Faculty of Engineering
and Applied Science

Engineering 8074 - Design for Ocean and Ice Environments
Engineering 9096 – Sea Ice Engineering

FINAL EXAMINATION

Date: Thurs., Apr. 13, 2017 **Professor: Dr. C. Daley**
Time: 9:00 - 11:30 am

Answer all questions. Total 100 marks. Each question is worth marks indicated **[x].**
Watch your time.
Please answer on the question paper. Use the back of the sheets as needed.
Good luck.

NAME: ______________________________

STUDENT NUMBER: ______________________________

1. There are 5 question parts, worth 3 marks each. Fill in your answers in the spaces provided.
[15]

a) After about four inches of ice has formed on a pond, there is a heavy snowfall which sinks the 4 inches of ice and floods the lower layer of snow, which then freezes. When you take a core sample, a week later, and examine the crystal structure – what do you expect to see?

b) What happens to the salt in seawater when the sea freezes?

c) Compare the terms "multiyear ice" with "ancient glacial ice"

d) What will happen to worlds oceans if the whole of the Ross ice shelf in the Antarctic breaks off and floats up into the Pacific?

e) What is the main difference between sea ice and lake ice?

2. Ice Growth and Bearing Capacity [20]

a) Ice starts to form in a pond behind you house on December 12th. The daily average temperatures for the rest of December was -4C, and then -8 for January. That data was on a weather site at Environment Canada. There has been very little snow, which has blown off the pond, and the days in Dec and Jan have been mostly cloudy. You plan to go out on the pond on Feb 1. What do you expect the thickness to be?

b) How would your estimate have changes if there had been a lot of sunny days or a lot of snow?

3. The news story below was recently in the news. **[20]**

Husky Energy says one of their oil platforms had a close call with an iceberg off the coast of Newfoundland.

By: Staff The Canadian Press Published on Thu Mar 30 2017

ST. JOHN'S, N.L. — A floating oil platform off Newfoundland has had a near-miss with an iceberg the size of a small office building.

Husky Energy said a "medium size" iceberg came within 180 metres of the SeaRose FPSO at about 5:30 a.m. Wednesday.

"We had an iceberg pass close by our production facility," Husky's Colleen McConnell said Thursday.

"We've been monitoring this particular piece of ice for awhile. It changed direction at about two in the morning, and we obviously had to respond quickly."

The Canada-Newfoundland and Labrador Offshore Petroleum Board described the iceberg as 40 metres wide, 60 metres long and standing eight metres above the waterline.

The massive, 270-metre SeaRose is Husky's lone oil-producing asset in the area. Built in 2004, the red-hulled, ship-like vessel can also store up to 940,000 barrels.

The board says Husky de-pressurized production wells and flushed flowlines with treated seawater, while the crew mustered in preparation for a potential disconnect.

But the iceberg passed without incident and was more than 500 metres away by 6 a.m.

The board says it is discussing the near-miss with Husky.

Icebergs are monitored constantly in the area, about 350 kilometres east of St. John's in the Jeanne d'Arc Basin, McConnell said.

She said Husky uses satellite monitoring and dedicated surveillance flights to monitor for icebergs in what has been a busy season for them.

"There have been a number of icebergs in the area, we've been monitoring and managing those," she said. "Certainly, we've had a very busy number of days."

The company has several vessels in the area able to undertake ice-management measures, including towing and water cannons, she said.

McConnell wouldn't speculate on whether the SeaRose was in danger of being hit, or what might have happened if it had been.

The SeaRose took similar measures in 2015 during another near-miss. It has never had to disconnect, she said.

She said Husky decided Wednesday to move its nearby drilling rig, the Henry Goodrich, into an ice-free area. It had already suspended operations on Sunday because of ice and weather conditions.

a) Discuss how you would go about estimating the size of the impact that might have occurred but didn't in the case described above.
b) What information would you need? What would be a reasonable range of values for the needed information?
c) Can you provide an estimate of the possible impact force, with an estimate of its accuracy.

4. Resistance in ice (Linqvist) [15]
For a ship of the following dimensions, the owner wishes to be able to break 0.5m of ice at 4 m/s (8 knots). Assume characteristic length of ice is 15x thickness (in this case 7.5m).

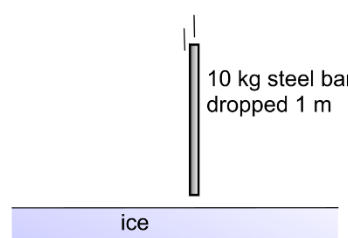
Ship length L: 100 m
Ship Breadth B: 20 m
Ships Draft T: 6 m
Ship's stem vertical angle $\emptyset$: 45 deg
Ship's stem waterline angle α: 45 deg
Ship's bow average waterline angle $\bar{\alpha}$: 30 deg

a) Describe Linqvist's assumptions and approach.

b) Compute the breaking component of resistance:

c) How would you go about verifying that value?

