MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Faculty of Engineering and Applied Science

Engineering 5003 - Ship Structures

FINAL EXAMINATION

Solutions

Date: Wednesday. April 9, 2014 **Professor: Dr. C. Daley**
Time: 1:00 - 3:30 pm
Maximum Marks: 100

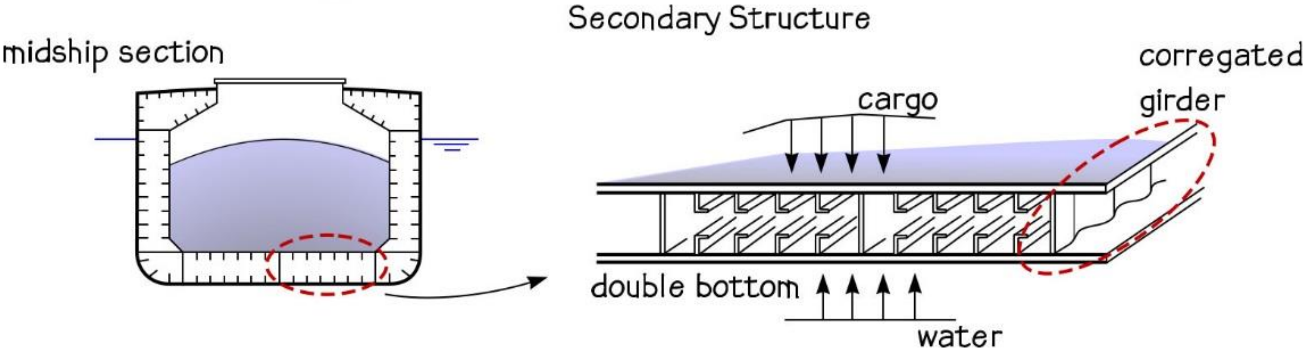
Instructions:
Please write/sketch clearly in the white answer book.
Answer all 8 questions.

This is a closed book exam. Some formulae are given at the end of the question paper.

1. Design ideas (10 marks)

The photograph below shows a recent novel design for a deep web girder for use in large aircraft.

Imagine such a section for use as double bottom girders in a ship. Discuss the pros and cons of this possible design change. Focus on structural behavior, though other considerations can be mentioned. Please be concise (ideally a half-page or less).



2. Still water bending (17 Marks: 2, 5, 8, 2 marks for a,b,c,d)

There is a floating block of wood as sketched below;

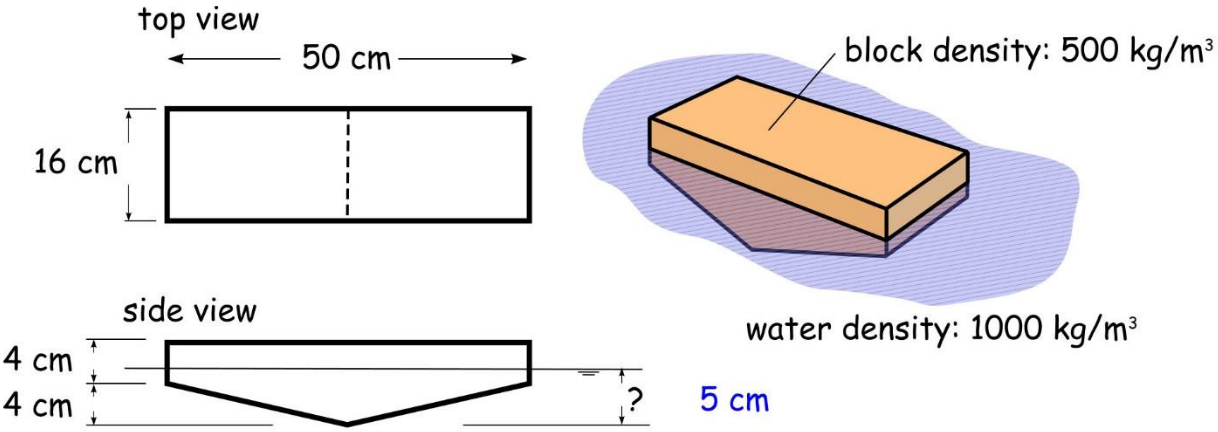
a) What is the draft at which the block will float?

