



Faculty of Engineering  
and Applied Science

**Engineering 8674 - Design for Ocean and Ice Environments**  
**Engineering 9096 – Sea Ice Engineering**

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**FINAL EXAMINATION**

**Date: Thur., Apr. 16, 2009**

**Professor: Dr. C. Daley**

**Time: 2:00 - 4:30 pm**

Answer all questions. Total 100 marks. Each question is worth marks indicated [x].

Watch your time. 150min/100 marks = 90 sec/mark!

Short, clear answers are best. If you are having a problem (ie a road block) assume something, write down the assumption, and continue. Use the back of the sheets as needed.

Good luck.

NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

1. There are 10 questions worth 1 mark each. Fill in the answers (or complete the phrase) in the spaces provided.

[10]

1. The Kara Sea is located; north of Russia.
2. A cantilever beam test measures; flexural strength.
3. In addition to ice science (glaciology) and Newtonian and continuum mechanics, the field of ice engineering is supported by (based upon); field and lab observations.
4. The place name “Lachine” used by the explorer Cartier refers to China.
5. Which of the following is an area of ice engineering interest:  
Beaufort Peninsula, Prudhoe Sea, Melville Sea, Bohai Strait, Caspian Sea: Caspian Sea.
6. The chemical formula for ice is: H<sub>2</sub>O.
7. The density of pure water is about: 1000 kg/m<sup>3</sup>.
8. The energy needed to change the phase of ice from solid to liquid is called latent heat of fusion.
9. The initial ice crystals formed from ice slurry are designated with the letter: P.
10. During secondary creep deformation the ice stress is: constant.

2. Make a sketch of a map of the arctic ocean, naming some of its seas and neighboring lands [15]  
A sketch with some of the features of:



**3. Ice Growth**

[15]

Consider the following scenario. Late in the evening on Jan.11<sup>th</sup> a man was walking home and crossed a pond. He fell through the ice and drowned. A search was started on the 12<sup>th</sup> but he wasn't discovered until January 18<sup>th</sup>, when the typical ice thickness in the pond was 25cm. The weather records for mean daily temperature are tabulated below.

From this data, estimate the ice thickness when the man fell through the ice.

| Date   | Mean Daily Temperature C |
|--------|--------------------------|
| Jan 12 | -8                       |
| Jan 13 | -10                      |
| Jan 14 | -12                      |
| Jan 15 | -9                       |
| Jan 16 | -18                      |
| Jan 17 | -22                      |
| Jan 18 | -12                      |

Using  $h = .025\sqrt{\text{FDD}}$  the total FDD to get 25cm (.25m) is 100.

The sum of the FDD from the 12th to the 18th is 91 FDD, so that there were 9 FDD before the 12th.

9 FDD will produce an ice thickness of;

$$h = .025 \sqrt{9} = .075 \text{ m.}$$

The ice was approximately 7.5cm (2.9in.) when the man fell through the ice.

&lt;=ANS

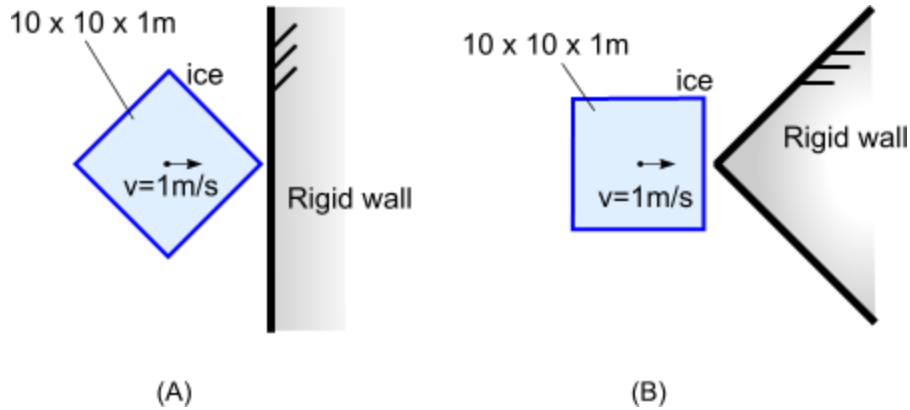
4. Sea ice has a salinity of 3 ppt, and is at -10 C.

What is the brine volume ratio?