5. Ice impact **[15 total]**
Consider the case of impact where a round steel bar, dropped from a height of 1m (1m from base of bar to ice surface – also the distance that the center of gravity drops prior to impact), strikes a sheet of ice. The bar weights 10 kg has a diameter of 0.025 m. The bar penetrates 4cm into the ice.
The area of a circle is $\pi/4$ dia^2.



a) Can you estimate the ice crushing strength? State your assumptions and describe your reasoning.

b) What would you expect to happen if you dropped the bar from a height of 2m?

6. Pressure area. **[15]**

Explain the differences between ‘spatial’ and ‘process’ pressure-area models.

Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m^3
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·1O^{-6} / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i} \qquad (1)$$

Where Q is heat flux in W/m^2, k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i} \qquad (2)$$

Where t is time, and ρ_i is density of the ice.

the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS} \qquad (6)$$

Converted to freezing degree days:

$$h_i = .037\sqrt{FDD} \quad \text{[m]} \qquad (7)$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025\sqrt{FDD} \quad \text{[m]} \qquad (8)$$

$$\nu_b = .001 \cdot S\left(.53 - \frac{49.2}{T}\right) \qquad (13)$$

where T is temperature (°C), S is salinity (ppt) and ν_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65\left(1 - \sqrt{\frac{\nu_b}{275}}\right) \qquad (15)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24\left(1 - \sqrt{\frac{\nu_b}{33}}\right) \qquad (16)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82\left(1 - \sqrt{\frac{\nu_b}{142}}\right) \quad \text{case a) across grain} \qquad (17)$$

$$\sigma_t = 1.54\left(1 - \sqrt{\frac{\nu_b}{311}}\right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2} \qquad (21)$$

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2} \qquad (22)$$

1.1 Beam on Elastic Foundation

$$EI\frac{d^4 v}{dx^4} + kv = 0 \qquad (23)$$

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, $k = \rho g w$. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k}\right)^{1/4} \qquad (24)$$

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection: $$v = \frac{2P}{\lambda k} \cdot e^{-x/\lambda} \cdot \cos x/\lambda \qquad (25)$$

Slope: $$\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda)) \qquad (26)$$

Moment: $$M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda) \qquad (27)$$

Shear: $$Q = -P \cdot e^{-\beta x} \cdot (\cos(x/\lambda) - \sin(x/\lambda)) \qquad (28)$$

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4}\lambda \qquad (28)$$

$$\lambda = \left(\frac{E}{3\rho g}\right)^{1/4} h^{3/4} \qquad (29)$$

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4} \qquad (30)$$

$$\ell = \sqrt[4]{D/k} \qquad (55)$$
$$\omega = w/\ell \qquad (56)$$
$$x = r/\ell \qquad (57)$$
$$p' = p/k \qquad (58)$$

For a concentrated load the solution is;

$$w = \frac{p\ell^2}{2\pi D} kei(r/\ell) \qquad (60)$$

$$w_{\max} = \frac{p\ell^2}{8D} \qquad (61)$$

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}} \qquad (62)$$

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log\left[\frac{E \cdot h^3}{k \cdot c^4}\right]} \sigma_{flex} \cdot h^2 \qquad (67)$$
$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \qquad (70)$$

Where

- I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors
- m - shape factor (above)
- k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling)
- σ_c – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\overline{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \qquad (71)$$

<u>kinetic energy</u> (KE):

$$KE = \tfrac{1}{2} m \cdot v^2 \quad \text{(for head-on collisions)(72)}$$

$$KE = \tfrac{1}{2} m_e \cdot v_n^{\;2} \quad \text{(for oblique collisions)(73)}$$

m_e is the 'effective' mass

v_n is the 'normal' velocity

<u>crushing energy</u> (IE):

$$IE = \int_0^x F(x)dx \qquad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \qquad (75)$$

$$V = N_c(\cos\theta - \mu\sin\theta) \qquad (76)$$

$$H = N_c(\sin\theta + \mu\cos\theta) \qquad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6}t^2} \qquad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \qquad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left(60\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} - 1\right)} \qquad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{(60 \cdot (\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta))} \qquad (84)$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \qquad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta)/60)} \qquad (86)$$

