MEMORIAL UNIVERSITY Faculty of Engineering and Applied Science

Engineering 8674 - Design for Ocean and Ice Environments

FINAL EXAMINATION

With Solutions

Date: Thurs., Apr. 12, 2007 **Professor: Dr. C. Daley**
Time: 2:00 - 4:30 pm

Answer all questions. Total 100 marks. Each question is worth marks indicated **[x].**
Watch your time. 150min/100 marks = 90 sec/mark!
Short, clear answers are best. If you are having a problem (ie a road block) assume something, write down the assumption, and continue. Use the back of the sheets as needed.
Good luck.

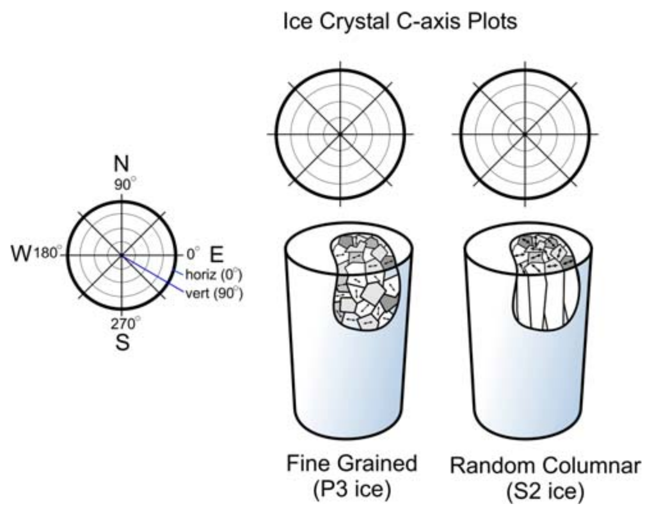
NAME: ______________________________

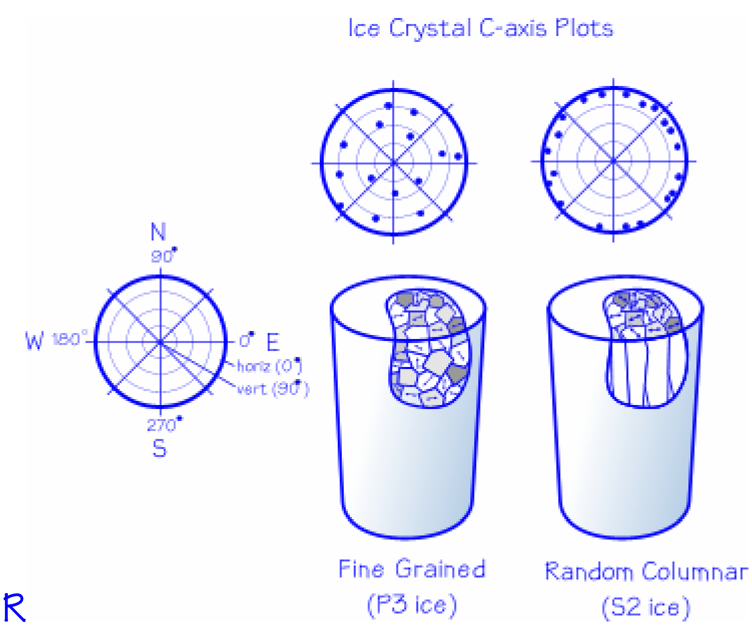
NUMBER: ______________________________

1. There are 10 questions worth 1 mark each. Fill in the answers in the spaces provided. **[10]**

1. In the notes the third 'leg' of ice engineering is said to be: field observations + guesswork.
2. Who came first, Franklin, Hudson or Davis? Davis.
3. What was the key artic 'event' of the 19th century? Loss of Franklin.
4. The word Albedo refers to: reflectivity.
5. 60 cm thick sea ice is termed: thin First Year Ice (2nd stage).
6. Dark Nilas is: thin new sea ice around 5cm thick.
7. Comminution means Breakup of a solid by itself.
8. The development of pressure/area with indentation is called: process pressure-area.
9. The highest Baltic Ice Class is called: 1ASuper.
10. Hobson's Choice is: An Ice Island where the medium scale ice experiments took place.