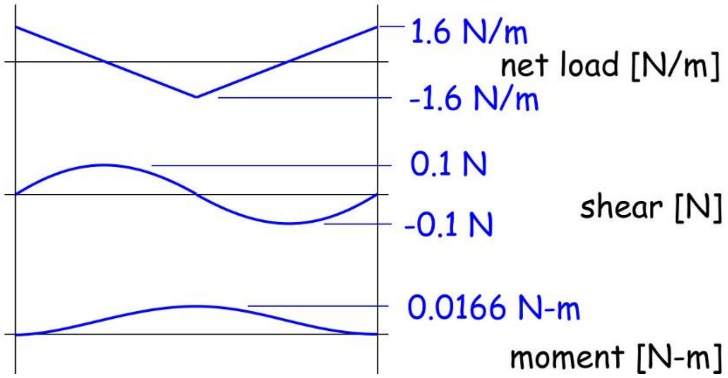
b) Plot the 'net load' curve for the block. Show numerical values.

c) Plot the shear force and bending moment diagrams. Show numerical values.

d) If you changed the density of the fluid, what value would cause the bending moment to become zero?

YOU CAN USE 10 m/s^2 for gravity to keep values simple.





d) If density was 1000 (or 999.999)

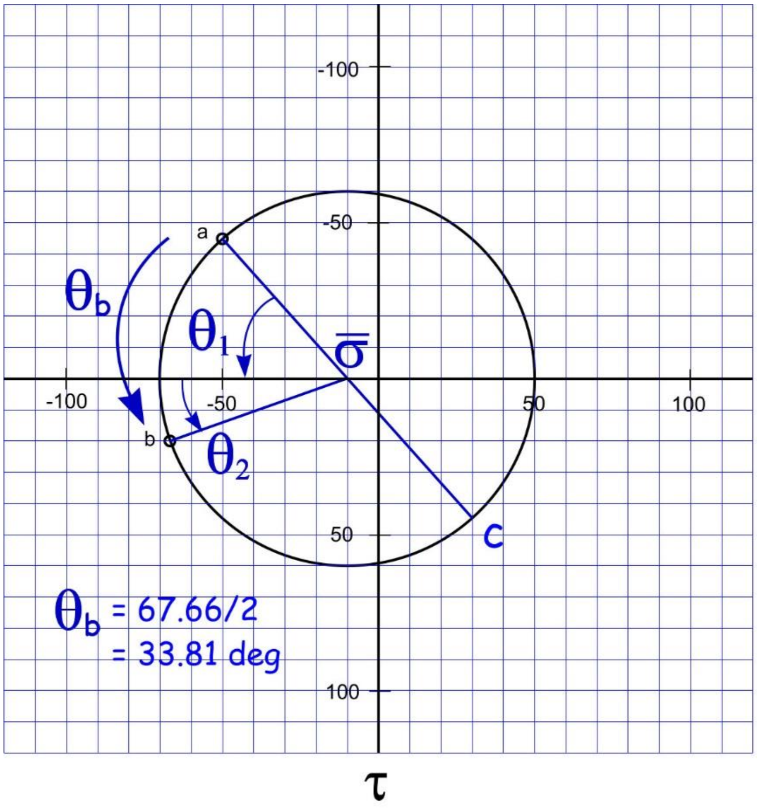
3. Stress (12 Marks 5, 5, 2 marks for a,b,c)

The plot shows a state of stress. The principal stresses are 50 MPa tension and 70 MPa compression.

a) sketch the stresses on an element of material (a small square) where the state of stress at 'a' is the stress on a vertical cut (a vertical face) (90 deg. from horizontal).

b) What is the orientation (from horizontal) of the plane that experiences the stress values marked 'b' ?

c) What is the von-Mises equivalent stress for this state of stress.



θ_b = 67.66/2
= 33.81 deg

$\bar{\sigma}$ = -10 MPa
r = 60 MPa

σ_a = -50 MPa

$\tau_a = -\sqrt{60^2-(50-10)^2}$
= -44.72 MPa

σ_c = 30 MPa

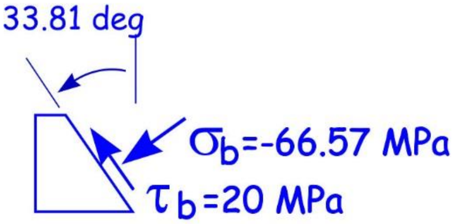
τ_c = 44.72 MPa

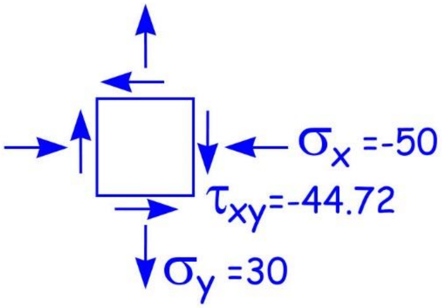
$\sigma_b = -\sqrt{60^2-20^2}-10$
= -66.57 MPa

