



Faculty of Engineering
and Applied Science

Engineering 8074 - Design for Ocean and Ice Environments
Engineering 9096 – Sea Ice Engineering

FINAL EXAMINATION

Solutions

Date: Wed., Apr. 10, 2019

Professor: Dr. C. Daley

Time: 9:00 - 11:30 am

Answer all questions. Total 100 marks. Each question is worth marks indicated [x].

Watch your time.

Please answer on the question paper. Use the back of the sheets as needed.

Good luck.

NAME: _____

STUDENT NUMBER: _____

1. There are 10 questions worth 2 marks each. Fill in the answers (or complete the phrase) in the spaces provided.

[20]

1. List two ice class systems? ...

IACS Polar Class Rules, and Finnish-Sweedish Ice Class Rules

2. In ice mechanics, a cantilever beam is used to...

Flexural strength of the whole ice sheet

3. Icebreakers have bows which are ...

Shaped to break and clear ice. Normally very large angles from vertical

4. The term “FDD” refers to ...

Freezing Degree Days

5. Latent Heat refers to ...

The energy need to change ice from solid to liquid

6. Popov is from Russia and Lynqvist is from Finland

7. Primary ice is ...

A surface layer of polycrystalline ice that forms initially out of frazil

8. Ice floats because ...

It is less dense than water.

9. The term “pressure ridge” refers to ...

A linear region of rubble ice that forms between two converging ice floes

10. Croasdale is known for ...

His limit load concepts

2. Describe ice at the atomic scale, including a sketch.

[10]

3. Explain what Multi-year ice is and its importance to shipping.

[10]

4. Ice Indentation

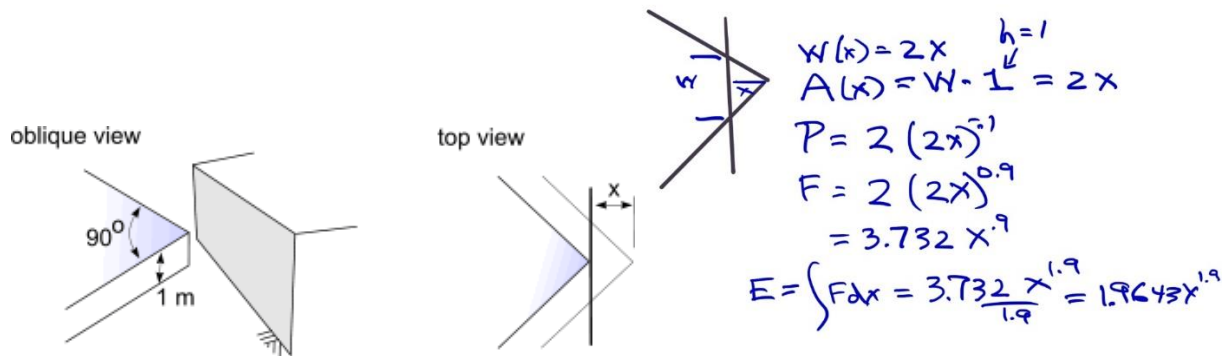
[20]

The sketch below shows a 90 deg ice edge, 1m thick, coming into contact with a solid wall.

Assuming that the ice crushing can be described with a process pressure-area relation of;

$$P_{av} = P_o A^{-0.1}, P_o = 2 \text{ MPa}$$

Find the contact Area, Force and crushing energy, all as functions of the penetration depth x .

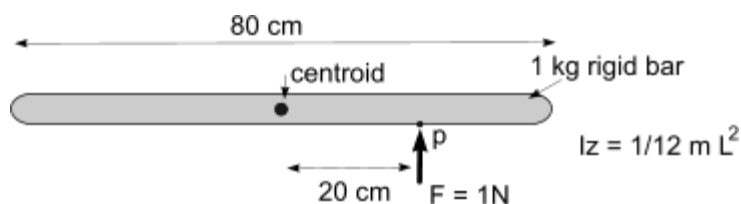


5. Effective Mass

[20]

A force of 1 N is applied to the side of an 80cm bar as shown, causing the bar to accelerate.

