



Faculty of Engineering
and Applied Science

Engineering 8674 - Design for Ocean and Ice Environments
Engineering 9096 – Sea Ice Engineering

F I N A L E X A M I N A T I O N

Date: Sat., Apr. 12, 2008
Time: 9:00 - 11:30 am

Professor: Dr. C. Daley

Answer all questions. Total 100 marks. Each question is worth marks indicated [x].

Watch your time. 150min/100 marks = 90 sec/mark!

Short, clear answers are best. If you are having a problem (ie a road block) assume something,
write down the assumption, and continue. Use the back of the sheets as needed.

Good luck.

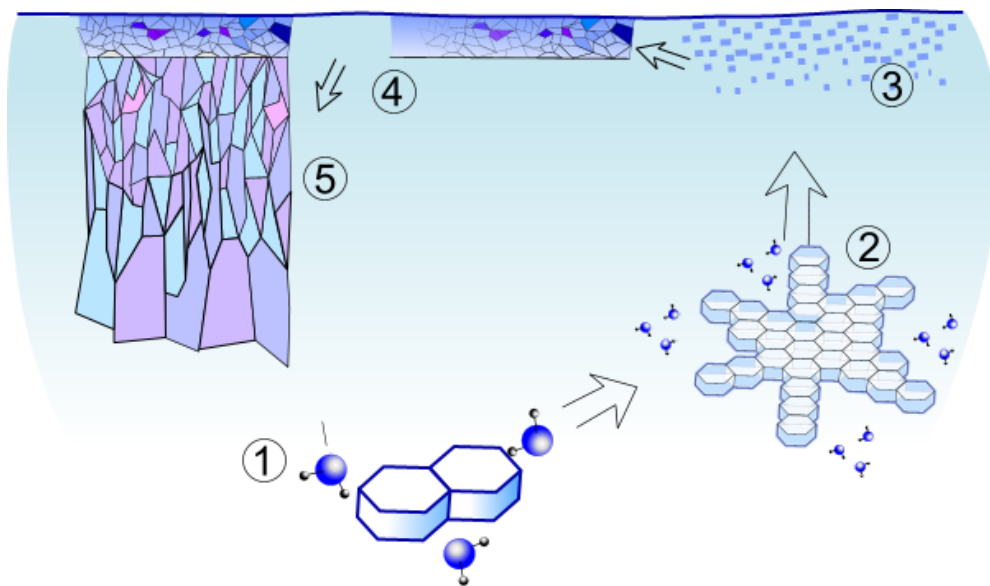
NAME: _____

NUMBER: _____

1. There are 5 questions worth 1 mark each. Fill in the answers (or complete the phrase) in the spaces provided. [5]

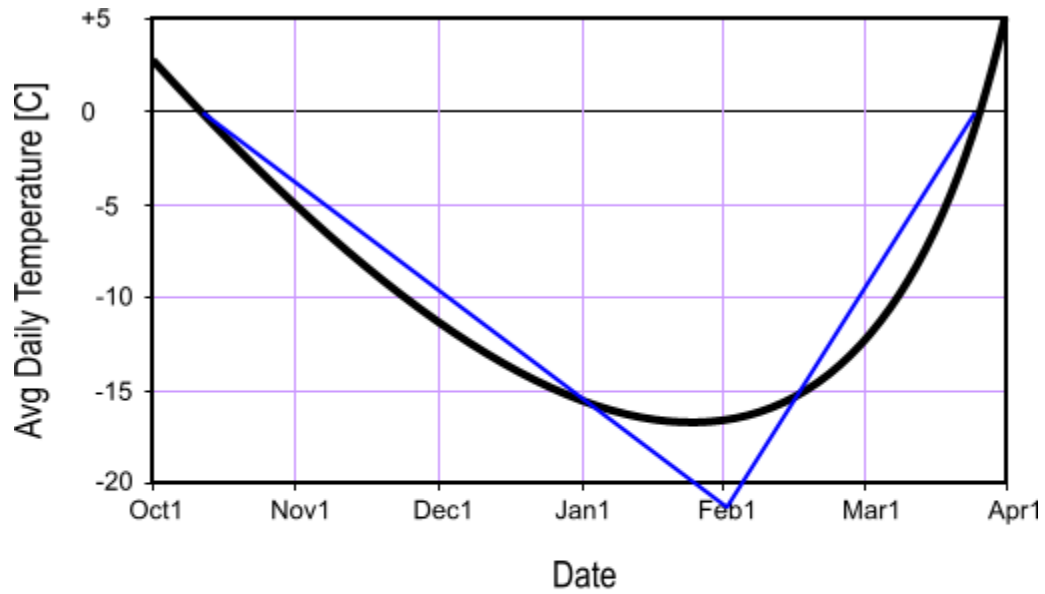
1. No models have ever really explained the mess that is ice crushing.
2. There are two types of Nilas. Which is the thicker type? light nilas.
3. Consolidation occurs in which ice feature? pressure ridges
4. The surface of ice doesn't melt evenly on a warm sunny day because the melt pools attract heat and melt more, while the unmelted ice reflects heat and stays cold
5. What ice strength tests can be used to estimate the tensile strength ice ice? direct tension (hourglass), ring tension tests and flexural tests