τ_b = 20 MPa

$\theta_1 = \sin^{-1}(44.72/60)$
= 48.19 deg

$\theta_2 = \sin^{-1}(20/60)$
= 19.47 deg

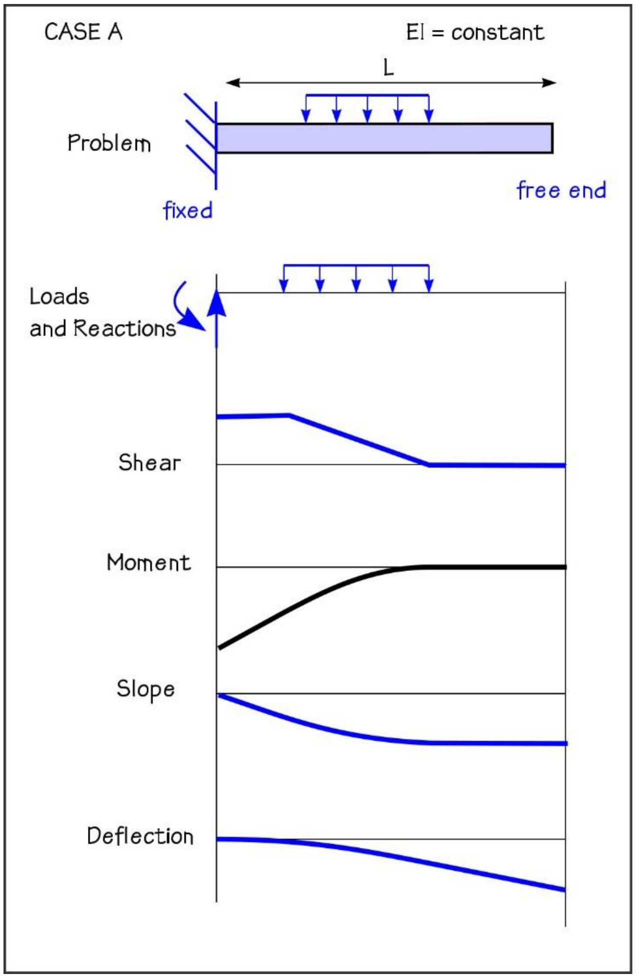


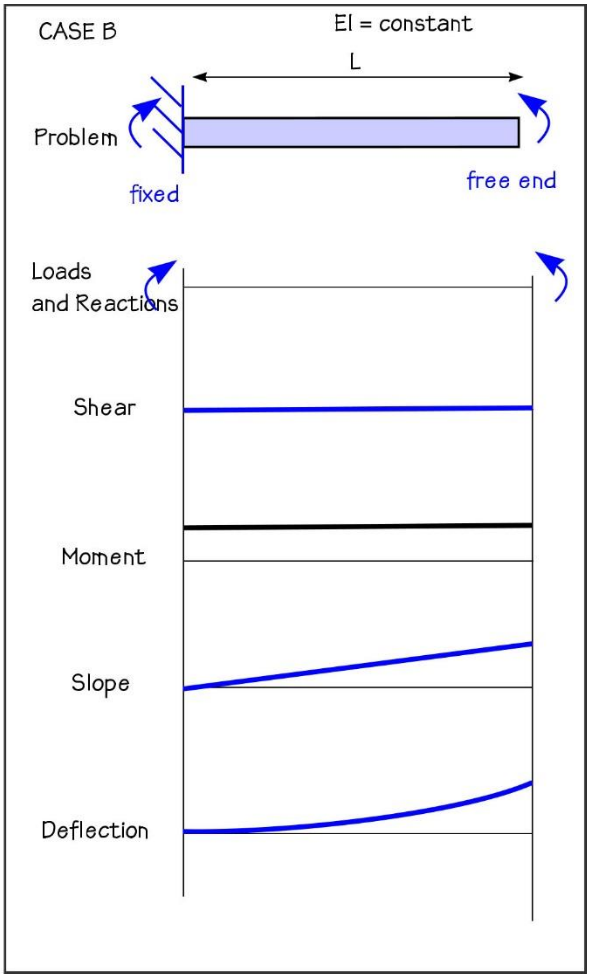


$\sigma_{vm} = \sqrt{70^2-(-70)50+50^2}$
= 104.4 MPa

4. Beam Responses (10 Marks 5 for each case)

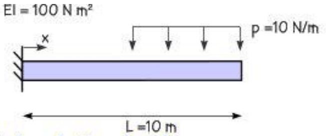
For the two cases shown below, the bending moment diagrams are sketched. Show a possible set of supports and loading. Sketch the shear, slope and deflection patterns. No numerical values are required. (5 marks each problem) (answer in the answer book- not on this sheet)





5. Direct Integration or Macaulay Method (12 Marks)

A cantilever beam has a uniform load on half the beam as shown. Using either direct integration or the Macaulay method, solve for the shear, bending, slope and deflection, as functions of x. Give the maximum numerical values, indicating where they occur.

