$\rho_{air} = 1.15$ kg/m^3
$U_{avg.} = 9.6$ m/s.

Bin No.	(m/s) V avg.	(kW) P-out	(kW P-available in wind	C_p per bin
1	—	0		
2	6.5	175	794	0.220
3	7	318	991	0.321
4	7.5	460	1219	0.377
5	8	603	1480	0.407
6	8.5	745	1775	0.420
7	9	949	2107	0.450
8	9.5	1153	2478	0.461
9	10	1316	2890	0.455
10	10.5	1479	3,346	0.442
11	11	1642	3,847	0.427
12	11.5	1805	4395	0.411
13	12	1968	4994	0.394
14	12.5	2120	5645	0.376
15	13	2263	6349	0.356
16	13.5	2385	7110	0.335
17	14	2500	7930	0.315
18	17.25	2500 ③	14,834	0.169

Power available in wind $= P = 0.5 \rho U^3 A$

Sample calculation for Bin 9:

$$P = 0.5\left(1.15 \text{ kg/m}^3\right)\left(10 \text{ m/s}\right)^3 \left(\pi \times 40^2 \text{m}^2\right)$$

$$= 0.5(1.15)(1000)(5026.5)$$

$$= 2,890,238 \text{ watts} = 2890 \text{ kW}$$

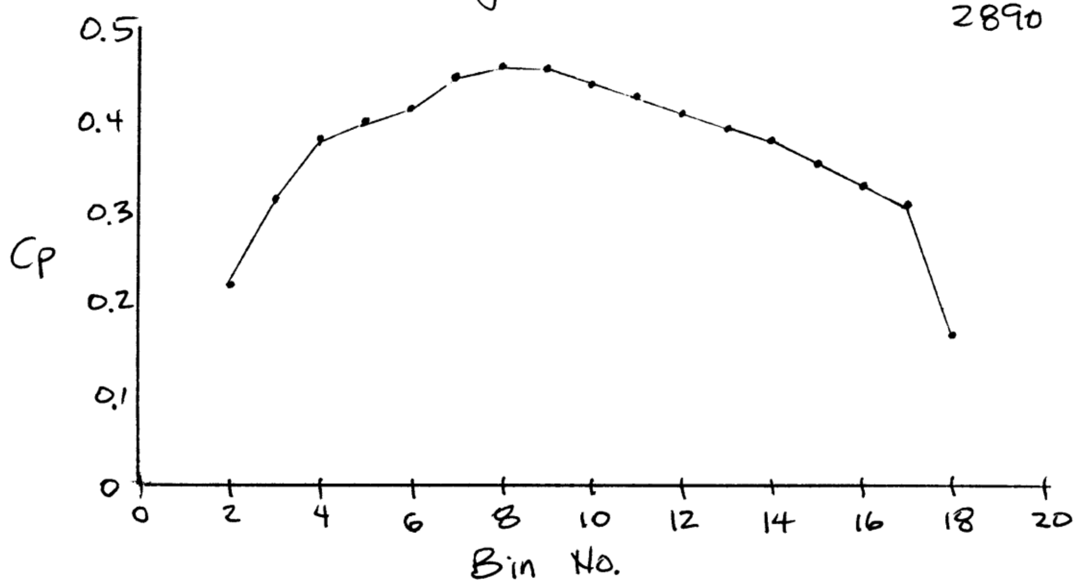
For other bins, power available in wind

$$= 2.89 \left(V_{avg}\right)^3 \text{ kW.}$$

13-2) a. continued.

Power coefficient C_p = $\frac{\text{power extracted}}{\text{power available}}$.

Sample calculation for bin 9: $C_p = \frac{1316}{2890} = 0.455$.



b. $U_{avg.} = 9.6$ m/s.

$$P = 0.5 \rho U^3 A$$

$$= 0.5 (1.15 \text{ kg/m}^3)(9.6 \text{ m/s})^3 (5026.5)$$

$$= 2{,}557{,}097 \text{ watts} = 2{,}557 \text{ kW}.$$

or to improve accuracy $2{,}557 \text{ kW} \times 6/\pi = 4{,}884$ kW.