2. Explain what is occurring (#s 1 to 5) in the figure below: [15]



- ① water molecules form ice particles
- ② larger ice flakes form
- ③ frazil forms (a slurry of ice flakes)
- ④ primary ice forms as water calms and slurry floats to a cold surface (P3b ice - fine grain random c-axis ice)
- ⑤ ice grows from the bottom of the primary ice forming secondary ice (S2b ice- large columnar grains with horizontal c-axis)

3. Using the temperature plot below, find the estimated maximum ice thickness for the sheltered ocean bay where this record was taken? Show your calculations and explain your assumptions. **[10]**



From Oct 10 to March 23 (est) is 164 days.

Avg temp. = -11°C

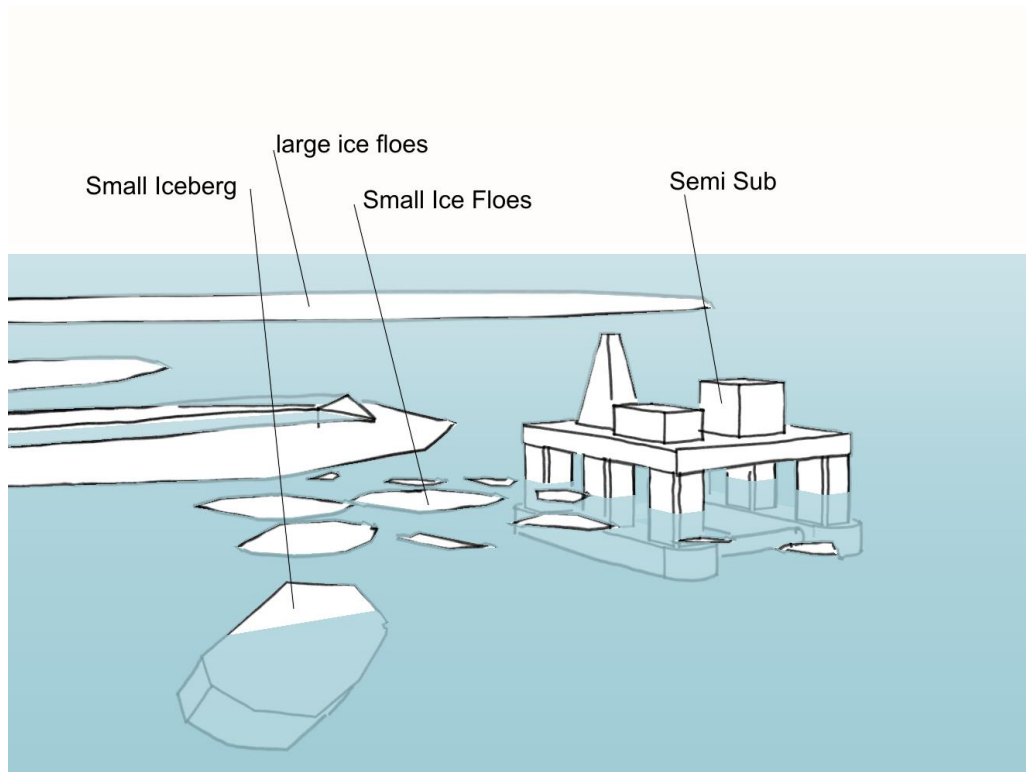
total FDD = 1804 deg-days

Using $h_{ice} = .025 \sqrt{FDD} = 1.06\text{m}$

4. Sea ice has a salinity of 4 ppt, and is at -8°C.
 What is the brine volume ratio?
 What is the uniaxial crushing strength likely to be?