- what is the vertical acceleration at the centroid?
- what is the angular acceleration of the bar?
- what is the vertical acceleration at point p (where the force is) ?
- what is the effective mass of point p ?



$$I = \frac{1}{12} 1 \times 0.8^2 = 0.0533 \text{ kg m}^2$$

$$a_{vc} = \frac{F}{m} = \frac{1}{1} = 1 \text{ m/s}^2$$

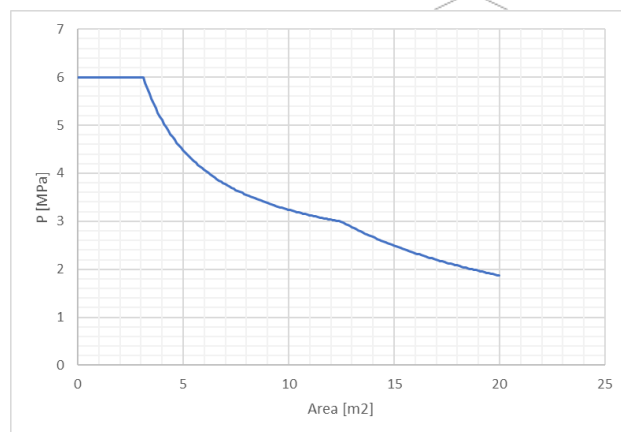
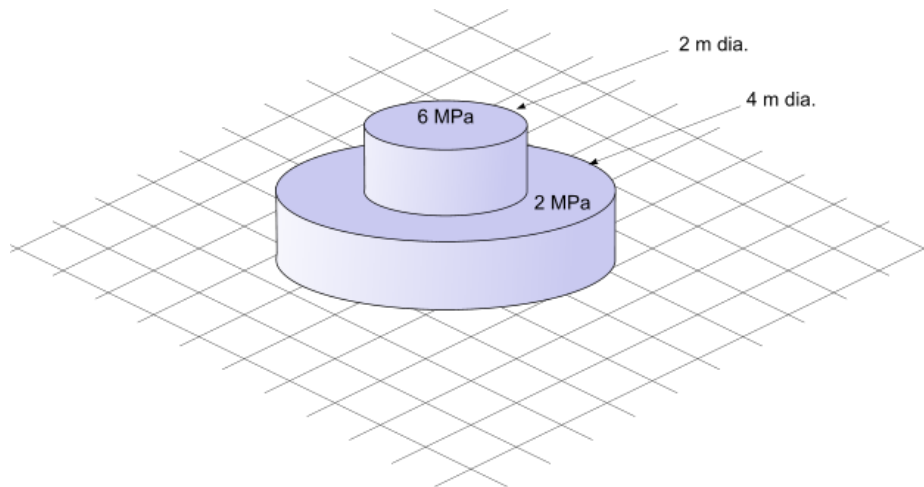
$$\alpha_c = \frac{M}{I} = \frac{1 \times 0.2}{0.0533} = 3.75 \text{ r/s}^2$$

$$\begin{aligned}
 a_p &= a_{vc} + \alpha_c \cdot l \\
 &= 1 + 3.75 \times 0.2 = 1.75
 \end{aligned}$$

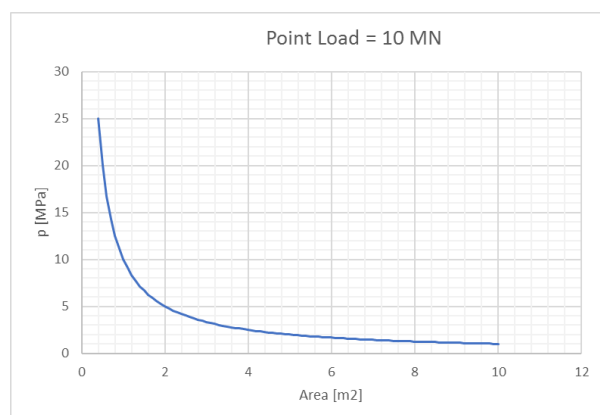
$$m_{\text{eff}} = \frac{F}{a_p} = \frac{1}{1.75} = \underline{\underline{0.571 \text{ kg}}}$$

6. Pressure Area

a) A load patch consists of a circular region of 6 MPa surrounded by a larger region of 2 MPa. Sketch the spatial pressure-area curve for this load. [10]



b) what would be the spatial pressure-area curve for a point load (over the area range of 0 – 10 m²) [10]



Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m³
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·10 ⁻⁶ / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i}$$
(1)

Where Q is heat flux in W/m², k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i}$$
(2)