The net forces applied to the ice are;

$$V = N_c(\cos\theta - \mu\sin\theta) - W\sin\theta \cdot (\sin\theta + \mu\cos\theta) \qquad (87)$$

$$H = N_c(\sin\theta + \mu\cos\theta) + W\cos\theta \cdot (\sin\theta + \mu\cos\theta) \qquad (88)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice:

$$V = \frac{\sigma_t t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (90)$$

With (87), the required normal force Nc is;

$$N_c = \frac{V + W\sin\theta \cdot (\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad (91)$$

$$P = C \cdot A^{-0.5}$$

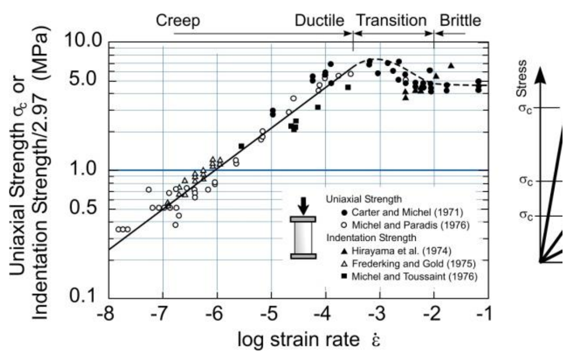


Figure 1. Influence of strain rate on uniaxial strength.

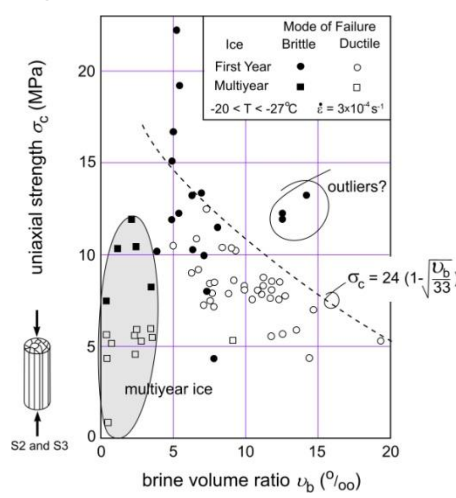


Figure 2. Compressive strength of Beaufort Sea ice.

The projected normal area is for spherical indentation;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \qquad (93)$$

The force is;:

$$F_n = p_o A_n^{ex} \cdot A_n$$
$$= p_o(2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \qquad (94)$$

The indentation energy is:

$$IE = \frac{p_o}{(2+ex)}(2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \qquad (95)$$

Which can be expressed in a more general form as :

$$IE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \quad (96)$$

$$fx = 2 + ex \quad (97)$$

$$fa = (2\pi \cdot R)^{fx-1} \quad (98)$$

$$KE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \quad (101)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa}\right)^{\frac{1}{fx}} \quad (102)$$

The normal force can be found by substituting eqn. (102) into (96), with (97) and (98) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa}\right)^{\frac{fx-1}{fx}} \quad (103)$$

Which in our spherical case becomes:

$$F_n = \left[p_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)}\right]^{\frac{1}{2+ex}} \cdot \left(\frac{1}{2} M_e \cdot V_n^2 (2+ex)\right)^{\frac{1+ex}{2+exx}}$$

(104)

Lindqvist's resistance equation

$$R_{ice} = (R_C + R_B)(1 + 1.4 \cdot v / \sqrt{g \cdot H_{ice}}) + R_S \cdot (1 + 9.4 \cdot v / \sqrt{g \cdot L})$$

(105)

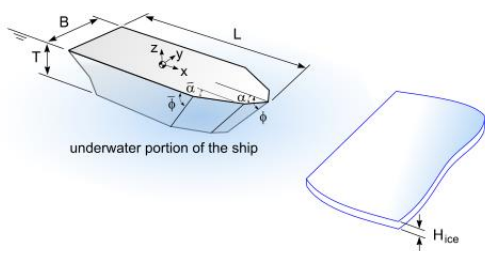


Figure 3. Linqvist's hull form with angles and dimensions.

1.1.1 Ice Crushing Term R_C

Linqvist starts by estimating the vertical force needed to fail an ice edge in flexure:

$$F_v = 0.5 \cdot \sigma_b \cdot H_{ice}^2 \quad (106)$$

Where σ_b is the flexural strength of ice. He then assumes that this force is generated (continuously) on each side of the stem and resolves the x (along ship) component, which is the resistance that the ship will feel;

$$R_C = F_v \frac{\tan\phi + \mu \cdot \cos\phi / \cos\psi}{1 - \mu \cdot \sin\phi / \cos\psi} \quad (107)$$

Where μ is the friction factor, ϕ is the stem angle, α is the waterline angle and;

$$\psi = \arctan(\tan\phi / \sin\alpha) \quad (108)$$

These equations are dimensionally consistent, so any units will work.