[10]

5. You want to design an ice-capable semi-submersible rig for the Grand Banks. . [30]
The situation is sketched below.
How would you approach this design problem? What features would you change or add on the semi?
How would you determine the loads?



6. For the ship described below calculate the level ice resistance force (Lindqvist). **[30]**

Assume:

$$V = 3 \text{ m/s}$$

$$L = 100\text{m}, B=20\text{m}, T=5\text{m}, \text{Block Coefficient } C_B = 0.8 \text{ (recall : mass}=\rho L B T C_B)$$

$$\text{use } \rho = 1025 \text{ kg/m}^3$$

$$\alpha = 25^\circ$$

$$\phi = 30^\circ$$

$$H_{ice} = 1\text{m}$$

Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m ³
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·10 ⁻⁶ / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i}$$

(1)

Where Q is heat flux in W/m², k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i}$$

(2)

Where t is time, and ρ_i is density of the ice.

the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$

Converted to freezing degree days:

$$h_i = .037 \sqrt{FDD} \quad [\text{m}]$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025 \sqrt{FDD} \quad [\text{m}]$$

$$v_b = .001 \cdot S \left(.53 - \frac{49.2}{T} \right)$$

where T is temperature (°C), S is salinity (ppt) and v_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65 \left(1 - \sqrt{\frac{v_b}{275}} \right)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24 \left(1 - \sqrt{\frac{v_b}{33}} \right)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82 \left(1 - \sqrt{\frac{v_b}{142}} \right) \quad \text{case a) across grain}$$

$$\sigma_t = 1.54 \left(1 - \sqrt{\frac{v_b}{311}} \right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2}$$

(21)

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2}$$

(22)

1.1 Beam on Elastic Foundation

$$EI \frac{d^4 v}{dx^4} + kv = 0$$

(23)

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, k = ρgw. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k} \right)^{1/4}$$

(24)

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection:
$$v = \frac{2P}{\lambda k} e^{-x/\lambda} \cdot \cos x/\lambda$$

Slope:
$$\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$$

Moment:
$$M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$$

Shear:
$$Q = -P \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$$

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4} \lambda$$

(8)

$$\lambda = \left(\frac{E}{3\rho g} \right)^{1/4} h^{3/4}$$

(29) (13)

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4}$$

(30)

$$\ell = \sqrt[4]{D/k}$$

(55)

$$\omega = w/\ell$$

(56)

$$x = r/\ell$$

(57)

$$p' = p/k$$

(58) (15)

For a concentrated load the solution is;

$$w = \frac{p \ell^2}{2\pi D} kei(r/\ell)$$

(16)

$$w_{\max} = \frac{p \ell^2}{8D}$$

(61)

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}}$$

(62) (17)

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log \left[\frac{E \cdot h^3}{k \cdot c^4} \right]} \sigma_{flex} \cdot h^2 \quad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \quad (70)$$

Where

I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors

m - shape factor (above)

k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling)

σ_c – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\bar{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \quad (71)$$

kinetic energy (KE):

$$KE = \frac{1}{2} m \cdot v^2 \quad (\text{for head-on collisions}) \quad (72)$$

$$KE = \frac{1}{2} m_e \cdot v_n^2 \quad (\text{for oblique collisions}) \quad (73)$$

m_e is the 'effective' mass

v_n is the 'normal' velocity

crushing energy (IE):

$$IE = \int_0^x F(x) dx \quad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \quad (75)$$

$$V = N_c (\cos \theta - \mu \sin \theta) \quad (76)$$

$$H = N_c (\sin \theta + \mu \cos \theta) \quad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6} t^2} \quad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \quad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t} \left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left(60 \frac{\cos \theta - \mu \sin \theta}{\sin \theta + \mu \cos \theta} - 1 \right)} \quad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{(60 \cdot (\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta))}$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \quad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta) / 60)}$$

The net forces applied to the ice are;

$$V = N_c (\cos \theta - \mu \sin \theta) - W \sin \theta \cdot (\sin \theta + \mu \cos \theta)$$

$$H = N_c (\sin \theta + \mu \cos \theta) + W \cos \theta \cdot (\sin \theta + \mu \cos \theta)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t} \left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice;

$$V = \frac{\sigma_t t}{\left(60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (90)$$