2. Fill in the C-axis plots with plausible examples of grain orientations for the P3 and S2 ice samples: **[10]**





ANSWER

3. Sea Ice is 1cm (or 5cm) thick. Its -25C. How long until the ice will be 10cm thick? **[10]**

use the equation:

$$h_i = .025\sqrt{FDD} \qquad \text{m, deg C}$$

$$FDD = \left(\frac{h_i}{.025}\right)^2 = 1600\,h_i^2$$

to reach 1cm , it takes:

$FDD = 1600\,(.01)^2 = .16$ FDD

@-25C = .0064 days = .15 hrs

to reach 10cm , it takes:

$FDD = 1600\,(.1)^2 = 16$ FDD

@-25C = .64 days = 15.36 hrs

to gor from 1cm to 10cm takes:

15.2 hrs <= ANS

or

use the equation:

$$h_i = .025\sqrt{FDD} \qquad \text{m, deg C}$$

$$FDD = \left(\frac{h_i}{.025}\right)^2 = 1600\,h_i^2$$

to reach 5cm , it takes:

$FDD = 1600\,(.05)^2 = 4$ FDD

@-25C = .16 days = 3.84 hrs

to reach 10cm , it takes:

$FDD = 1600\,(.1)^2 = 16$ FDD

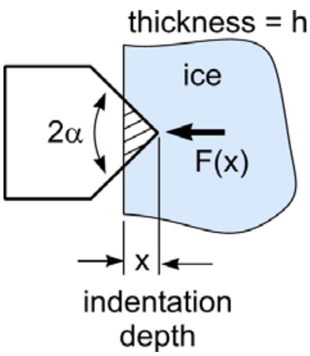
@-25C = .64 days = 15.36 hrs

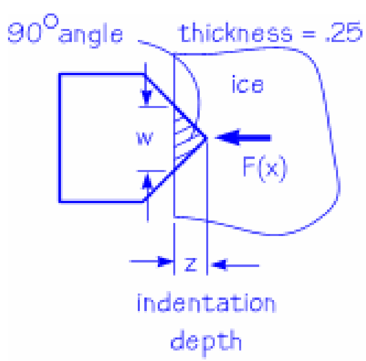
to gor from 5cm to 10cm takes:

11.52 hrs <= ANS

4. Write a paragraph describing the best aspect of the book you read for the course. **[15]**

5. An ice floe with dimensions 8m x 8m by 25 cm (fresh water ice) hits a bridge pier as pictured below, at 1 m/s. The pier is vertical, with 2α = 90deg. The ice pressure (process pressure) can be described with $P=2xA^{-.5}$ MPa. What is the impact force on the bridge pier. Once this is done, compare the answer to Korzhavin's equation (with σ_c = 5 MPa). **[20]**





ice mass = 8x8x.25x1000 = 16000kg

ice kinetic energy = 16000kg x 1 m^2/s^2

KE = 16000 N-m = .016 MN-m

w = 2 tan45 z = 2 z

A = w h = .5 z

$P = 2\, A^{-.5}$

$F = 2\, A^{-.5} \times A = 2\, A^{.5} = 1.414\, z^{.5}$ (MN)