Energy in one year $= 4{,}884 \text{ kW} \times 8760 \text{ h} = 42{,}781$ kWh.

13-4) Turbine rated capacity = 10 KW. (max. possible power)

at $U \leq 4$ m/s $P_i = 0$.

at $4 \text{ m/s} \leq U \leq 10 \text{ m/s}$ $P_i = (10/6)(U-4)$ KW.

at $10 \text{ m/s} \leq U \leq 20 \text{ m/s}$ $P_i = 10$ KW

at $U \geq 20$ m/s $P_i = 0$

Use wind conditions from Table 13-1.

Find annual output in KWh.

Bin	Bin. Avg. V, (m/s)	P_i KW	hours/y	Energy KWh
1	0	0	80	0
2	0.5	0	204	0
3	1.5	0	496	0
4	2.5	0	806	0
5	3.5	0	1211	0
6	4.5	0.833	1254	1,045
7	5.5	2.50	1246	3115
8	6.5	4.17	1027	4283
9	7.5	5.83	709	4133
10	8.5	7.50	549	4118
11	9.5	9.17	443	4062
12	10.5	10	328	3280
13	11.5	10	221	2210
14	12.5	10	124	1240
15	13.5	10	60	600
16	—	10	2	20

Bin no. 16 has no upper bound on velocity of wind given.

Bin no. 16 can be included or left out in summing column (5) because it is negligible when working to 3 significant figures.

Sample calculation for Bin 9:

$$P_i = (10/6)(7.5-4) \text{ KW} = 5.83 \text{ KW}.$$

Energy KWh/bin is column (3) × column (4)

Sum column (5) = 28,106 KWh = 28.1 MWh = annual output.

13-5) Turbine has 3 blades, $B = 3$

Pitch at tip $= -2°$, $\beta_0 = -2$

Tip speed ratio, TSR of $\lambda = 7$

Rated wind speed (gives maximum capacity) $= 11.62$ m/s.

for $\alpha < 12°$, $C_L = 0.368 + 0.0942\alpha$

$C_D = 0.00994 + 0.000529\alpha + 0.0001055\alpha^2$

Sect.	r/R	(m) r	c, (m) Chord	θ_T (°) Twist	σ'	x	β (°)	ϕ
1	0.05	0.248	Hub	Hub	–	0.35	–	
2	0.15	0.743	0.428	35.3	0.275	1.05	33.3	38.29
3	0.25	1.238	0.415	20.65	0.160	1.75	18.65	24.64
4	0.35	1.734	0.347	13.05	0.0956	2.45	11.05	17.73
5	0.45	2.229	0.285	8.9	0.0611	3.15	6.90	13.87
6	0.55	2.724	0.241	5.95	0.0422	3.85	3.95	11.29
7	0.65	3.219	0.207	3.6	0.0307	4.55	1.60	9.43
8	0.75	3.715	0.180	2.05	0.0231	5.25	0.05	8.332
9	0.85	4.210	0.152	0.9	0.0172	5.95	−1.10	7.29
10	0.95	4.705	0.123	0.2	0.0124	6.65	−1.80	6.78
Tip	1.0	4.953	0.107	0	0.00103	7.00	−2.00	7.99

a.) Find for each section
angle of attack, α
angle of relative wind ϕ
lift and drag coefficients C_L and C_D.
local contribution to C_P, C_{Pi}

Turbine blade length $= 4.953$ m.

local solidity, $\sigma' = \dfrac{Bc}{2\pi r} = \dfrac{3c}{2\pi r} = 0.4775\,\dfrac{c}{r}$

local speed ratio, $x = (r/R)\lambda = r/R(7) = 7$ (column (2))

element pitch, $\beta = \theta_T + \beta_0 = \theta_T - 2° =$ column (5) $- 2°$

13-5) a. continued

$$C_L = \frac{4 \sin\phi\,(\cos\phi - x \sin\phi)}{\sigma'(\sin\phi + x\cos\phi)}$$

and for $\alpha < 12^\circ$

$$C_L = 0.368 + 0.0942\alpha = 0.368 + 0.0942(\phi - \beta)$$

$$0.368 + 0.0942(\phi - \beta) = \frac{4 \sin\phi\,(\cos\phi - x \sin\phi)}{\sigma'(\sin\phi + x\cos\phi)}$$

For each section σ', x, and β are known and the only unknown is ϕ. So can solve above equation for ϕ.