With (87), the required normal force N_c is;

$$N_c = \frac{V + W \sin \theta \cdot (\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)} \quad (91)$$

$$P = C \cdot A^{-0.5}$$

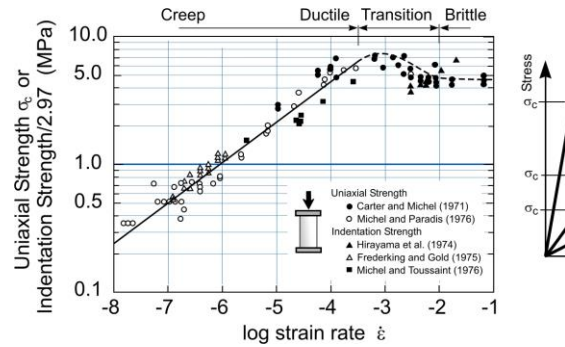


Figure 1. Influence of strain rate on uniaxial strength.

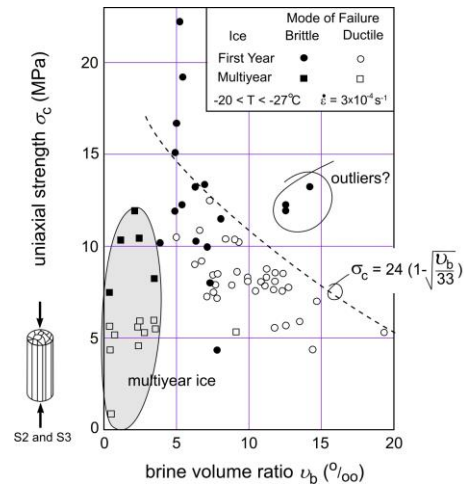


Figure 2. Compressive strength of Beaufort Sea ice.

The projected normal area is for spherical indentation;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \quad (93)$$

The force is;;

$$F_n = p_o A_n^{ex} \cdot A_n \quad (84)$$

$$= p_o (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \quad (94)$$

The indentation energy is:

$$IE = \frac{P_o}{(2+ex)} (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \quad (95)$$

Which can be expressed in a more general form as :

$$IE_e = \frac{P_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (96)$$

$$f_x = 2 + ex \quad (97)$$

$$f_a = (2\pi \cdot R)^{f_x-1} \quad (98)$$

$$KE_e = \frac{P_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (101)$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot f_x}{p_o \cdot f_a} \right)^{\frac{1}{f_x}} \quad (102)$$

The normal force can be found by substituting eqn. (102) into (96), with (97) and (98) to give

$$F_n = p_o \cdot f_a \cdot \left(\frac{KE_e \cdot f_x}{p_o \cdot f_a} \right)^{\frac{f_x-1}{f_x}} \quad (103)$$

Which in our spherical case becomes:

$$F_n = \left[p_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)} \right]^{\frac{1}{2+ex}} \cdot \left(\frac{1}{2} M_e \cdot V_n^2 (2+ex) \right)^{\frac{1+ex}{2+ex}}$$

Lindqvist's resistance equation

$$R_{ice} = (R_C + R_B)(1 + 1.4 \cdot v / \sqrt{g \cdot H_{ice}}) + R_S \cdot (1 + 9.4 \cdot v / \sqrt{g \cdot L}) \quad (105)$$

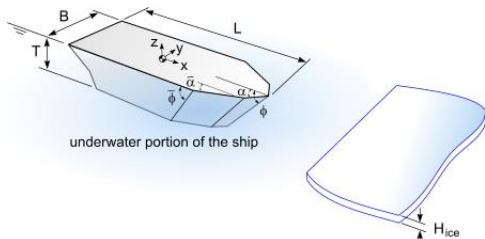


Figure 3. Lindqvist's hull form with angles and dimensions.

1.1.1 Ice Crushing Term R_C

Lindqvist starts by estimating the vertical force needed to fail an ice edge in flexure:

$$F_v = 0.5 \cdot \sigma_b \cdot H_{ice}^2 \quad (106)$$

Where σ_b is the flexural strength of ice. He then assumes that this force is generated (continuously) on each side of the stem and resolves the x (along ship) component, which is the resistance that the ship will feel;

$$R_C = F_v \frac{\tan \phi + \mu \cdot \cos \phi / \cos \psi}{1 - \mu \cdot \sin \phi / \cos \psi} \quad (107)$$

Where μ is the friction factor, ϕ is the stem angle, α is the waterline angle and;

$$\psi = \arctan(\tan \phi / \sin \alpha) \quad (108)$$

These equations are dimensionally consistent, so any units will work.