$IE = \int F\, dz = 1.414/1.5\ z^{1.5} = .943\, z^{1.5}$

KE=IE : $.943\, z^{1.5} = .016$ -> $z = (.016/.943)^{.666} = .066$m

$F = 1.414\ .066^{.5} = .363$ (MN) = 363 kN <=ANS

Korzhavin

$P_{ice} = I\ \ m\ \ k\ \ \sigma_c$

Where

I - indentation (confinement) factor, say3.0

m - shape factor, say 1.0

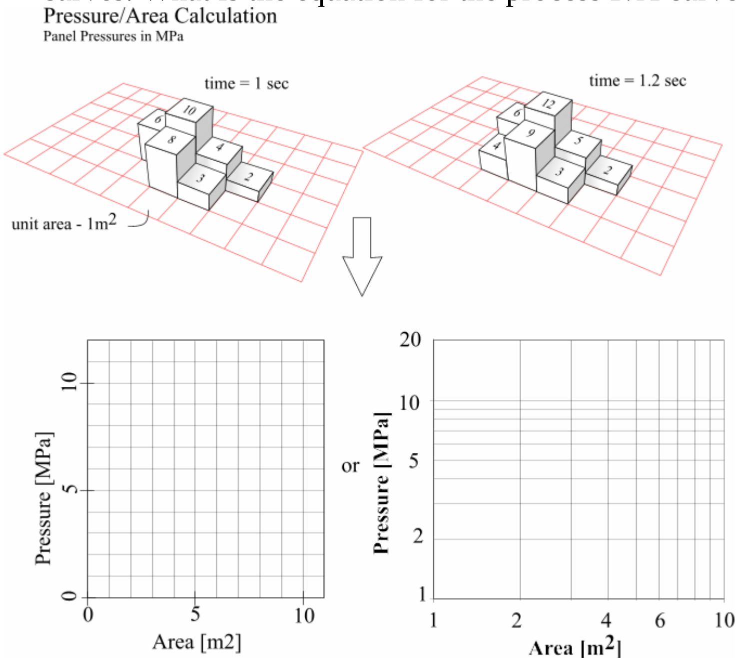
k - contact factor say 0.8

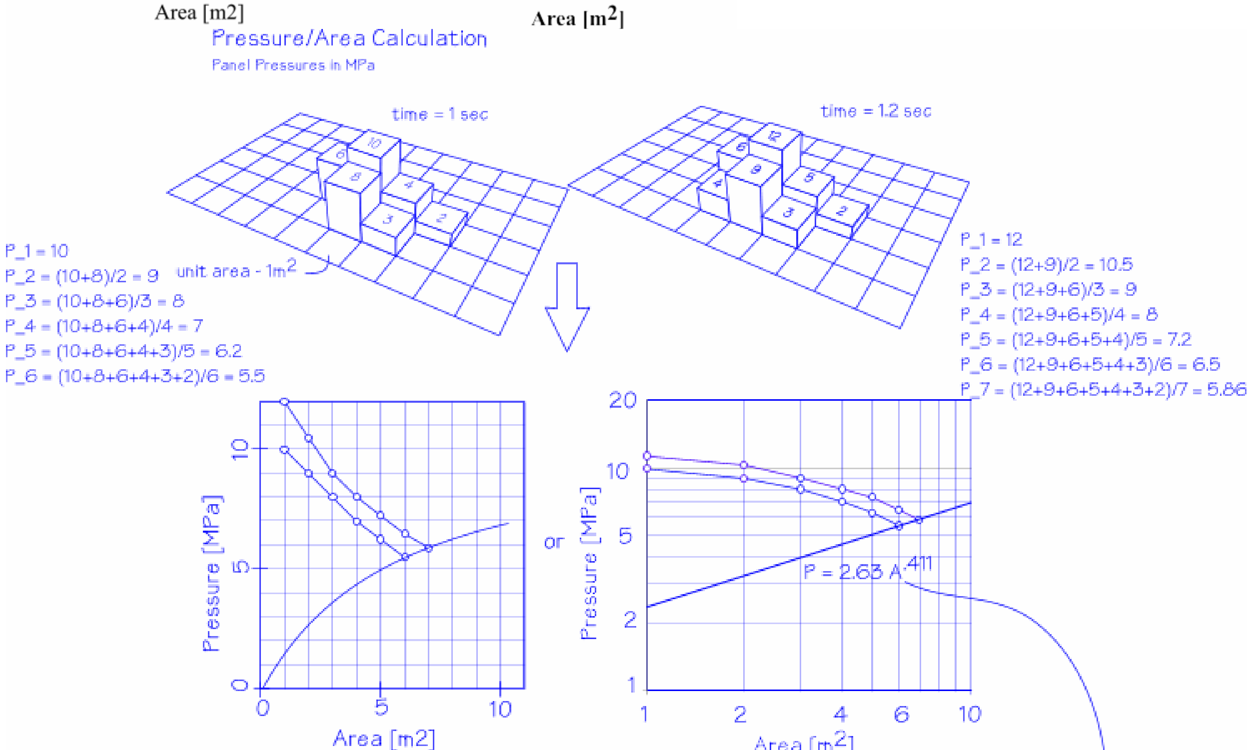
σ_c – uniaxial crushing strength say 5

$P_{ice} = 3 \times 1 \times 0.8 \times 5 = 12$ MPa

F = 2 x .066 x .25 x 12 = .396 MN = 396 kN <=ANS (very close!)