What is the uniaxial crushing strength likely to be?

**[10]**

5. Consider the two cases of ice-structure impact (A and B). The geometry of the two cases is very similar. In both cases an ice indentation of 1m would result in a nominal contact shape that would be 2m wide and 1 m high. 10, 10 and 5 marks each part, **[25 total]**
- a) Explain how you would solve these scenarios to get the max. collision force.
- b) Explain why the two scenarios would give the same result (e.g. what assumptions in the solution would cause the results to be the same?).
- c) Discuss why the real scenarios might have different results.

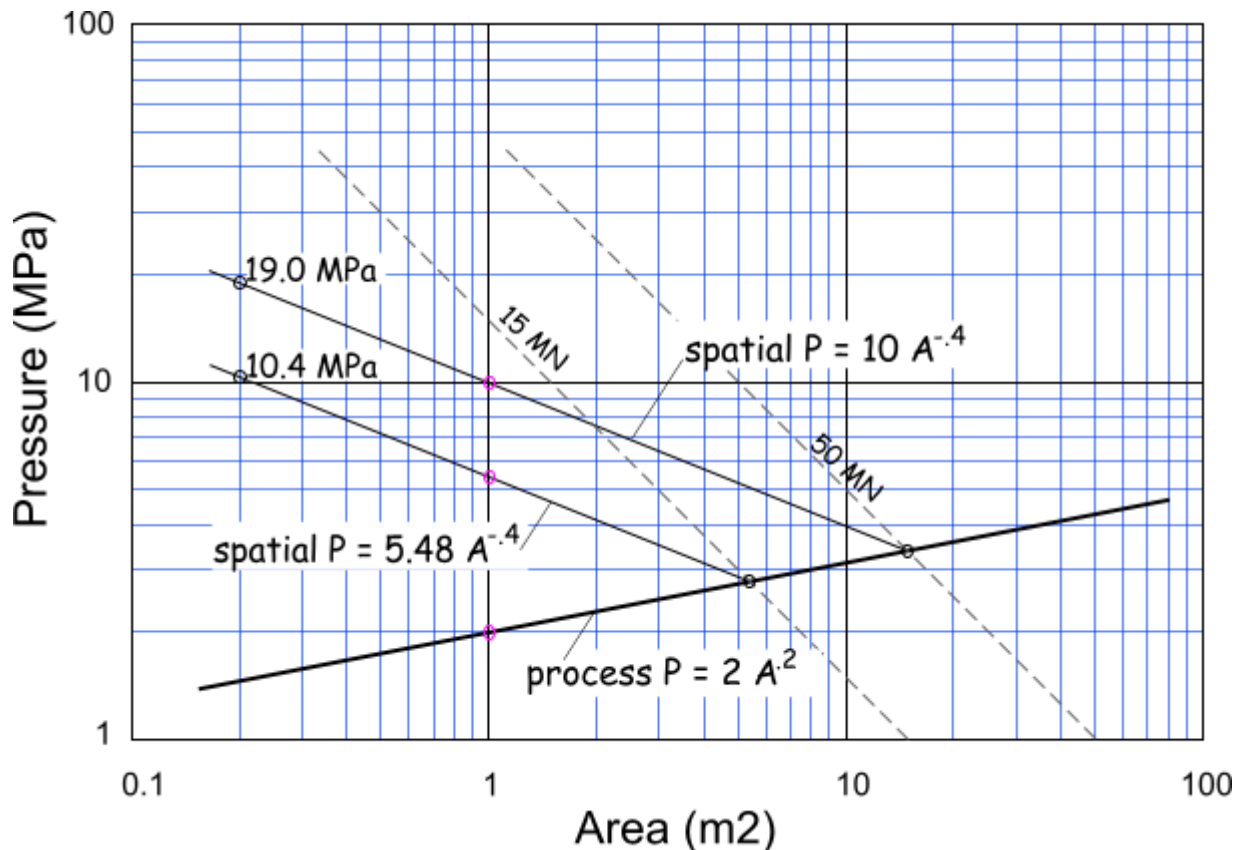


## 6. Pressure area Curves.

5 marks per part [25 total]

Consider a design problem involving icebergs and rigid offshore structures. Assuming that the collision process can be modeled with a “process” pressure-area curve of the form  $P_p = 2 A^{-2}$ , and that the local pressures are modeled with a “spatial” pressure-area curve of the form  $P_s = C A^{-4}$ . Assume that the two curves are connected, meaning that the (right hand) ends of both curves are at the same spot.