Where t is time, and ρ_i is density of the ice.

the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$
(6)

Converted to freezing degree days:

$$h_i = .037 \sqrt{FDD} \quad [\text{m}]$$
(7)

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025 \sqrt{FDD} \quad [\text{m}]$$
(8)

$$v_b = .001 \cdot S \left(.53 - \frac{49.2}{T} \right)$$
(13)

where T is temperature (°C), S is salinity (ppt) and v_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65 \left(1 - \sqrt{\frac{v_b}{275}} \right)$$
(15)

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24 \left(1 - \sqrt{\frac{v_b}{33}} \right)$$
(16)

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82 \left(1 - \sqrt{\frac{v_b}{142}} \right) \quad \text{case a) across grain}$$
(17)

$$\sigma_t = 1.54 \left(1 - \sqrt{\frac{v_b}{311}} \right) \quad \text{case b) along grain}$$
(18)

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2}$$
(21)

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2}$$
(22)

1.1 Beam on Elastic Foundation

$$EI \frac{d^4 v}{dx^4} + kv = 0$$
(23)

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, k = ρgw. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k} \right)^{1/4}$$
(24)

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection:
$$v = \frac{2P}{\lambda k} \cdot e^{-x/\lambda} \cdot \cos(x/\lambda)$$
(25)

Slope:
$$\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$$
(26)

Moment:
$$M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$$
(27)

Shear:
$$Q = -P \cdot e^{-\beta x} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$$
(28)

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4} \lambda$$
(28)

$$\lambda = \left(\frac{E}{3\rho g} \right)^{1/4} h^{3/4}$$
(29)

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4}$$
(30)

$$\ell = \sqrt[4]{D/k}$$
(55)

$$\omega = w/\ell$$
(56)

$$x = r/\ell$$
(57)

$$p' = p/k$$
(58)

For a concentrated load the solution is;

$$w = \frac{p\ell^2}{2\pi D} kei(r/\ell)$$
(60)

$$w_{\max} = \frac{p\ell^2}{8D}$$
(61)

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}}$$
(62)

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log \left[\frac{E \cdot h^3}{k \cdot c^4} \right]} \sigma_{flex} \cdot h^2 \quad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \quad (70)$$

Where

I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors
 m - shape factor (above)
 k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling)
 σ_c - uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\bar{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \quad (71)$$

kinetic energy (KE):

$$KE = \frac{1}{2} m \cdot v^2 \quad (\text{for head-on collisions})(72)$$

$$KE = \frac{1}{2} m_e \cdot v_n^2 \quad (\text{for oblique collisions})(73)$$

m_e is the 'effective' mass

v_n is the 'normal' velocity

crushing energy (IE):

$$IE = \int_0^x F(x) dx \quad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \quad (75)$$

$$V = N_c (\cos \theta - \mu \sin \theta) \quad (76)$$

$$H = N_c (\sin \theta + \mu \cos \theta) \quad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6} t^2} \quad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \quad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t} \left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left(60 \frac{\cos \theta - \mu \sin \theta}{\sin \theta + \mu \cos \theta} - 1 \right)} \quad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{\left(60 \cdot (\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta) \right)} \quad (84)$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \quad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta) / 60)} \quad (86)$$

The net forces applied to the ice are;

$$V = N_c (\cos \theta - \mu \sin \theta) - W \sin \theta \cdot (\sin \theta + \mu \cos \theta) \quad (87)$$

$$H = N_c (\sin \theta + \mu \cos \theta) + W \cos \theta \cdot (\sin \theta + \mu \cos \theta) \quad (88)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t} \left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice:

$$V = \frac{\sigma_t t}{\left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (90)$$

With (87), the required normal force N_c is;

$$N_c = \frac{V + W \sin \theta \cdot (\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)} \quad (91)$$

$$P = C \cdot A^{-0.5}$$

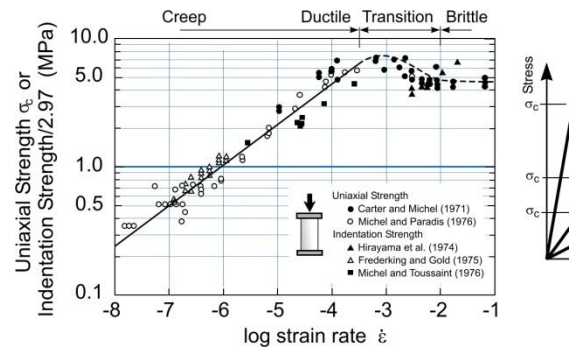


Figure 1. Influence of strain rate on uniaxial strength.