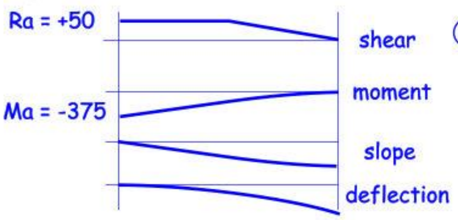


① Find Reactions

Ra = 10 x 5 = 50 N +Q

Ma = 10 x 5 x 7.5 = 375 N m (- due to sense) +M

② Sketch solution (est)



③ Macaulay

$$\text{Load}(x)[N/m] = \underbrace{R_A\langle x-0\rangle^{-1} - 10\langle x-5\rangle^{0}}_{\text{vertical forces}} + \underbrace{M_A\langle x-0\rangle^{-2}}_{\text{point moments}}$$

Shear $Q(x)[N] = R_A\langle x-0\rangle^{0} - 10\langle x-5\rangle^{1}$ (integrate just the vertical forces)

Moment $M(x)[Nm] = M_A\langle x-0\rangle^{0} + R_A\langle x-0\rangle^{1} - 5\langle x-5\rangle^{2}$ (now include the moments)

Slope $\theta(x)[Nm^2/Nm^2 = rad] = \cancel{\theta(0)} + 1/EI(M_A\langle x-0\rangle^{1} + R_A/2\langle x-0\rangle^{2} - 5/3\langle x-3\rangle^{3})$

Deflection $v(x)[Nm^3/Nm^2 = m] = \cancel{v(0)} + \cancel{\theta(0)}x + 1/EI(M_A/2\langle x-0\rangle^{2} + R_A/6\langle x-0\rangle^{3} - 5/12\langle x-5\rangle^{4})$

Shear $Q(x)[N] = 50\langle x-0\rangle^{0} - 10\langle x-5\rangle^{1}$

Moment $M(x)[Nm] = -375\langle x-0\rangle^{0} + 50\langle x-0\rangle^{1} - 5\langle x-5\rangle^{2}$

Slope $\theta(x)[rad] = -3.75\langle x-0\rangle^{1} + 0.25\langle x-0\rangle^{2} - .01666\langle x-5\rangle^{3}$

Deflection $v(x)[m] = -1.875\langle x-0\rangle^{2} + .0833\langle x-0\rangle^{3} - .004166\langle x-5\rangle^{4}$

Max Values $Q(0)[N] = 50$

$M(0)[Nm] = -375$

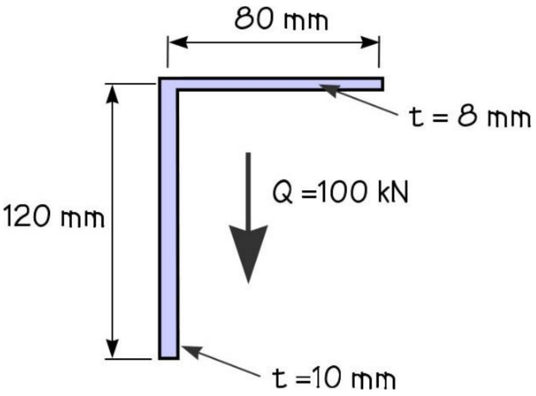
$\theta(10)[rad] = -3.75\ 10^{1} + 0.25\ 10^{2} - .01666(10-5)^{3} = -37.5 + 25 - 2.0832 = -14.58$

$v(10)[m] = -1.875\ 10^{2} + .0833\ 10^{3} - .004166\ 5^{4} = -106.8$

6. Shear Flow (15 Marks 10, 5 marks for a,b)

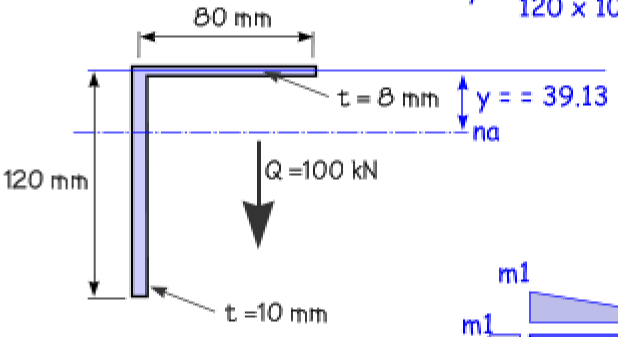
A cross section is shown below. A vertical shear force of 100kN is applied.

a) Solve the shear flow, plot it and then also show the shear stress values (9).