- Plot the process pressure-area curve,
- Plot the point (pressure and area) on the process pressure-area curve that corresponds to a force of 15MN **at  $P=2.8 \text{ MPa}$  ,  $A= 5.36 \text{ m}^2$**
- Find the value of C for a spatial pressure-area curve so that the right hand end of the spatial curve meets the process curve at the 15MN spot.  **$C = 5.48$**
- What is the predicted local pressure (from the spatial pressure-area curve) at an area of  $0.2 \text{ m}^2$ ?  **$10.4 \text{ MPa}$**
- What would the local pressure (as found in ‘d’ above) become if the force in ‘c’ above were to be 50MN?  **$19 \text{ MPa}$**



Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

|                                 |                          |
|---------------------------------|--------------------------|
| Density                         | 917 kg /m <sup>3</sup>   |
| Melting point                   | 0°C                      |
| Specific heat (heat capacity)   | 2.01 kJ / kg /°C         |
| Latent heat of fusion (or melt) | 334 kJ / kg              |
| Thermal conductivity            | 2.2 W / m / °C           |
| Linear expansion coefficient    | 55·10 <sup>-6</sup> / °C |
| Vapor pressure                  | 610.7 Pa                 |
| Refractive index                | 1.31                     |
| Acoustic velocity               |                          |
| longitudinal wave               | 1928 m/s                 |
| transverse wave                 | 1951 m/s                 |

$$Q = \frac{k_i \cdot \Delta T}{h_i}$$

(1)

Where Q is heat flux in W/m², k<sub>i</sub> is thermal conductivity in W / m / °C, h<sub>i</sub> is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i}$$

(2)

Where t is time, and ρ<sub>i</sub> is density of the ice.

the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$

Converted to freezing degree days:

$$h_i = .037 \sqrt{FDD} \quad [\text{m}]$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025 \sqrt{FDD} \quad [\text{m}]$$

$$v_b = .001 \cdot S \left( .53 - \frac{49.2}{T} \right)$$

where T is temperature (°C ), S is salinity (ppt) and v<sub>b</sub> is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65 \left( 1 - \sqrt{\frac{v_b}{275}} \right)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24 \left( 1 - \sqrt{\frac{v_b}{33}} \right)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82 \left( 1 - \sqrt{\frac{v_b}{142}} \right) \quad \text{case a) across grain}$$

$$\sigma_t = 1.54 \left( 1 - \sqrt{\frac{v_b}{311}} \right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2}$$

(21)

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2}$$

(22)

1.1 Beam on Elastic Foundation

$$EI \frac{d^4 v}{dx^4} + kv = 0$$

(23)

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, k = ρgw. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left( \frac{4EI}{k} \right)^{1/4}$$

(24)

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection:

$$v = \frac{2P}{\lambda k} e^{-x/\lambda} \cdot \cos x/\lambda$$

Slope:

$$\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$$

Moment:

$$M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$$

Shear:

$$Q = -P \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$$

The maximum bending moment occurs at a distance x<sub>c</sub> from the free end:

$$x_c = \frac{\pi}{4} \lambda$$

(8)

$$\lambda = \left( \frac{E}{3\rho g} \right)^{1/4} h^{3/4}$$

(29) (13)

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4}$$

(30)

$$\ell = \sqrt[4]{D/k}$$

(55)

$$w = w/\ell$$

(56)

$$x = r/\ell$$

(57)

$$p' = p/k$$

(58) (15)

For a concentrated load the solution is;

$$w = \frac{p \ell^2}{2\pi D} kei(r/\ell)$$

(16)

$$w_{\max} = \frac{p \ell^2}{8D}$$

(61)

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}}$$

(62) (17)

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log \left[ \frac{E \cdot h^3}{k \cdot c^4} \right]} \sigma_{flex} \cdot h^2 \quad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \quad (70)$$

Where

$I$  - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors

$m$  - shape factor (above)

$k$  - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling)

$\sigma_c$  – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\bar{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \quad (71)$$

kinetic energy (KE):

$$KE = \frac{1}{2} m \cdot v^2 \quad (\text{for head-on collisions}) \quad (72)$$

$$KE = \frac{1}{2} m_e \cdot v_n^2 \quad (\text{for oblique collisions}) \quad (73)$$