6. For the two measured sets of pressure given below, plot the spatial and process pressure-area curves. What is the equation for the process P/A curve? **[15]**





Process P/A $P = Po\, A^{ex}$ $\log P = \log Po + ex \log A$

$\log 5.5 = \log Po + ex \log 6$

$\log 5.86 = \log Po + ex \log 7$

solve 2 equations:

$ex = .411$

$Po = 2.63$

$P = 2.63\, A^{.411}$ <= ANS

7. For the ship shown below calculate the collision force on the bow, assuming Popov mechanics, and a pressure-area curve of $P=1\ A^{-1}$. **[20]**

Assume:

V = 3 m/s

L = 100m, B=20m, T=5m, Block Coefficient C_B = 0.8 (recall : mass=ρ L B T C_B)

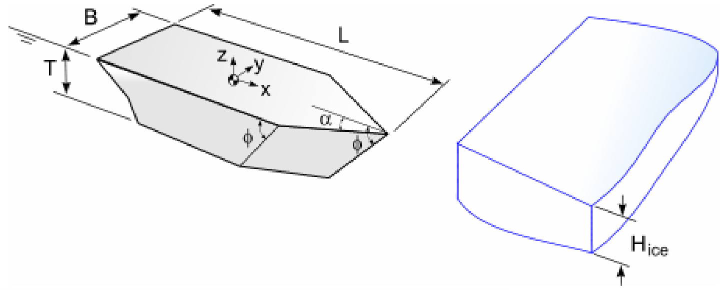
use ρ = 1000 kg/m3 (for simplicity)

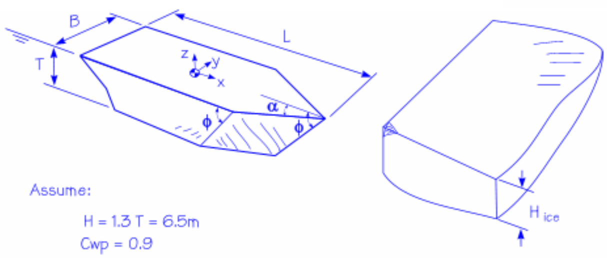
α = 25°

ϕ = 30°

impact coordinates: x = 45m, y= 5m, z=0m

for ice , assume wedge angle is 90° (very thick)





Assume:

H = 1.3 T = 6.5m

Cwp = 0.9

Cm = 0.9

$$F_n = \left[P_o\left(\frac{\tan\phi/2}{\sin\beta\ \cos^2\beta'}\right)^{(1+ex)}\right]^{\frac{1}{(3+2ex)}} \left[1/2\ Me\ Vn^2\ (3+2ex)\right]^{\frac{(2+2ex)}{(3+2ex)}}$$

Ship Main Parameters

			units
Ship Name	SN	M.V.Test	text
Length	Loa	100.0	m
Beam	B	20.00	m
Draft	T	5.00	m
Height	H	6.50	m
Block Coef.	CB	0.8	nd
Waterplane Coef	Cwp	0.9	nd
Midship Coefficient	Cm	0.9	nd
Mass	M	8200	tonnes
Initial Values			
Ship Speed	Vs	3.0	m/s

Hull Angles and coordinates

Alpha	α	25 deg	0.4363	rad
Beta	β	38.93 deg	0.6794	rad
Beta prime	β'	36.20 deg	0.6319	rad
gamma	γ	60.0 deg	1.0472	rad
x coordinate	x	45.0 m		
y coordinate	y	5.0 m		
z coordinate	z	0 m		

Ice Crushing Terms

Ice strength term	Po	1	Mpa
Ice exponent (process PA)	ex	0.1	
Wedge angle	ϕ	90 deg	1.57 rad
Ice Thickness	hi	10.00	m
Ice flex. Strength	sf	10.00	MPa
form factor 1	fx	3.20	
form factor 2	fa	2.86	
Flexural for limit	Ff	2031.62	MN
Flex limit on pen	ζnlim	19.77	m