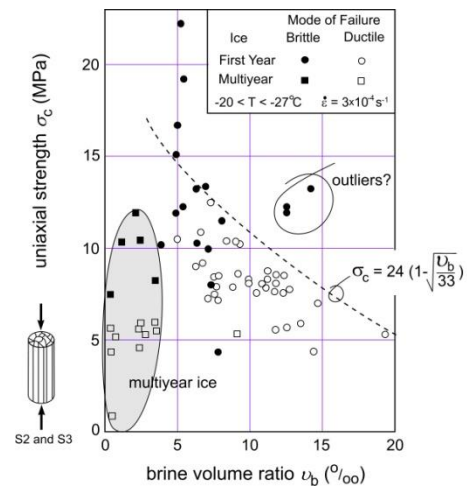


Figure 2. Compressive strength of Beaufort Sea ice.

The projected normal area is for spherical indentation;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \quad (93)$$

The force is;:

$$F_n = p_o A_n^{ex} \cdot A_n = p_o (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \quad (94)$$

The indentation energy is:

$$IE = \frac{p_o}{(2+ex)} (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \quad (95)$$

Which can be expressed in a more general form as :

$$IE_e = \frac{p_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (96)$$

$$f_x = 2 + ex \quad (97)$$

$$f_a = (2\pi \cdot R)^{f_x - 1} \quad (98)$$

$$KE_e = \frac{p_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (101)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot f_x}{p_o \cdot f_a} \right)^{\frac{1}{f_x}} \quad (102)$$

The normal force can be found by substituting eqn. (102) into (96), with (97) and (98) to give

$$F_n = p_o \cdot f_a \cdot \left(\frac{KE_e \cdot f_x}{p_o \cdot f_a} \right)^{\frac{f_x - 1}{f_x}} \quad (103)$$

Which in our spherical case becomes:

$$F_n = \left[p_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)} \right]^{\frac{1}{2+ex}} \cdot \left(\frac{1}{2} M_e \cdot V_n^2 (2+ex) \right)^{\frac{1+ex}{2+ex}} \quad (104)$$

Lindqvist's resistance equation

$$R_{ice} = (R_C + R_B)(1 + 1.4 \cdot v / \sqrt{g \cdot H_{ice}}) + R_S \cdot (1 + 9.4 \cdot v / \sqrt{g \cdot L}) \quad (105)$$

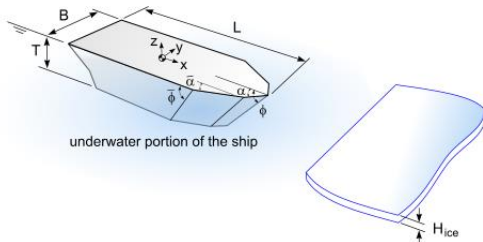


Figure 3. Lindqvist's hull form with angles and dimensions.

1.1.1 Ice Crushing Term R_C

Lindqvist starts by estimating the vertical force needed to fail an ice edge in flexure:

$$F_v = 0.5 \cdot \sigma_b \cdot H_{ice}^2 \quad (106)$$

Where σ_b is the flexural strength of ice. He then assumes that this force is generated (continuously) on each side of the stem and resolves the x (along ship) component, which is the resistance that the ship will feel;

$$R_C = F_v \frac{\tan \phi + \mu \cdot \cos \phi / \cos \psi}{1 - \mu \cdot \sin \phi / \cos \psi} \quad (107)$$

Where μ is the friction factor, ϕ is the stem angle, α is the waterline angle and;

$$\psi = \arctan(\tan \phi / \sin \alpha) \quad (108)$$

These equations are dimensionally consistent, so any units will work.