Sect.	α (°)	C_L	C_D	$\Delta\lambda$	C_D/C_L	$8\Delta\lambda/\lambda^2$	x	ϕ
1	—			—				
2	4.99	0.838	0.0152	0.7	0.0181	0.1143	1.05	38.3
3	5.99	0.932	0.0169	0.7	0.0181	0.1143	1.75	24.6
4	6.68	0.997	0.0182	0.7	0.0183	0.1143	2.45	17.7
5	6.97	1.025	0.0188	0.7	0.0183	0.1143	3.15	13.9
6	7.34	1.059	0.0195	0.7	0.0184	0.1143	3.85	11.3
7	7.83	1.106	0.0206	0.7	0.0186	0.1143	4.55	9.43
8	8.282	1.148	0.0216	0.7	0.0188	0.1143	5.25	8.33
9	8.39	1.158	0.0218	0.7	0.0188	0.1143	5.95	7.29
10	8.58	1.176	0.0222	0.7	0.0189	0.1143	6.65	6.78
Tip.	9.99	1.309	0.0258	0.35	0.0197	0.0571	7.00	7.99

$$\phi = \alpha + \beta; \quad \alpha = \phi - \beta$$

$$C_L = 0.368 + 0.0942\alpha$$

$$C_D = 0.00994 + 0.000529\alpha + 0.0001055\alpha^2$$

$$\Delta\lambda = x_i - x_{i-1}$$

13-5) a. continued

$$\Delta\lambda = x_i - x_{i-1}$$

$$C_{P,i} = \left(\frac{8\Delta\lambda}{\lambda^2}\right)\sin^2\phi_i\left(\cos\phi_i - x\sin\phi_i\right) \times \left(\sin\phi_i + x\cos\phi_i\right)\left[1 - \left(\frac{C_D}{C_L}\right)\cot\phi_i\right]x^2$$

Sect.	$C_{P,i}$
1	
2	0.00145
3	0.0211
4	0.0328
5	0.0428
6	0.0531
7	0.0632
8	0.0743
9	0.0794
10	0.0828
Tip	0.0057

<u>Comments</u>: In setting $C_L = C_L$ using the two separate equations it was found that both a positive value of ϕ and a negative value of ϕ were possible.

From Fig. 13-23, showing that $\alpha + \beta = \phi$, it is clear that $\phi > \beta$ and so only the positive result is reasonable.

If difficulty arises, one can guess ϕ and solve for σ', x, or β, to find ϕ

b. Overall power coefficient $= \Sigma C_{P_i} = 0.45665$

c. At rated wind speed $= 11.62$ m/s

Extractable wind power, assume $\rho = 1.15$ kg/m^3.

$$P(\lambda) = 0.5\,\rho\,A\,C_P\,U^3$$

$$= 0.5\left(1.15\,\frac{kg}{m^3}\right)\left(\pi \times 4.953^2\right)\left(0.45665\right)\left(11.62\,\frac{m}{s}\right)^3$$

($\pi \times 4.953^2$ = 77.07 m^2)

$$= 31{,}750 \text{ watts} = 31.75 \text{ kW}$$

13-6) Turbine to deliver 250 KW at design speed of 10 m/s.
$C_L = 1$ and $C_D = 0$

Divide blade into 10 segments. each with uniform aerodynamic characteristics.
Overall solidity = 0.088, $\lambda = 6$, $B = 3$.