Popov Terms

x dirn cosine	l	0.3410	nd
y dirn cosine	m	0.7313	nd
z dirn cosine	n	0.5907	nd
roll moment arm	λl	2.9533	m
pitch moment arm	μl	-26.5798	m
yaw moment arm	ηl	31.2042	m
Surge Added Mass	Amx	0.0000	nd
Sway Added Mass	Amy	0.5000	nd
Heave Added Mass	Amz	1.4211	nd
Roll Added Mass	Amrol	0.2500	nd
Pitch Added Mass	Ampit	1.5873	nd
Yaw Added Mass	Amyaw	0.5500	nd
roll gyrad(squared)	rx2	38.6086	m2
pitch gyrad(squared)	ry2	630.00	m2
yaw gyrad(squared)	rz2	625.00	m2
Mass Reduction Coef.	Co	2.2362	nd
Effective mass	Me	3666917	kg
Normal Speed	Ve	1.023	m/s
Kinetic Energy	KEe	1918979	kg-m2/s2
Impulse	Ie	3751463	kg-m/s

Results

pen(n)	ζn	1.270	m
Normal Force	Fn	4.84	MN

Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m^3
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·1O^{-6} / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i} \qquad (1)$$

Where Q is heat flux in W/m^2, k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i} \qquad (2)$$

Where t is time, and ρ_i is density of the ice.

the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$

Converted to freezing degree days:

$$h_i = .037\sqrt{FDD} \quad \text{[m]}$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025\sqrt{FDD} \quad \text{[m]}$$

$$\nu_b = .001 \cdot S\left(.53 - \frac{49.2}{T}\right)$$

where T is temperature (°C), S is salinity (ppt) and v_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65\left(1 - \sqrt{\frac{\nu_b}{275}}\right)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24\left(1 - \sqrt{\frac{\nu_b}{33}}\right)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82\left(1 - \sqrt{\frac{\nu_b}{142}}\right) \quad \text{case a) across grain}$$

$$\sigma_t = 1.54\left(1 - \sqrt{\frac{\nu_b}{311}}\right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2} \qquad (21)$$

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2} \qquad (22)$$

1.1 Beam on Elastic Foundation

$$EI\frac{d^4 v}{dx^4} + kv = 0 \qquad (23)$$

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, $k = \rho g w$. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k}\right)^{1/4} \qquad (24)$$

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection: $v = \frac{2P}{\lambda k} \cdot e^{-x/\lambda} \cdot \cos x/\lambda$ (6)

Slope: $\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$

Moment: $M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$

Shear: $Q = -P \cdot e^{-\beta x} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$ (7)

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4}\lambda \qquad (8)$$

$$\lambda = \left(\frac{E}{3\rho g}\right)^{1/4} h^{3/4} \qquad (29) \qquad (13)$$

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4} \qquad (30)$$

$$\ell = \sqrt[4]{D/k} \qquad (55)$$
$$\omega = w/\ell \qquad (56)$$
$$x = r/\ell \qquad (57)$$
$$p' = p/k \qquad (58) \qquad (15)$$

For a concentrated load the solution is;

$$w = \frac{p\ell^2}{2\pi D} kei(r/\ell) \qquad (16)$$

$$w_{\max} = \frac{p\ell^2}{8D} \qquad (61)$$

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}} \qquad (62) \qquad (17)$$

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log\left[\frac{E \cdot h^3}{k \cdot c^4}\right]} \sigma_{flex} \cdot h^2 \qquad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \qquad (70)$$

Where

I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors

m - shape factor (above)

k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling)

σ_c – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\overline{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \qquad (71)$$

<u>kinetic energy</u> (KE):

$$KE = \frac{1}{2} m \cdot v^2 \quad \text{(for head-on collisions)(72)}$$

$$KE = \frac{1}{2} m_e \cdot v_n^{\,2} \quad \text{(for oblique collisions)(73)}$$

m_e is the 'effective' mass

v_n is the 'normal' velocity

<u>crushing energy</u> (IE):

$$IE = \int_0^x F(x)dx \qquad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \qquad (75)$$

$$V = N_c(\cos\theta - \mu\sin\theta) \qquad (76)$$

$$H = N_c(\sin\theta + \mu\cos\theta) \qquad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6}t^2} \qquad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \qquad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left(60\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} - 1\right)} \qquad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{(60 \cdot (\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta))}$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \qquad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta)/60)}$$