$m_e$  is the 'effective' mass

$v_n$  is the 'normal' velocity

crushing energy (IE):

$$IE = \int_0^x F(x) dx \quad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \quad (75)$$

$$V = N_c (\cos \theta - \mu \sin \theta) \quad (76)$$

$$H = N_c (\sin \theta + \mu \cos \theta) \quad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6} t^2} \quad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \quad (80)$$

Combining the above and letting  $L = 10t$  gives;

$$\sigma_t = \frac{V}{t} \left( 60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left( 60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left( 60 \frac{\cos \theta - \mu \sin \theta}{\sin \theta + \mu \cos \theta} - 1 \right)} \quad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{\left( 60 \cdot (\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta) \right)}$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \quad (85)$$

The ratio of the  $N_c$  force to  $V_{vert}$  is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos \theta - \mu \sin \theta) - (\sin \theta + \mu \cos \theta) / 60)}$$

The net forces applied to the ice are;

$$V = N_c (\cos \theta - \mu \sin \theta) - W \sin \theta \cdot (\sin \theta + \mu \cos \theta)$$

$$H = N_c (\sin \theta + \mu \cos \theta) + W \cos \theta \cdot (\sin \theta + \mu \cos \theta)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t} \left( 60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) \quad (89)$$

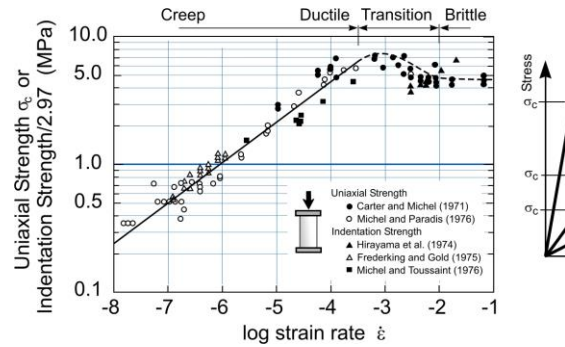
This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice;

$$V = \frac{\sigma_t t}{\left( 60 - \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)} \quad (90)$$

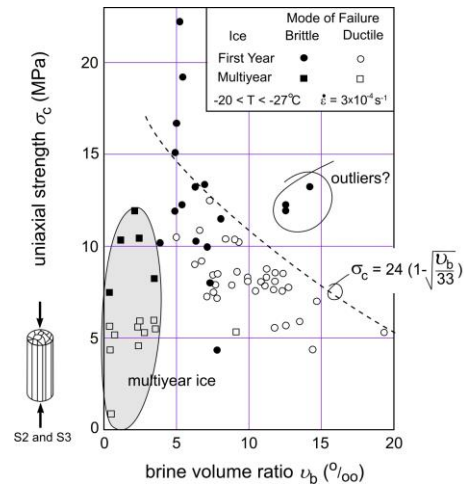
With (87), the required normal force  $N_c$  is;

$$N_c = \frac{V + W \sin \theta \cdot (\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)} \quad (91)$$

$$P = C \cdot A^{-0.5}$$



**Figure 1. Influence of strain rate on uniaxial strength.**



**Figure 2. Compressive strength of Beaufort Sea ice.**

The projected normal area is for spherical indentation;

$$A_n = 2 \cdot \pi \cdot R \cdot \zeta_n \quad (93)$$

The force is;;

$$F_n = p_o A_n^{ex} \cdot A_n \quad (84)$$

$$= p_o (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{1+ex} \quad (94)$$

The indentation energy is:

$$IE = \frac{P_o}{(2+ex)} (2 \cdot \pi \cdot R)^{1+ex} \cdot \zeta_n^{2+ex} \quad (95)$$

Which can be expressed in a more general form as :

$$IE_e = \frac{P_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (96)$$

$$f_x = 2 + ex \quad (97)$$

$$f_a = (2\pi \cdot R)^{f_x-1} \quad (98)$$

$$KE_e = \frac{P_o}{f_x} f_a \cdot \zeta_n^{f_x} \quad (101)$$

Solving for the normal indentation:

$$\zeta_n = \left( \frac{KE_e \cdot f_x}{P_o \cdot f_a} \right)^{\frac{1}{f_x}} \quad (102)$$