1.1.2 Ice Breaking Term R_B

The breaking resistance term (see appendix in Lindqvist 1989 for derivation) is as follows;

$$R_B = k \cdot B \cdot \left(\frac{H_{ice}^3}{l_c^2} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi) \quad (109)$$

Where k is a constant, B is the beam of the ship, the angles are as shown previously and l_c is the characteristic length of the ice sheet;

$$l_c = 4 \sqrt{\frac{E \cdot H_{ice}^3}{12 \rho_w g (1 - \nu^2)}} \quad (110)$$

Lindqvist makes the following assumptions to simplify the equation. Poisons ratio is 0.3. The elastic modulus is 2 GPa. He assumes the cusp length is 1/3 of the

characteristic length and the shear strength is equal to the flexural strength.

$$R_B = .003 \cdot \sigma_b \cdot B \cdot \left(\frac{H_{ice}^{1.5}}{\sqrt{m}} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi) \quad (111)$$

1.1.3 Ice Submergence Term R_S

The submergence resistance term (see appendix in Lindqvist 1989 for derivation) is derived from energy consideration and is as follows;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot (B \cdot T \cdot (B + T) / (B + 2T) + \mu (A_u + \cos \phi \cdot \cos \psi \cdot A_f)) \quad (113)$$

Where $\delta \rho$ is the density difference between ice and water (the cause of buoyancy) A_u is the area of the flat of bottom and A_f is the area of the bow. With approximations for the areas involved, Lindqvist proposed the alternate formula;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot B \cdot \left(T \frac{B+T}{B+2T} + \mu \left(.7 \cdot L - \frac{T}{\tan \phi} - \frac{0.25 \cdot B}{\tan \alpha} + T \cdot \cos \phi \cdot \cos \psi \cdot \sqrt{\frac{1}{\sin^2 \phi} + \frac{1}{\tan^2 \alpha}} \right) \right) \quad (114)$$

POPOV

$$M_e = \frac{F_n}{a_n} \quad (115)$$

The effective mass is;

$$M_e = \frac{M_{ship}}{C_o} \quad (116)$$

where C_o is the mass reduction coefficient.

The inertial properties of the vessel are as follows,

Hull angles at point:

α = waterline angle

β = frame angle

β' = normal frame angle

γ = sheer angle

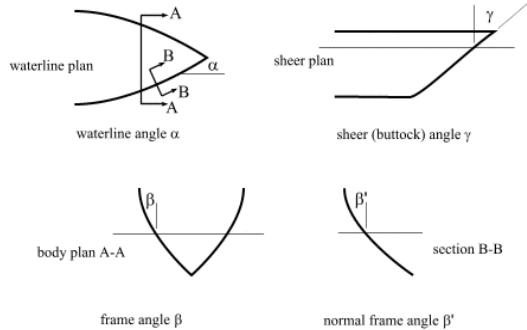


Figure 4. Hull angle definitions.

The various angles are related as follows,

$$\tan \beta = \tan \alpha \cdot \tan \gamma \quad (117)$$

$$\tan \beta' = \tan \beta \cdot \cos \alpha \quad (118)$$

Based on these angles, the direction cosines, l, m, n are

$$l = \sin \alpha \cdot \cos \beta' \quad (119)$$

$$m = \cos \alpha \cdot \cos \beta' \quad (120)$$

$$n = \sin \beta' \quad (121)$$

and the moment arms are

$$\lambda_1 = n \cdot y_p - m \cdot z_p \quad (\text{roll moment arm}) \quad (122)$$

$$\mu_1 = l \cdot z_p - n \cdot x_p \quad (\text{pitch moment arm}) \quad (123)$$

$$\eta_1 = m \cdot x_p - l \cdot y_p \quad (\text{yaw moment arm}) \quad (124)$$

The added mass terms are as follows;

$$AM_x = \text{added mass factor in surge} = 0 \quad (125)$$

$$AM_y = \text{added mass factor in sway} = 2 T/B \quad (126)$$

$$AM_z = \text{added mass factor in heave} = 2/3 (B Cwp^2) / (T(Cb(1+Cwp))) \quad (127)$$

$$AM_{rol} = \text{added mass factor in roll} = 0.25 \quad (128)$$

$$AM_{pit} = \text{added mass factor in pitch} = B / (T(3-2Cwp)(3-Cwp)) \quad (129)$$

$$AM_{yaw} = \text{added mass factor in yaw} = 0.3 + 0.05 L/B \quad (130)$$

The mass radii of gyration (squared) are;

$$rx^2 = Cwp B^2 / (11.4 Cm) + H^2 / 12 \quad (\text{roll}) \quad (131)$$

$$ry^2 = 0.07 Cwp L^2 \quad (\text{pitch}) \quad (132)$$

$$rz^2 = L^2 / 16 \quad (\text{yaw}) \quad (133)$$

With the above quantities defined, the mass reduction coefficient is

$$C_o = l^2/(1+AM_x) + m^2/(1+AM_y) + n^2/(1+AM_z) + \lambda l^2/(rx^2(1+AM_{rol}) + \mu l^2/(ry^2(1+AM_{pit})) + \eta l^2/(rz^2(1+AM_{yaw})) \tag{134}$$