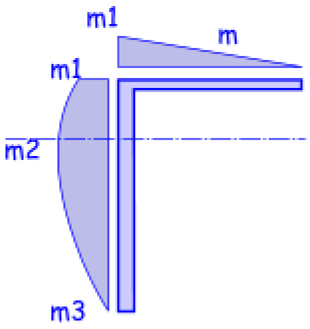
$A = 120 \times 10 + 80 \times 8 = 1840$

$$y = \frac{120 \times 10 \times 60}{120 \times 10 + 80 \times 8}$$



$$I_b = \frac{1}{3} 10\ 120^3$$
$$= 5,760,000$$

$$I_{na} = 5,760,000 - 1840 \times 39.13^2$$
$$= 2,942,609$$



$$m = \int y\ da \quad = \quad 39.13 \times 8 \int_0^s ds$$

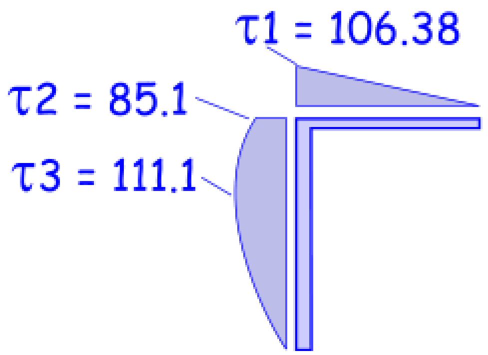
$$m1 = 39.13 \times 8 \times 80$$
$$= 25043\ mm^3$$

$$m2 = 25043 + \int_{39.13}^{0} y\ \ 10\ dy$$

$$m2 = 25043 + \ 10/2\ \times 39.13^2$$

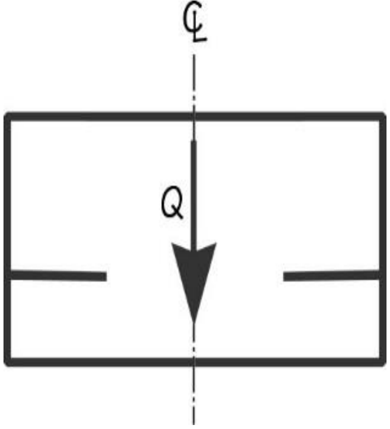
m2 = 32699

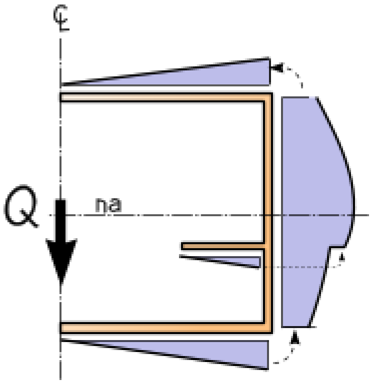
m3 = 0



$$\tau = \frac{Q\ m}{I\ t} = \frac{100,000\ m}{2,942,609\ t}$$

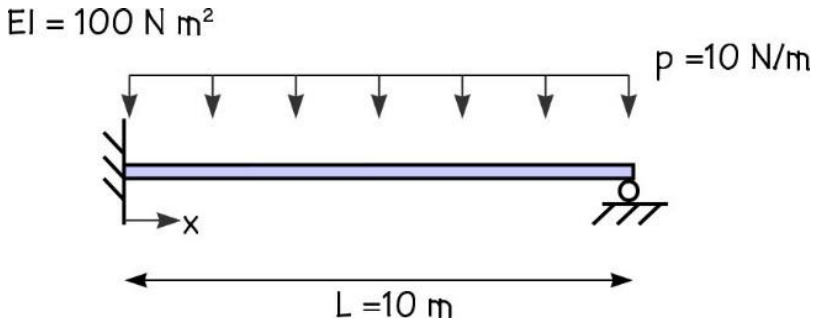
b) Sketch the shear flow pattern for the section shown below (no numbers needed) (6).