1.1.2 Ice Breaking Term R_B

The breaking resistance term (see appendix in Lindqvist 1989 for derivation) is as follows;

$$R_B = k \cdot B \cdot \left(\frac{H_{ice}^3}{l_c^2} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi) \quad (109)$$

Where k is a constant, B is the beam of the ship, the angles are as shown previously and l_c is the characteristic length of the ice sheet;

$$l_c = 4 \sqrt{\frac{E \cdot H_{ice}^3}{12 \rho_w g (1 - \nu^2)}} \quad (110)$$

Lindqvist makes the following assumptions to simplify the equation.

Poisons ratio is 0.3. The elastic modulus is 2 GPa. He assumes the cusp length is 1/3 of the characteristic length and the shear strength is equal to the flexural strength.

$$R_B = .003 \cdot \sigma_b \cdot B \cdot \left(\frac{H_{ice}^{1.5}}{\sqrt{m}} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi)$$

1.1.3 Ice Submergence Term R_S

The submergence resistance term (see appendix in Lindqvist 1989 for derivation) is derived from energy consideration and is as follows;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot (B \cdot T \cdot (B + T) / (B + 2T) + \mu (A_u + \cos \phi \cdot \cos \psi \cdot A_f))$$

Where $\delta \rho$ is the density difference between ice and water (the cause of buoyancy) A_u is the area of the flat of bottom and A_f is the area of the bow. With approximations for the areas involved, Lindqvist proposed the alternate formula;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot B \cdot \left(T \frac{B+T}{B+2T} + \mu \left(.7 \cdot L - \frac{T}{\tan \phi} - \frac{0.25 \cdot B}{\tan \alpha} + T \cdot \cos \phi \cdot \cos \psi \cdot \sqrt{\frac{1}{\sin^2 \phi} + \frac{1}{\tan^2 \alpha}} \right) \right) \quad (114)$$

POPOV

$$M_e = \frac{F_n}{a_n} \quad (115)$$

The effective mass is;

$$M_e = \frac{M_{ship}}{C_o} \quad (116)$$

where C_o is the mass reduction coefficient.

The inertial properties of the vessel are as follows,

Hull angles at point:

α = waterline angle

β = frame angle

β' = normal frame angle

γ = sheer angle

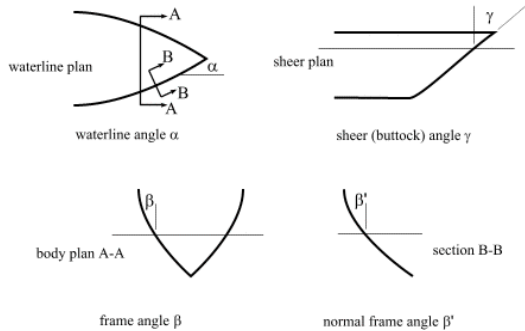


Figure 4. Hull angle definitions.

The various angles are related as follows,

$$\tan \beta = \tan \alpha \cdot \tan \gamma \quad (117)$$

$$\tan \beta' = \tan \beta \cdot \cos \alpha \quad (118)$$

Based on these angles, the direction cosines, l, m, n are

$$l = \sin \alpha \cdot \cos \beta' \quad (119)$$

$$m = \cos \alpha \cdot \cos \beta' \quad (120)$$

$$n = \sin \beta' \quad (121)$$

and the moment arms are

$$\lambda_1 = n \cdot y_p - m \cdot z_p \quad (\text{roll moment arm}) \quad (122)$$

$$\mu_1 = l \cdot z_p - n \cdot x_p \quad (\text{pitch moment arm}) \quad (123)$$

$$\eta_1 = m \cdot x_p - l \cdot y_p \quad (\text{yaw moment arm}) \quad (124)$$

The added mass terms are as follows;

$$AM_x = \text{added mass factor in surge} = 0 \quad (125)$$

$$AM_y = \text{added mass factor in sway} = 2 T/B \quad (126)$$

$$AM_z = \text{added mass factor in heave} = 2/3 (B Cwp^2) / (T(Cb(1+Cwp))) \quad (127)$$

$$AM_{rol} = \text{added mass factor in roll} = 0.25 \quad (128)$$

$$AM_{pit} = \text{added mass factor in pitch} = B / (T(3-2Cwp)(3-Cwp)) \quad (129)$$