1.1.4 Contact with General Wedge (Normal to hull)

a general wedge-shaped edge indentation (normal to hull).
The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta'')} \tag{135}$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta'') \cos(\beta'')} \tag{136}$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta'') \cos^2(\beta'')} \tag{137}$$

The pressure-area approach is used again. Substituting (137) into (75) we arrive at:

$$F_n = p_o A_n^{ex} \cdot A_n = p_o \left(\frac{\tan(\phi/2)}{\sin(\beta'') \cos^2(\beta'')} \right)^{1+ex} \cdot \zeta_n^{2+2ex} \tag{138}$$

Note: this can be expressed in a general form as:
$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \tag{139}$$

Where $fx = 3 + 2ex$ (140)

$$fa = \left(\frac{\tan(\phi/2)}{\sin(\beta'') \cos^2(\beta'')} \right)^{1+ex} \tag{141}$$

The indentation energy is found by substituting (140) and (141) into (96), to give:

$$IE = \frac{p_o}{(3 + 2ex)} \left(\frac{\tan(\phi/2)}{\sin(\beta'') \cos^2(\beta'')} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{142}$$

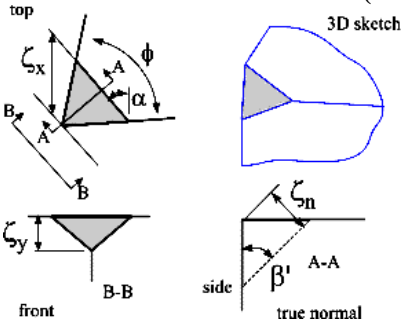


Figure 5. General Wedge-shaped Edge (normal to hull).

1.1.5 Popov Force Calculation

The Popov collision scenarios can be analyzed by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \tag{143}$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \tag{144}$$

which ,using equation (96) can be stated as;

$$KE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \tag{145}$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \tag{146}$$

The normal force can be found by substituting eqn. (146) into (139) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \tag{147}$$

$$V_n = V_{ship} \cdot lx \tag{149}$$

where V_n is the normal velocity at the point of impact

V_{ship} is the x-direction velocity (all others are zero)
 lx is the x-direction cosine

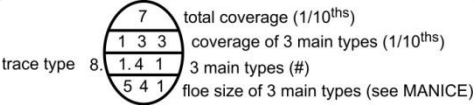
By combining the above equations, a single closed-form expression for the collision force is found;

$$F_n := p_o \cdot \left[\frac{\tan\left(\frac{1}{2} \cdot \phi\right)}{\left(\sin(\beta'') \cdot \cos(\beta'')\right)^2} \right]^{(1+ex)} \cdot \frac{1}{3+2 \cdot ex} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3+2 \cdot ex) \right]^{\frac{(2+2 \cdot ex)}{(3+2 \cdot ex)}} \tag{151}$$

Color	Ice Type	Thickness	Egg Code #
	Ice Free		
	< 1/10 unspecified ice		
	New Ice	< 10 cm	1,2
	Grey Ice	10 - 15 cm	4
	Grey-White Ice	15 - 30 cm	5
	Thin FY	First-year ice (FY)	7
	Medium FY	30 - 70 cm	8
	Thick FY	70 - 120 cm	9
	Second-year Ice	> 120 cm	1.
	Multi-year Ice	Old Ice	4.
			8.
			9.

Floe Size/Grandeur des floes (F_aF_bF_c)

Description/Élément	Width/Extension	Code
Pancake ice/Glace en crêpes		0
Small ice cake, brash ice/Petit glaçons, sarrasins	<2 m	1
Ice cake/Glaçons	2-20 m	2
Small floe/Petits floes	20-100 m	3
Medium floe/Floes moyens	100-500 m	4
Big floe/Grands floes	500-2000 m	5
Vast floe/Floes immenses	2-10 km	6
Giant floe/Floes géants	>10 km	7



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.