The normal force can be found by substituting eqn. (102) into (96), with (97) and (98) to give

$$F_n = P_o \cdot f_a \cdot \left( \frac{KE_e \cdot f_x}{P_o \cdot f_a} \right)^{\frac{f_x-1}{f_x}} \quad (103)$$

Which in our spherical case becomes:

$$F_n = \left[ P_o \cdot (2 \cdot \pi \cdot R)^{(1+ex)} \right]^{\frac{1}{2+ex}} \cdot \left( \frac{1}{2} M_e \cdot V_n^2 (2+ex) \right)^{\frac{1+ex}{2+ex}}$$

Lindqvist's resistance equation

$$R_{ice} = (R_C + R_B)(1 + 1.4 \cdot v / \sqrt{g \cdot H_{ice}}) + R_S \cdot (1 + 9.4 \cdot v / \sqrt{g \cdot L}) \quad (105)$$

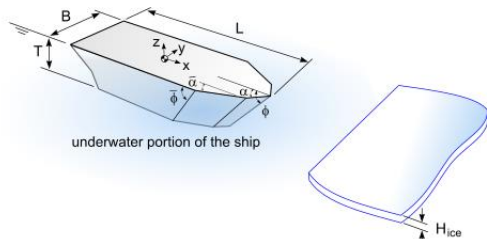


Figure 3. Linqvist's hull form with angles and dimensions.

### 1.1.1 Ice Crushing Term $R_C$

Linqvist starts by estimating the vertical force needed to fail an ice edge in flexure:

$$F_v = 0.5 \cdot \sigma_b \cdot H_{ice}^2 \quad (106)$$

Where  $\sigma_b$  is the flexural strength of ice. He then assumes that this force is generated (continuously) on each side of the stem and resolves the x (along ship) component, which is the resistance that the ship will feel;

$$R_C = F_v \frac{\tan \phi + \mu \cdot \cos \phi / \cos \psi}{1 - \mu \cdot \sin \phi / \cos \psi} \quad (107)$$

Where  $\mu$  is the friction factor,  $\phi$  is the stem angle,  $\alpha$  is the waterline angle and;

$$\psi = \arctan(\tan \phi / \sin \alpha) \quad (108)$$

These equations are dimensionally consistent, so any units will work.

### 1.1.2 Ice Breaking Term $R_B$

The breaking resistance term (see appendix in Linqvist 1989 for derivation) is as follows;

$$R_B = k \cdot B \cdot \left( \frac{H_{ice}^3}{l_c^2} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi) \quad (109)$$

Where k is a constant, B is the beam of the ship, the angles are as shown previously and  $l_c$  is the characteristic length of the ice sheet;

$$l_c = 4 \sqrt{\frac{E \cdot H_{ice}^3}{12 \rho_w g (1 - \nu^2)}} \quad (110)$$

Linqvist makes the following assumptions to simplify the equation.

Poisons ratio is 0.3. The elastic modulus is 2 GPa. He assumes the cusp length is 1/3 of the characteristic length and the shear strength is equal to the flexural strength.

$$R_B = .003 \cdot \sigma_b \cdot B \cdot \left( \frac{H_{ice}^{1.5}}{\sqrt{m}} \right) (\tan \psi + \mu \cos \phi / (\sin \alpha \cdot \cos \psi)) (1 + 1 / \cos \psi)$$

### 1.1.3 Ice Submergence Term $R_S$

The submergence resistance term (see appendix in Linqvist 1989 for derivation) is derived from energy consideration and is as follows;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot (B \cdot T \cdot (B + T) / (B + 2T) + \mu (A_u + \cos \phi \cdot \cos \psi \cdot A_f))$$

Where  $\delta \rho$  is the density difference between ice and water (the cause of buoyancy)  $A_u$  is the area of the flat of bottom and  $A_f$  is the area of the bow. With approximations for the areas involved, Linqvist proposed the alternate formula;

$$R_S = \delta \rho \cdot g \cdot H_{ice} \cdot B \cdot \left( T \frac{B+T}{B+2T} + \mu \left( .7 \cdot L - \frac{T}{\tan \phi} - \frac{0.25 \cdot B}{\tan \alpha} + T \cdot \cos \phi \cdot \cos \psi \cdot \sqrt{\frac{1}{\sin^2 \phi} + \frac{1}{\tan^2 \alpha}} \right) \right) \quad (114)$$

### POPOV

$$M_e = \frac{F_n}{a_n} \quad (115)$$

The effective mass is;

$$M_e = \frac{M_{ship}}{C_o} \quad (116)$$

where  $C_o$  is the mass reduction coefficient.