The net forces applied to the ice are;

$$V = N_c(\cos\theta - \mu\sin\theta) - W\sin\theta \cdot (\sin\theta + \mu\cos\theta)$$

$$H = N_c(\sin\theta + \mu\cos\theta) + W\cos\theta \cdot (\sin\theta + \mu\cos\theta)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice:

$$V = \frac{\sigma_t t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (90)$$

With (87), the required normal force Nc is;

$$N_c = \frac{V + W\sin\theta \cdot (\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad (91)$$

$$P = C \cdot A^{-0.5}$$

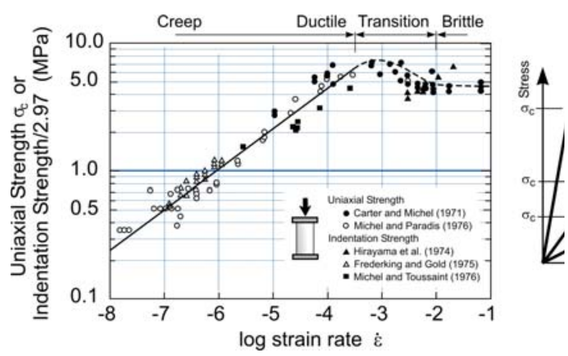


Figure 1. Influence of strain rate on uniaxial strength.

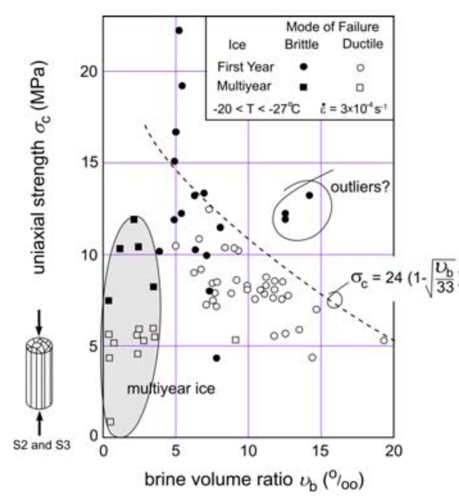


Figure 2. Compressive strength of Beaufort Sea ice.

The projected normal area is for spherical indentation;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \qquad (93)$$

The force is;:

$$F_n = p_o A_n^{ex} \cdot A_n \qquad (84)$$

$$= p_o (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \qquad (94)$$

The indentation energy is:

$$IE = \frac{p_o}{(2+ex)}(2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \qquad (95)$$

Which can be expressed in a more general form as : (86)

$$IE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \qquad (96)$$

$$fx = 2 + ex \qquad (97)$$

$$fa = (2\pi \cdot R)^{fx-1} \quad (98)$$

$$KE_e = \frac{p_o}{fx}\, fa \cdot \zeta_n^{fx} \quad (101)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa}\right)^{\frac{1}{fx}} \quad (102)$$

The normal force can be found by substituting eqn. (102) into (96), with (97) and (98) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa}\right)^{\frac{fx-1}{fx}} \quad (103)$$

Which in our spherical case becomes:

$$F_n = \left[p_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)}\right]^{\frac{1}{2+ex}} \cdot \left(\frac{1}{2} M_e \cdot V_n^2 (2+ex)\right)^{\frac{1+ex}{2+exx}}$$

Lindqvist's resistance equation

$$R_{ice} = (R_C + R_B)(1 + 1.4 \cdot v / \sqrt{g \cdot H_{ice}}) + R_S \cdot (1 + 9.4 \cdot v / \sqrt{g \cdot L}) \quad (105)$$

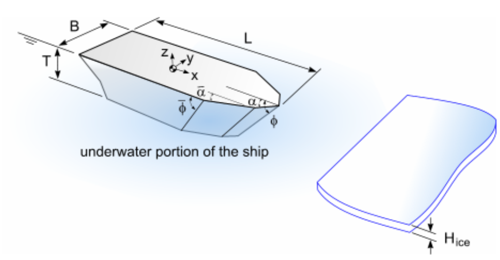


Figure 3. Linqvist's hull form with angles and dimensions.