1.1.2 Ice Breaking Term R_B

The breaking resistance term (see appendix in Linqvist 1989 for derivation) is as follows;

$$R_B = k \cdot B \cdot \left(\frac{H_{ice}^3}{l_c^2}\right)(\tan\psi + \mu\cos\phi / (\sin\alpha \cdot \cos\psi))(1 + 1/\cos\psi)$$

(109)

Where k is a constant, B is the beam of the ship, the angles are as shown previously and l_c is the characteristic length of the ice sheet;

$$l_c = \sqrt[4]{\frac{E \cdot H_{ice}^3}{12\rho_w g(1-\upsilon^2)}} \quad (110)$$

Lindqvist makes the following assumptions to simplify the equation. Poisons ratio is 0.3. The elastic modulus is 2 GPa. He assumes the cusp length is 1/3 of the characteristic length and the shear strength is equal to the flexural strength.

$$R_B = .003 \cdot \sigma_b \cdot B \cdot \left(\frac{H_{ice}^{1.5}}{\sqrt{m}}\right)(\tan\psi + \mu\cos\phi / (\sin\alpha \cdot \cos\psi))(1 + 1/\cos\psi)$$

(111)

1.1.3 Ice Submergence Term R_S

The submergence resistance term (see appendix in Linqvist 1989 for derivation) is derived from energy consideration and is as follows;

$$R_S = \delta\rho \cdot g \cdot H_{ice} \cdot \left(B \cdot T \cdot (B+T)/(B+2T) + \mu(A_u + \cos\phi \cdot \cos\psi \cdot A_f)\right)$$

(113)

Where $\delta\rho$ is the density difference between ice and water (the cause of buoyancy) A_u is the area of the flat of bottom and A_f is the area of the bow. With approximations for the areas involved, Linqvist proposed the alternate formula;

$$R_S = \delta\rho \cdot g \cdot H_{ice} \cdot B \cdot \left(T\frac{B+T}{B+2T} + \mu\left(.7 \cdot L - \frac{T}{\tan\phi} - \frac{0.25 \cdot B}{\tan\alpha} + T \cdot \cos\phi \cdot \cos\psi \cdot \sqrt{\frac{1}{\sin^2\phi} + \frac{1}{\tan^2\alpha}}\right)\right)$$

(114)

POPOV

$$M_e = \frac{F_n}{a_n} \quad (115)$$

The effective mass is;

$$M_e = \frac{M_{ship}}{C_O} \quad (116)$$

where *Co* is the mass reduction coefficient.

The inertial properties of the vessel are as follows,

Hull angles at point:
- α = waterline angle
- β = frame angle
- β' = normal frame angle
- γ = sheer angle

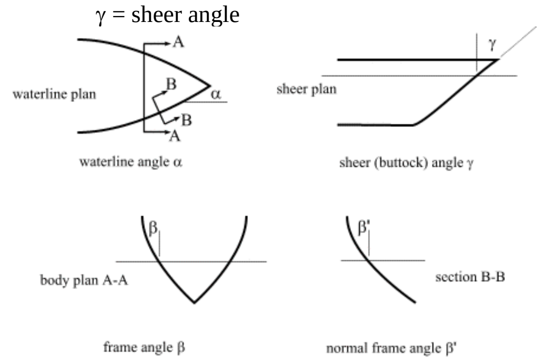


Figure 4. Hull angle definitions.

The various angles are related as follows,

$$\tan\beta = \tan\alpha \cdot \tan\gamma \quad (117)$$

$$\tan\beta' = \tan\beta \cdot \cos\alpha \quad (118)$$

Based on these angles, the direction cosines, *l,m,n* are

$$l = \sin\alpha \cdot \cos\beta' \quad (119)$$

$$m = \cos\alpha \cdot \cos\beta' \quad (120)$$

$$n = \sin\beta' \quad (121)$$

and the moment arms are

$$\lambda 1 = n \cdot y_p - m \cdot z_p \text{ (roll moment arm)} \quad (122)$$

$$\mu 1 = l \cdot z_p - n \cdot x_p \text{ (pitch moment arm)} \quad (123)$$

$$\eta 1 = m \cdot x_p - l \cdot y_p \text{ (yaw moment arm)} \quad (124)$$

The added mass terms are as follows;

AMx = added mass factor in surge = 0 (125)
AMy = added mass factor in sway = 2 T/B (126)
AMz = added mass factor in heave = 2/3 (B Cwp^2)/(T(Cb(1+Cwp)) (127)
AMrol = added mass factor in roll = 0.25 (128)
AMpit = added mass factor in pitch = B/((T(3-2Cwp)(3-Cwp)) (129)
AMyaw = added mass factor in yaw = 0.3 + 0.05 L/B (130)