The inertial properties of the vessel are as follows,

Hull angles at point:

$\alpha$  = waterline angle

$\beta$  = frame angle

$\beta'$  = normal frame angle

$\gamma$  = sheer angle

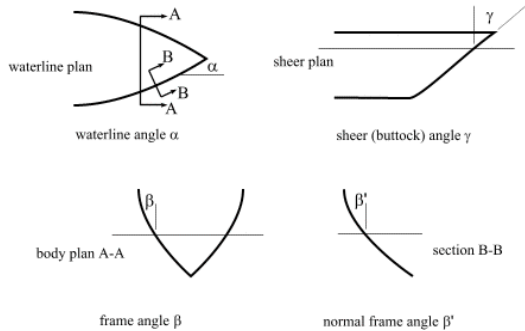


Figure 4. Hull angle definitions.

The various angles are related as follows,

$$\tan \beta = \tan \alpha \cdot \tan \gamma \quad (117)$$

$$\tan \beta' = \tan \beta \cdot \cos \alpha \quad (118)$$

Based on these angles, the direction cosines,  $l, m, n$  are

$$l = \sin \alpha \cdot \cos \beta' \quad (119)$$

$$m = \cos \alpha \cdot \cos \beta' \quad (120)$$

$$n = \sin \beta' \quad (121)$$

and the moment arms are

$$\lambda_1 = n \cdot y_p - m \cdot z_p \quad (\text{roll moment arm}) \quad (122)$$

$$\mu_1 = l \cdot z_p - n \cdot x_p \quad (\text{pitch moment arm}) \quad (123)$$

$$\eta_1 = m \cdot x_p - l \cdot y_p \quad (\text{yaw moment arm}) \quad (124)$$

The added mass terms are as follows;

$$AM_x = \text{added mass factor in surge} = 0 \quad (125)$$

$$AM_y = \text{added mass factor in sway} = 2 T/B \quad (126)$$

$$AM_z = \text{added mass factor in heave} = 2/3 (B Cwp^2) / (T(Cb(1+Cwp))) \quad (127)$$

$$AM_{rol} = \text{added mass factor in roll} = 0.25 \quad (128)$$

$$AM_{pit} = \text{added mass factor in pitch} = B / (T(3-2Cwp)(3-Cwp)) \quad (129)$$

$$AM_{yaw} = \text{added mass factor in yaw} = 0.3 + 0.05 L/B \quad (130)$$

The mass radii of gyration (squared) are;

$$rx^2 = Cwp B^2 / (11.4 Cm) + H^2 / 12 \quad (\text{roll}) \quad (131)$$

$$ry^2 = 0.07 Cwp L^2 \quad (\text{pitch}) \quad (132)$$

$$rz^2 = L^2 / 16 \quad (\text{yaw}) \quad (133)$$

With the above quantities defined, the mass reduction coefficient is

$$Co = l^2/(1+AMx) + m^2/(1+AMy) + n^2/(1+AMz) + \lambda l^2/(rx^2(1+AMrol)) + \mu l^2/(ry^2(1+AMpit)) + \eta l^2/(rz^2(1+AMyaw)) \tag{134}$$

1.1.4 Contact with General Wedge (Normal to hull)

a general wedge-shaped edge indentation (normal to hull).  
The projected areas, vertical, horizontal and normal are;

$$A_v = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\cos^2(\beta')} \tag{135}$$

$$A_h = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta')\cos(\beta')} \tag{136}$$

$$A_n = \frac{\zeta_n^2 \cdot \tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \tag{137}$$

The pressure-area approach is used again. Substituting (137) into (75) we arrive at:

$$F_n = p_o A_n^{ex} \cdot A_n = p_o \left( \frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{138}$$

Note: this can be expressed in a general form as:

$$F_n = p_o \cdot fa \cdot \zeta_n^{fx-1} \tag{139}$$

Where  $fx = 3 + 2ex$  (140)

$$fa = \left( \frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \tag{141}$$