1.1.1 Ice Crushing Term R_C

Linqvist starts by estimating the vertical force needed to fail an ice edge in flexure:

$$F_v = 0.5 \cdot \sigma_b \cdot H_{ice}^2 \quad (106)$$

Where σ_b is the flexural strength of ice. He then assumes that this force is generated (continuously) on each side of the stem and resolves the x (along ship) component, which is the resistance that the ship will feel;

$$R_C = F_v \frac{\tan\phi + \mu \cdot \cos\phi / \cos\psi}{1 - \mu \cdot \sin\phi / \cos\psi} \quad (107)$$

Where μ is the friction factor, ϕ is the stem angle, α is the waterline angle and;

$$\psi = \arctan(\tan\phi / \sin\alpha) \quad (108)$$

These equations are dimensionally consistent, so any units will work.

1.1.2 Ice Breaking Term R_B

The breaking resistance term (see appendix in Linqvist 1989 for derivation) is as follows;

$$R_B = k \cdot B \cdot \left(\frac{H_{ice}^3}{l_c^2}\right)(\tan\psi + \mu\cos\phi / (\sin\alpha \cdot \cos\psi))(1 + 1/\cos\psi) \quad (109)$$

Where k is a constant, B is the beam of the ship, the angles are as shown previously and l_c is the characteristic length of the ice sheet;

$$l_c = \sqrt[4]{\frac{E \cdot H_{ice}^3}{12\rho_w g(1-\upsilon^2)}} \quad (110)$$

Linqvist makes the following assumptions to simplify the equation. Poisons ratio is 0.3. The elastic modulus is 2 GPa. He assumes the cusp length is 1/3 of the characteristic length and the shear strength is equal to the flexural strength.

$$R_B = .003 \cdot \sigma_b \cdot B \cdot \left(\frac{H_{ice}^{1.5}}{\sqrt{m}}\right)(\tan\psi + \mu\cos\phi / (\sin\alpha \cdot \cos\psi))(1 + 1/\cos\psi)$$

1.1.3 Ice Submergence Term R_S

The submergence resistance term (see appendix in Linqvist 1989 for derivation) is derived from energy consideration and is as follows;

$$R_S = \delta\rho \cdot g \cdot H_{ice} \cdot \left(B \cdot T \cdot (B+T)/(B+2T) + \mu(A_u + \cos\phi \cdot \cos\psi \cdot A_f)\right)$$

Where $\delta\rho$ is the density difference between ice and water (the cause of buoyancy) A_u is the area of the flat of bottom and A_f is the area of the bow. With approximations for the areas involved, Linqvist proposed the alternate formula;

$$R_S = \delta\rho \cdot g \cdot H_{ice} \cdot B \cdot \left(T \frac{B+T}{B+2T} + \mu\left(.7 \cdot L - \frac{T}{\tan\phi} - \frac{0.25 \cdot B}{\tan\alpha} + T \cdot \cos\phi \cdot \cos\psi \cdot \sqrt{\frac{1}{\sin^2\phi} + \frac{1}{\tan^2\alpha}}\right)\right) \quad (114)$$

POPOV (104)

$$M_e = \frac{F_n}{a_n} \quad (115)$$

The effective mass is;

$$M_e = \frac{M_{ship}}{C_O} \quad (116)$$

where *Co* is the mass reduction coefficient.

The inertial properties of the vessel are as follows,

Hull angles at point:
α = waterline angle
β = frame angle
β' = normal frame angle
γ = sheer angle

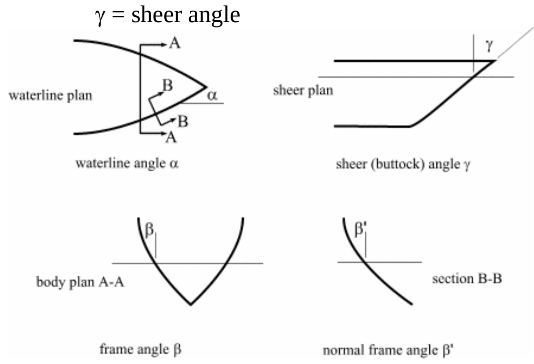


Figure 4. Hull angle definitions.