Sect.	r/R	ϕ (°)	c/R	r (m)	c (m)	σ'	x	C_{Pi}
1	0.05			0.83			0.3	
2	0.15	32.0	0.191	2.49	3.17	0.608	0.9	0.0145
3	0.25	22.5	0.159	4.15	2.64	0.304	1.5	0.0272
4	0.35	17.0	0.128	5.81	2.12	0.174	2.1	0.0396
5	0.45	13.5	0.105	7.47	1.74	0.111	2.7	0.0518
6	0.55	11.2	0.088	9.13	1.46	0.0764	3.3	0.0639
7	0.65	9.6	0.076	10.79	1.26	0.0558	3.9	0.0759
8	0.75	8.4	0.067	12.45	1.11	0.0426	4.5	0.0879
9	0.85	7.4	0.059	14.11	0.980	0.0332	5.1	0.0999
10	0.95	6.6	0.053	15.77	0.880	0.0266	5.7	0.1118
11	1.0							

Extractable wind power $P(\lambda) = 0.5 \rho U^3 A C_p$

where C_p = theoretical power factor $\simeq 0.5$ (O.K. preliminary)

$P(\lambda) = 0.5 (1.15\ Kg/m^3)(10\ m/s)^3 (A) C_p = 250$ KW.
(1.15 assumed)

$$A = \frac{250{,}000 \text{ Watts}}{0.5\,(1.15)(1000)(0.5)} = 869.6\ m^2 = \pi R^2$$

$R^2 = 276.8$, $R = 16.64$ m. $\simeq 16.6$ m.

If air density of $1.225\ Kg/m^3$ is assumed then $R = 16.1$m

Use $R = 16.6$ m. to solve for r and c.

13-6) continued

local solidity $\sigma' = \frac{Bc}{2\pi r} = \frac{3c}{2\pi r} = 0.4775 \frac{c}{r}$

$$C_L = \frac{4 \sin\phi \, (\cos\phi - x \sin\phi)}{\sigma' \, (\sin\phi + x \cos\phi)} = 1$$

Can solve for x from the above equation. but also $x = (r/R)\lambda$

Find x from $(r/R)\lambda$ and feed in equation for C_L.

All values of $C_L \simeq 1.0$ so calculated values of x are correct.

C_D is negligible so $C_D \simeq 0$. so $C_D/C_L \simeq 0$.

$\Delta\lambda = x_i - x_{i-1} = 0.6$, except between sections 10 and 11

There is not much information at the tip and its contribution to C_p can be ignored. as it is small

$$C_{p_i} = \left(\frac{8\Delta\lambda}{\lambda^2}\right) \sin^2\phi_i \,(\cos\phi_i - x \sin\phi_i) \times (\sin\phi_i + x\cos\phi_i)(x^2)$$

(Section in square brackets with (C_D/C_L) reduces to 1).

$$C_{p_i} = (0.1333)(\sin^2\phi_i)(\cos\phi_i - x\sin\phi_i)(\sin\phi_i + x\cos\phi_i)(x^2)$$

$\Sigma C_{p_i} = C_p = 0.5725$

C_p was assumed to be 0.5

Area of turbine might be reduced by $\frac{0.5}{0.5725}$

$$A = 869.6 \text{ m}^2 \left(\frac{0.5}{0.5725}\right) = 759.5 \text{ m}^2 = \pi R^2$$

$R = \left(\frac{759.5}{\pi}\right)^{0.5} = 15.55$, Use 15.6 m for R.

13-6) continued

repeat process of find C_p using $R = 15.6$ m.

Sect.	ϕ (°)	c (m)	r (m)	σ'	x
1			0.78		0.3
2	32.0	2.98	2.34	0.592	0.9
3	22.5	2.48	3.90	0.304	1.5
4	17.0	2.00	5.46	0.175	2.1
5	13.5	1.64	7.02	0.111	2.7
6	11.2	1.37	8.58	0.0770	3.3
7	9.6	1.19	10.14	0.0560	3.9
8	8.4	1.05	11.7	0.0429	4.5
9	7.4	0.920	13.3	0.0330	5.1
10	6.6	0.827	14.8	0.0266	5.7

local solidity $\sigma' = 0.4775\, c/r$ (values similar to before)

$x = (r/R)\lambda$, so same values as before, and so $\Delta\lambda$ same as before.

Values of σ', ϕ, and x used to find C_l remain the same.

Value of $\Delta\lambda$, and λ^2, remain the same and so C_{pi} values will be the same and C_p again $= 0.5725$.

So $R = 15.6$ m is good for preliminary design.