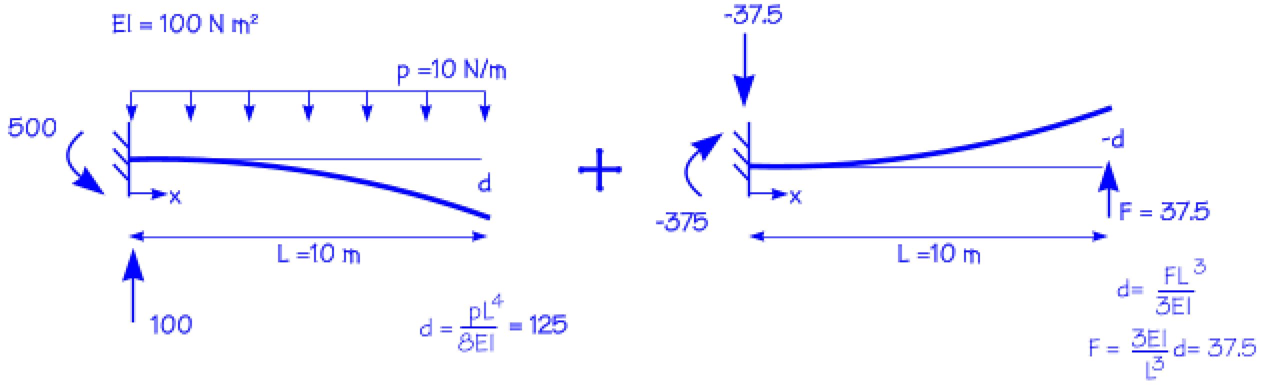




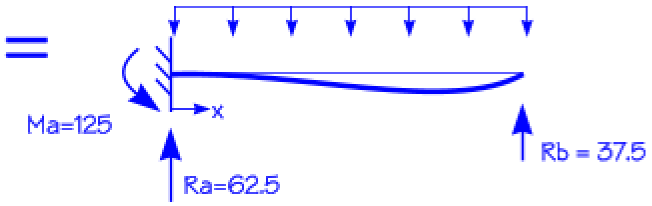
7. Force Method (12 Marks 8,4 marks for a,b)

a) Use the Force Method and the tables at the end to solve for the reactions in the problem below.
b) Then provide the equation for the deflected shape.





Soln with Reactions:



deflection:

$$v = \frac{10x^2}{2400}(600 - 40x + x^2) - \frac{37.5x^2}{600}(30 - x)$$

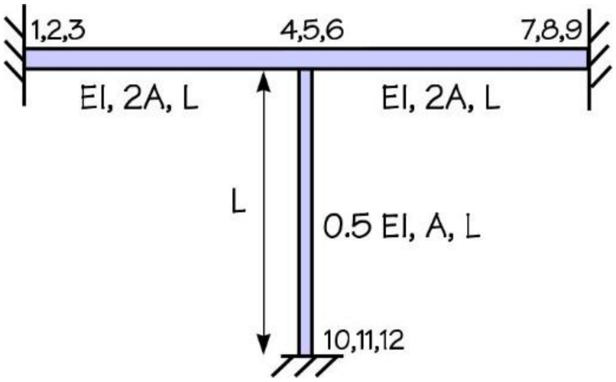
can be simplifies to (not necessary):

$$v = \frac{x^2}{240}(x-10)(x-15)$$

8. Matrix Structural Analysis (12 Marks 2, 10 marks for a,b)

a) Explain the difference between the terms “local coordinates” and "global coordinates".

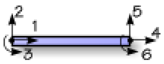
b) For the frame below, write the 12 terms that would be in the 5th row of the global stiffness matrix. Use the degree of freedom numbering as shown. The fifth row will have the terms $k_{1,5}$, $k_{2,5}$, etc.



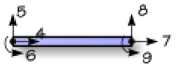
Local coordinates are for a single member and typically align with the member. There are multiple local systems. Global coordinates are a single world coordinate system uses for the whole problem.

Beam 1

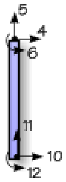
global k matrix



$$k_{global} = \begin{array}{c|cccccc} 1 & \frac{2AE}{L} & 0 & 0 & \frac{-2AE}{L} & 0 & 0 \\ 2 & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{-12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 3 & 0 & \frac{-6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & \frac{-6\,EI}{L^2} & \frac{2\,EI}{L} \\ 4 & \frac{-2AE}{L} & 0 & 0 & \frac{2AE}{L} & 0 & 0 \\ 5 & 0 & \frac{-12\,EI}{L^3} & \frac{-6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{-6\,EI}{L^2} \\ 6 & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{-6\,EI}{L^2} & \frac{4\,EI}{L} \end{array}$$



$$k_{global} = \begin{array}{c|cccccc} 4 & \frac{2AE}{L} & 0 & 0 & \frac{-2AE}{L} & 0 & 0 \\ 5 & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{-12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 6 & 0 & \frac{-6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & \frac{-6\,EI}{L^2} & \frac{2\,EI}{L} \\ 7 & \frac{-2AE}{L} & 0 & 0 & \frac{2AE}{L} & 0 & 0 \\ 8 & 0 & \frac{-12\,EI}{L^3} & \frac{-6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{-6\,EI}{L^2} \\ 9 & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{-6\,EI}{L^2} & \frac{4\,EI}{L} \end{array}$$