The various angles are related as follows,

$$\tan\beta = \tan\alpha \cdot \tan\gamma \quad (117)$$

$$\tan\beta' = \tan\beta \cdot \cos\alpha \quad (118)$$

Based on these angles, the direction cosines, *l,m,n* are

$$l = \sin\alpha \cdot \cos\beta' \quad (119)$$

$$m = \cos\alpha \cdot \cos\beta' \quad (120)$$

$$n = \sin\beta' \quad (121)$$

and the moment arms are

$$\lambda 1 = n \cdot y_p - m \cdot z_p \quad \textit{(roll moment arm)} \quad (122)$$

$$\mu 1 = l \cdot z_p - n \cdot x_p \quad \textit{(pitch moment arm)} \quad (123)$$

$$\eta 1 = m \cdot x_p - l \cdot y_p \quad \textit{(yaw moment arm)} \quad (124)$$

The added mass terms are as follows,

AMx = added mass factor in surge = 0 (125)
AMy = added mass factor in sway = 2 T/B (126)
AMz = added mass factor in heave = 2/3 (B Cwp^2)/(T(Cb(1+Cwp)) (127)
AMrol = added mass factor in roll = 0.25 (128)
AMpit = added mass factor in pitch
= B/((T(3-2Cwp)(3-Cwp)) (129)
AMyaw
= added mass factor in yaw = 0.3 + 0.05 L/B (130)

The mass radii of gyration (squared) are;

rx^2 = Cwp B^2/(11.4 Cm) + H^2/12 (roll) (131)
ry^2 = 0.07 Cwp L^2 (pitch) (132)
rz^2 = L^2/16 (yaw) (133)

With the above quantities defined, the mass reduction coefficient is

Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz)
+ $\lambda 1^2$/(rx^2(1+AMrol) + $\mu 1^2$/(ry^2 (1+AMpit)) + $\eta 1^2$/(rz^2 (1+AMyaw)) (134)

1.1.4 Contact with General Wedge (Normal to hull)

a general wedge-shaped edge indentation (normal to hull). The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta`)} \quad (135)$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos(\beta`)} \quad (136)$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \quad (137)$$

The pressure-area approach is used again. Substituting (137) into (75) we arrive at:

$$F_n = p_o A_n^{ex} \cdot A_n$$
$$= p_o \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{2+2ex} \quad (138)$$

Note: this can be expressed in a general form as:

$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \quad (139)$$

Where $fx = 3 + 2ex$ (140)

$$fa = \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \quad (141)$$

The indentation energy is found by substituting (140) and (141) into (96), to give:

$$IE = \frac{p_o}{(3+2ex)} \left(\frac{\tan(\phi/2)}{\sin(\beta`)\cos^2(\beta`)} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \quad (142)$$

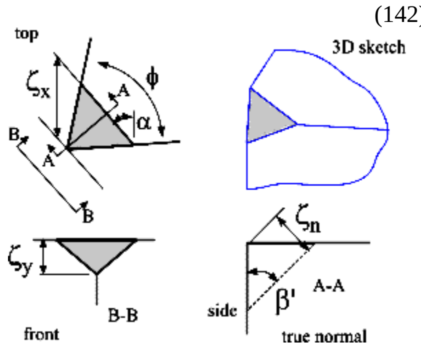


Figure 5. General Wedge-shaped Edge (normal to hull).

1.1.5 Popov Force Calculation

The Popov collision scenarios can be analyzed by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \quad (143)$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \quad (144)$$

which ,using equation (96) can be stated as;

$$KE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \quad (145)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \quad (146)$$

The normal force can be found by substituting eqn. (146) into (139) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \quad (147)$$

$$V_n = V_{ship} \cdot lx \quad (149)$$

where V_n is the normal velocity at the point of impact
V_{ship} is the x-direction velocity (all others are zero)
lx is the x-direction cosine

By combining the above equations, a single closed-form expression for the collision force is found;

$$F_n := \left[p_o \cdot \left[\frac{\tan\left(\frac{1}{2} \cdot \phi\right)}{\left(\sin(\beta') \cdot \cos(\beta')^2\right)} \right]^{(1+ex)} \right]^{\frac{1}{3+2 \cdot ex}} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3+2 \cdot ex) \right]^{\left[\frac{(2+2 \cdot ex)}{(3+2 \cdot ex)}\right]} \quad (151)$$