$$AM_{yaw} = \text{added mass factor in yaw} = 0.3 + 0.05 L/B \quad (130)$$

The mass radii of gyration (squared) are;

$$rx^2 = Cwp B^2 / (11.4 Cm) + H^2 / 12 \quad (\text{roll}) \quad (131)$$

$$ry^2 = 0.07 Cwp L^2 \quad (\text{pitch}) \quad (132)$$

$$rz^2 = L^2 / 16 \quad (\text{yaw}) \quad (133)$$

With the above quantities defined, the mass reduction coefficient is

$$Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz) + \lambda l^2/(rx^2(1+AMrol)) + \mu l^2/(ry^2(1+AMpit)) + \eta l^2/(rz^2(1+AMyaw)) \tag{134}$$

1.1.4 Contact with General Wedge (Normal to hull)

a general wedge-shaped edge indentation (normal to hull).
The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta')} \tag{135}$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta')\cos(\beta')} \tag{136}$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \tag{137}$$

The pressure-area approach is used again. Substituting (137) into (75) we arrive at:

$$F_n = p_o A_n^{ex} \cdot A_n = p_o \left(\frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{138}$$

Note: this can be expressed in a general form as:

$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \tag{139}$$

Where $fx = 3 + 2ex$ (140)

$$fa = \left(\frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \tag{141}$$

The indentation energy is found by substituting (140) and (141) into (96), to give:

$$IE = \frac{p_o}{(3 + 2ex)} \left(\frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{142}$$

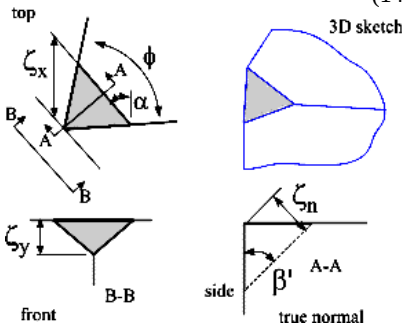


Figure 5. General Wedge-shaped Edge (normal to hull).

1.1.5 Popov Force Calculation

The Popov collision scenarios can be analyzed by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \tag{143}$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \tag{144}$$

which ,using equation (96) can be stated as;

$$KE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \tag{145}$$

Solving for the normal indentation:

$$\zeta_n = \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \tag{146}$$

The normal force can be found by substituting eqn. (146) into (139) to give

$$F_n = p_o \cdot fa \cdot \left(\frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \tag{147}$$

$$V_n = V_{ship} \cdot lx \tag{149}$$

where V_n is the normal velocity at the point of impact
 V_{ship} is the x-direction velocity (all others are zero)
 lx is the x-direction cosine

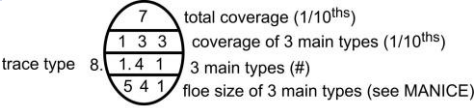
By combining the above equations, a single closed-form expression for the collision force is found;

$$F_n := p_o \cdot \left[\frac{\tan\left(\frac{1}{2}\phi\right)}{\left(\sin(\beta') \cdot \cos(\beta')^2\right)} \right]^{(1+ex)} \cdot \left[\frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3 + 2 \cdot ex) \right]^{\frac{1}{3+2 \cdot ex}} \cdot \left[\frac{(2 + 2 \cdot ex)}{(3 + 2 \cdot ex)} \right] \tag{151}$$

Color	Ice Type	Thickness	Egg Code #
	Ice Free		
	< 1/10 unspecified ice		
	New Ice	< 10 cm	1,2
	Grey Ice	10 - 15 cm	4
	Grey-White Ice	15 - 30 cm	5
	Thin FY	First-year ice (FY)	7
	Medium FY	30 - 70 cm	8
	Thick FY	70 - 120 cm	9
	Second-year Ice	>30 cm	1.
	Multi-year Ice	Old Ice	4.
			8.
			9.

Floe Size/Grandeur des floes (F_aF_bF_c)

Description/Élément	Width/Extension	Code
Pancake ice/Glace en crêpes		0
Small ice cake, brash ice/Petit glaçons, sarrazins	<2 m	1
Ice cake/Glaçons	2-20 m	2
Small floe/Petits floes	20-100 m	3
Medium floe/Floes moyens	100-500 m	4
Big floe/Grands floes	500-2000 m	5
Vast floe/Floes immenses	2-10 km	6
Giant floe/Floes géants	>10 km	7



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.