The indentation energy is found by substituting (140) and (141) into (96), to give:

$$IE = \frac{p_o}{(3 + 2ex)} \left( \frac{\tan(\phi/2)}{\sin(\beta')\cos^2(\beta')} \right)^{1+ex} \cdot \zeta_n^{3+2ex} \tag{142}$$

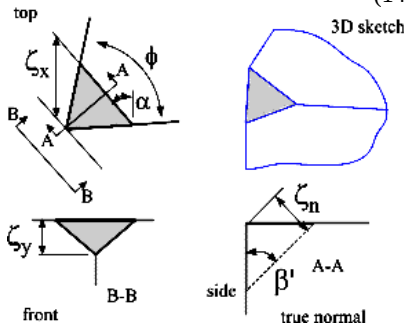


Figure 5. General Wedge-shaped Edge (normal to hull).

1.1.5 Popov Force Calculation

The Popov collision scenarios can be analyzed by equating the normal kinetic energy with the ice crushing energy.

$$KE_e = IE \tag{143}$$

where

$$KE_e = \frac{Me}{2} \cdot Vn^2 \tag{144}$$

which ,using equation (96) can be stated as;

$$KE_e = \frac{p_o}{fx} fa \cdot \zeta_n^{fx} \tag{145}$$

Solving for the normal indentation:

$$\zeta_n = \left( \frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{1}{fx}} \tag{146}$$

The normal force can be found by substituting eqn. (146) into (139) to give

$$F_n = p_o \cdot fa \cdot \left( \frac{KE_e \cdot fx}{p_o \cdot fa} \right)^{\frac{fx-1}{fx}} \tag{147}$$

$$V_n = V_{ship} \cdot lx \tag{149}$$

where  $V_n$  is the normal velocity at the point of impact  
 $V_{ship}$  is the x-direction velocity (all others are zero)  
 $lx$  is the x-direction cosine

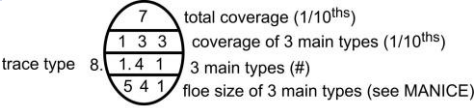
By combining the above equations, a single closed-form expression for the collision force is found;

$$F_n := p_o \cdot \left[ \frac{\tan\left(\frac{1}{2}\phi\right)}{\left(\sin(\beta') \cdot \cos(\beta')^2\right)} \right]^{(1+ex)} \cdot \left[ \frac{1}{2} \cdot Me \cdot Vn^2 \cdot (3 + 2 \cdot ex) \right]^{\frac{1}{3+2 \cdot ex}} \cdot \left[ \frac{(2 + 2 \cdot ex)}{(3 + 2 \cdot ex)} \right] \tag{151}$$

| Color | Ice Type               | Thickness           | Egg Code # |
|-------|------------------------|---------------------|------------|
|       | Ice Free               |                     |            |
|       | < 1/10 unspecified ice |                     |            |
|       | New Ice                | < 10 cm             | 1,2        |
|       | Grey Ice               | 10 - 15 cm          | 4          |
|       | Grey-White Ice         | 15 - 30 cm          | 5          |
|       | Thin FY                | First-year ice (FY) | 7          |
|       | Medium FY              | 30 - 70 cm          | 8          |
|       | Thick FY               | 70 - 120 cm         | 9          |
|       | Second-year Ice        | >30 cm              | 1.         |
|       | Multi-year Ice         | Old Ice             | 4.         |
|       |                        |                     | 8.         |
|       |                        |                     | 9.         |

Floe Size/Grandeur des floes (F<sub>a</sub>F<sub>b</sub>F<sub>c</sub>)

| Description/Élément                                | Width/Extension | Code |
|----------------------------------------------------|-----------------|------|
| Pancake ice/Glace en crêpes                        |                 | 0    |
| Small ice cake, brash ice/Petit glaçons, sarrazins | <2 m            | 1    |
| Ice cake/Glaçons                                   | 2-20 m          | 2    |
| Small floe/Petits floes                            | 20-100 m        | 3    |
| Medium floe/Floes moyens                           | 100-500 m       | 4    |
| Big floe/Grands floes                              | 500-2000 m      | 5    |
| Vast floe/Floes immenses                           | 2-10 km         | 6    |
| Giant floe/Floes géants                            | >10 km          | 7    |



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.