$$k_{global} = \begin{array}{c|cccccc} 4 & \frac{6\,EI}{L^3} & 0 & \frac{3\,EI}{L^2} & \frac{-6EI}{L^3} & 0 & \frac{3\,EI}{L^2} \\ 5 & 0 & \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 \\ 6 & \frac{-3\,EI}{L^2} & 0 & \frac{2EI}{L} & \frac{-3\,EI}{L^2} & 0 & \frac{1\,EI}{L} \\ 10 & \frac{-6EI}{L^3} & 0 & \frac{-3EI}{L^2} & \frac{6EI}{L^3} & 0 & \frac{-3\,EI}{L^2} \\ 11 & 0 & \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 \\ 12 & \frac{3\,EI}{L^2} & 0 & \frac{1\,EI}{L} & \frac{-3\,EI}{L^2} & 0 & \frac{2\,EI}{L} \end{array}$$

$$K_{global} = \begin{array}{c|cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 5 & 0 & \frac{-12\,EI}{L^3} & \frac{-6\,EI}{L^2} & 0 & \frac{24\,EI}{L^3} + \frac{AE}{L} & 0 & 0 & \frac{-12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{-AE}{L} & 0 \end{array}$$

Formulae

Weight of a Vessel:

$$W = \Delta = C_B \cdot L \cdot B \cdot T \cdot \gamma$$

Prohaska for parallel middle body : $\overline{W} = \frac{W_{hull}}{L}$ the values of a and b are ;

	$\frac{a}{\overline{\overline{W}}}$	$\frac{b}{\overline{\overline{W}}}$
Tankers (C_B = .85)	.75	1.125
Full Cargo Ships (C_B = .8)	.55	1.225
Fine Cargo Ships (C_B=.65)	.45	1.275
Large Passenger Ships (C_B=.55)	.30	1.35

$$\Delta lcg = \frac{x}{\overline{\overline{W}}} L \frac{7}{54}$$

Murray's Method

$$BM_B = \frac{1}{2}\left(\Delta_a g_a + \Delta_f g_f\right) = \frac{1}{2}\Delta \cdot \bar{x}$$

$$\bar{x} = L(a \cdot C_B + b)$$

Where

T/L	a	b
.03	.209	.03
.04	.199	.041
.05	.189	.052
.06	.179	.063

This table for a and b can be represented adequately by the equation;

$$a = .239 - T/L$$

b = 1.1 T/L -.003

Trochoidal Wave Profile

$$x = R\theta - r \sin \theta$$
$$z = r(1 - \cos \theta)$$

θ = rolling angle

2D Hooke's Law

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

von Mises

yield envelope: $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{yield}^2$

equivalent stress: $\sigma_{eqv} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$

Section Modulus Calculations

Ina = $1/12\ a\ d^2$
= $1/12\ t\ b^3 \cos^2\theta$

Family of Differential Equations Beam Bending

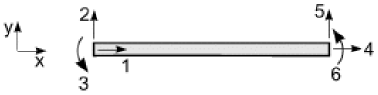
$v =$ deflection [m]
$v' = \theta =$ slope [rad]

$v''EI = M =$ bending moment [N-m]

$v'''EI = Q =$ shear force [N]

$v''''EI = P =$ line load [N/m]

Stiffness Terms



2D beam = 6 degrees of freedom

$$K = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

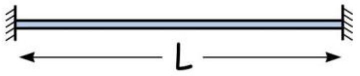
Shear flow: $q = \tau t, \quad q = Q\,m/I$

$m = \int yt\,ds$

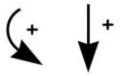
Torque: $Mx = 2qA$

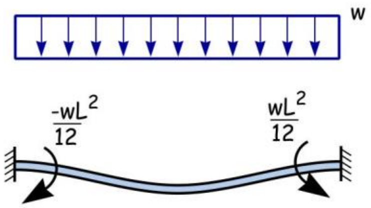
Fixed End Loads

fixed fixed beam, length L, constant EI:

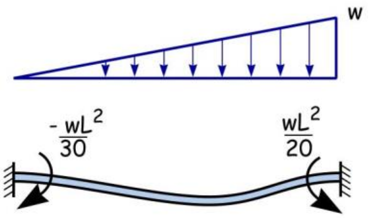


sign for moments and forces:

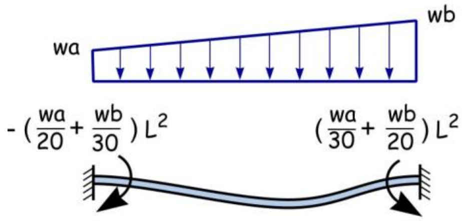




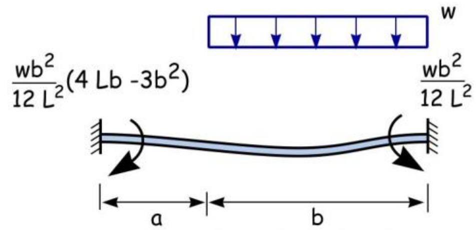
uniform load



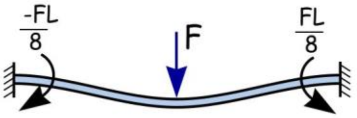
triangular load



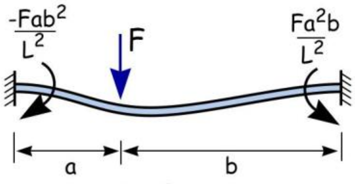
variable load



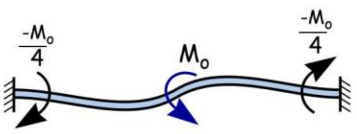
partial uniform load



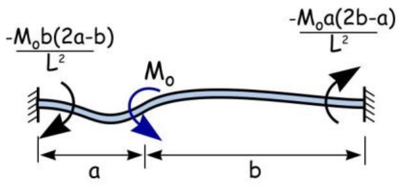
center force



point force

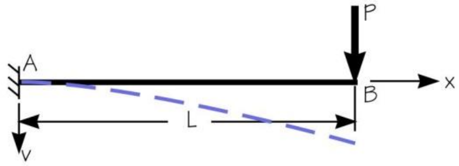
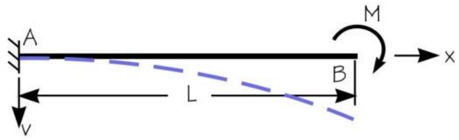
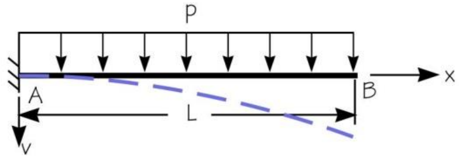
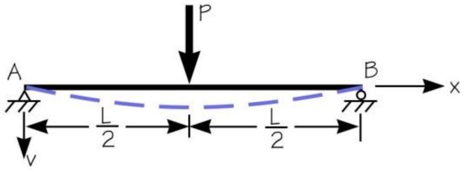
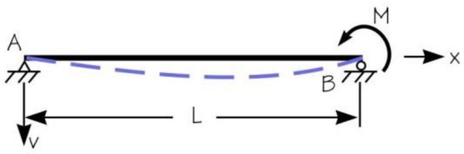
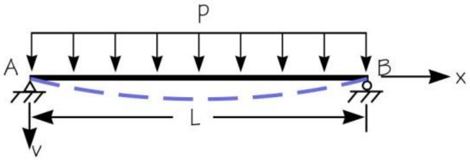
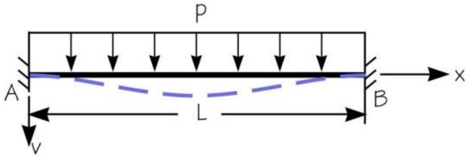
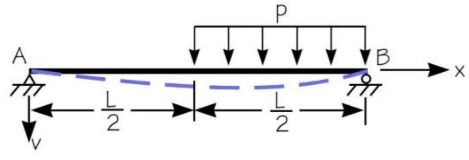


center moment



point moment

Deflection and Slopes of Beams

Loading	Deflection	Slope
	$v = \frac{Px^2}{6EI}(3L - x)$ $v_{max} = v_B = \frac{PL^3}{3EI}$	$\theta_B = \frac{PL^2}{2EI}$
	$v = \frac{Mx^2}{2EI}$ $v_{max} = v_B = \frac{ML^2}{2EI}$	$\theta_B = \frac{ML}{EI}$
	$v = \frac{px^2}{24EI}(6L^2 - 4Lx + x^2)$ $v_{max} = v_B = \frac{pL^4}{8EI}$	$\theta_B = \frac{pL^3}{6EI}$
	$v = \frac{Px^2}{48EI}(3L^2 - 4x^2)$ $v_{max} = \frac{PL^3}{48EI}$ @ x=L/2	$\theta_A = -\theta_B = \frac{PL^2}{16EI}$
	$v = \frac{Mx}{6EI\,L}(L^2 - x^2)$ $v_{max} = \frac{ML^2}{9\sqrt{3}\,EI}$ @ $x=L/\sqrt{3}$	$\theta_A = \frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$
	$v = \frac{px}{24EI}(L^3 - 2Lx^2 + x^3)$ $v_{max} = \frac{5\,pL^4}{384\,EI}$ @ x=L/2	$\theta_A = -\theta_B = \frac{pL^3}{24EI}$
	$v = \frac{px^2}{24EI}(L - x)^2$ $v_{max} = \frac{pL^4}{384\,EI}$ @ x=L/2	$\theta_A = \theta_B = 0$
	$v_{cent} = \frac{3\,pL^4}{256\,EI}$ @ x=L/2	$\theta_A = \frac{-7\,pL^3}{384\,EI}$ $\theta_B = \frac{3\,pL^3}{128\,EI}$