The mass radii of gyration (squared) are;

$rx^2 = Cwp\ B^2/(11.4\ Cm) + H^2/12$ (roll) (131)
$ry^2 = 0.07\ Cwp\ L^2$ (pitch) (132)
$rz^2 = L^2/16$ (yaw) (133)

With the above quantities defined, the mass reduction coefficient is

$$Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz) + \lambda 1^2/(rx^2(1+AMrol) + \mu 1^2/(ry^2\,(1+AMpit)) + \eta 1^2/(rz^2\,(1+AMyaw)) \qquad (134)$$

1.1.4 Contact with General Wedge (Normal to hull)

a general wedge-shaped edge indentation (normal to hull). The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta`)} \qquad (135)$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos(\beta`)} \qquad (136)$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \qquad (137)$$

The pressure-area approach is used again. Substituting (137) into (75) we arrive at:

$$F_n = p_o A_n^{ex} \cdot A_n$$
$$= p_o \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{2+2ex} \qquad (138)$$

Note: this can be expressed in a general form as:

$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \qquad (139)$$

Where $fx = 3 + 2ex$ (140)

$$fa = \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \qquad (141)$$

The indentation energy is found by substituting (140) and (141) into (96), to give:

$$IE = \frac{p_o}{(3+2ex)} \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \qquad (142)$$

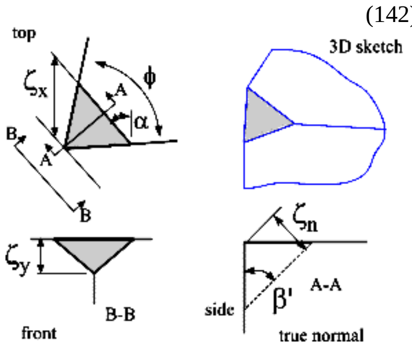


Figure 5. General Wedge-shaped Edge (normal to hull).

1.1.5 Popov Force Calculation

The Popov collision scenarios can be analyzed by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \qquad (143)$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \qquad (144)$$

which ,using equation (96) can be stated as;

$$KE_e = \frac{p_o}{fx}\, fa \cdot \zeta_n^{fx} \qquad (145)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \qquad (146)$$

The normal force can be found by substituting eqn. (146) into (139) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \qquad (147)$$

$$V_n = V_{ship} \cdot lx \qquad (149)$$

where V_n is the normal velocity at the point of impact
V_{ship} is the x-direction velocity (all others are zero)
lx is the x-direction cosine

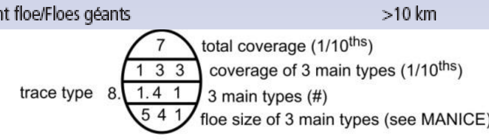
By combining the above equations, a single closed-form expression for the collision force is found;

$$F_n := \left[p_o \cdot \left[\frac{\tan\left(\frac{1}{2}\cdot\phi\right)}{\left(\sin(\beta')\cdot\cos(\beta')^2\right)} \right]^{(1+ex)} \right]^{\frac{1}{3+2\cdot ex}} \cdot \left[\frac{1}{2}\cdot Me\cdot Vn^2\cdot(3+2\cdot ex) \right]^{\left[\frac{(2+2\cdot ex)}{(3+2\cdot ex)}\right]} \qquad (151)$$

Color	Ice Type		Thickness		Egg Code #	
	Ice Free					
	< 1/10 unspecified ice					
	New Ice		< 10 cm		1,2	
	Grey Ice		10 - 15 cm		4	3
	Grey-White Ice		15 - 30 cm		5	
	Thin FY	First-year Ice (FY)	30 - 70 cm	>30 cm	7 8/9	6
	Medium FY		70 - 120 cm		1.	
	Thick FY		> 120 cm		4.	
	Second-year Ice	Old Ice			8.	7.
	Multi-year Ice				9.	

Floe Size/Grandeur des floes ($F_aF_bF_c$)

Description/Élément	Width/Extension	Code
Pancake ice/Glace en crêpes		0
Small ice cake, brash ice/Petit glaçons, sarrasins	<2 m	1
Ice cake/Glaçons	2-20 m	2
Small floe/Petits floes	20-100 m	3
Medium floe/Floes moyens	100-500 m	4
Big floe/Grands floes	500-2000 m	5
Vast floe/Floes immenses	2-10 km	6
Giant floe/Floes géants	>10 km	7



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.