Faculty of Engineering
and Applied Science

Engineering 8674 - Design for Ocean and Ice Environments
Engineering 9096 - Sea Ice Engineering

MID-TERM EXAMINATION

With Solutions

Date: Fri., Feb. 6, 2009 **Professor: Dr. C. Daley**
Time: 1:00 - 1:50 pm

Answer all questions. Total 50 marks. Each question is worth marks indicated **[x].**
Watch your time.
Short, clear answers are best. If you are having a problem (ie a road block) assume something, write down the assumption, and continue. Use the back of the sheets as needed.
Good luck.

NAME: ______________________________

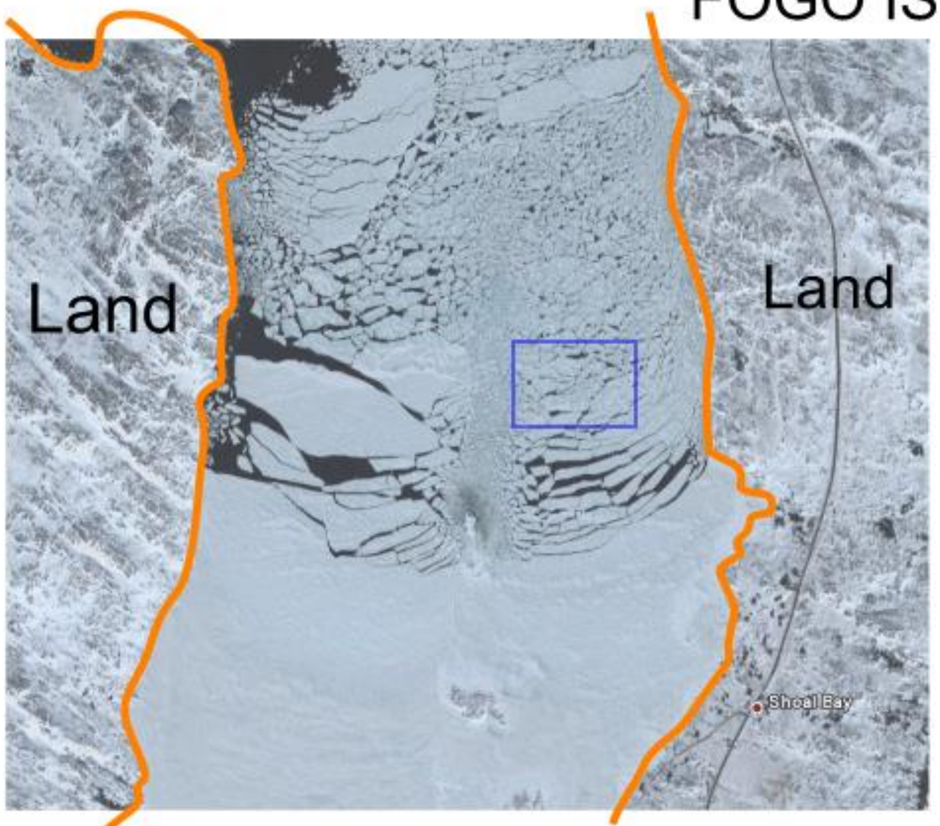
STUDENT NUMBER: ______________________________

There are 10 questions worth 1 mark each. Fill in the answers in the spaces provided. **[10]**

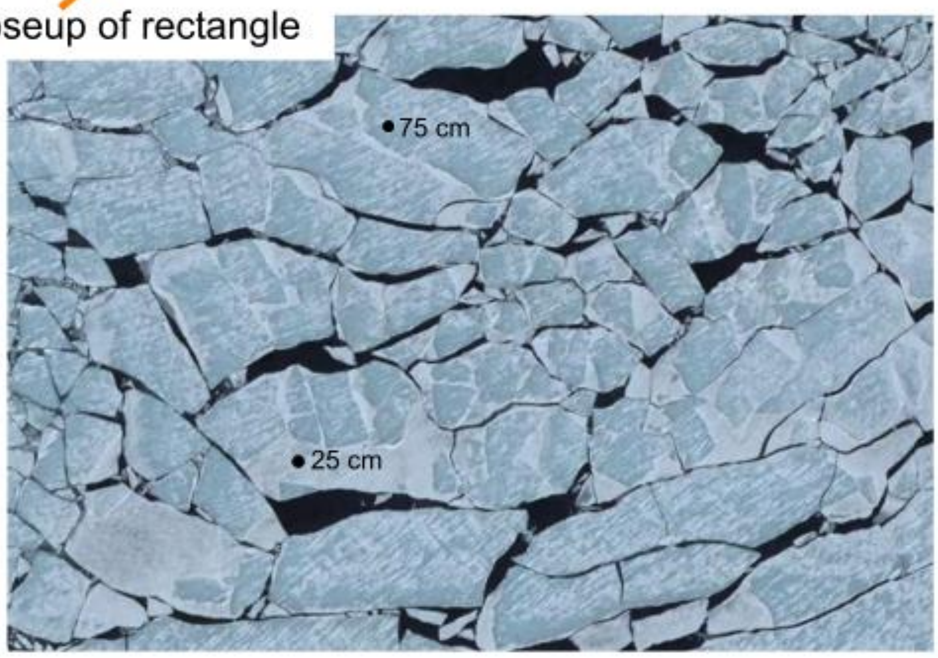
1. If you stand on thin ice and it starts to crack, the orientation of the first cracks will be: *radial*
2. A ring test measures *tension* .
3. Typical average salinity of first year sea ice is about: *5-10 ppt*.
4. Typical average salinity of old ice is about: *0.5 - 4 ppt* .
5. In 'P' ice the c-axis is oriented *randomly*.
6. Korzhavin's equation describes what type of ice failure? *crushing*
7. Ice creep stresses are mainly influenced by *temperature* and *strain rate*
8. A slurry of ice flakes suspended in water, is called *frazil*
9. The 'c' axis in 'Secondary' ice is: *horizontal*
10. Referring to ice strength, the phrase "Transition region" refers to: *change from ductile to brittle crushing*

2. On the following page there is an image of a bay on Fogo Island. There is a closeup of one part of the bay showing a couple of ice thickness measurements. At the bottom is a blank sketch of the bay. Create an ice chart in the blank sketch with ice regions and egg codes to describe each region. Make 4 regions and 3 egg codes. *See next page* **[15]**

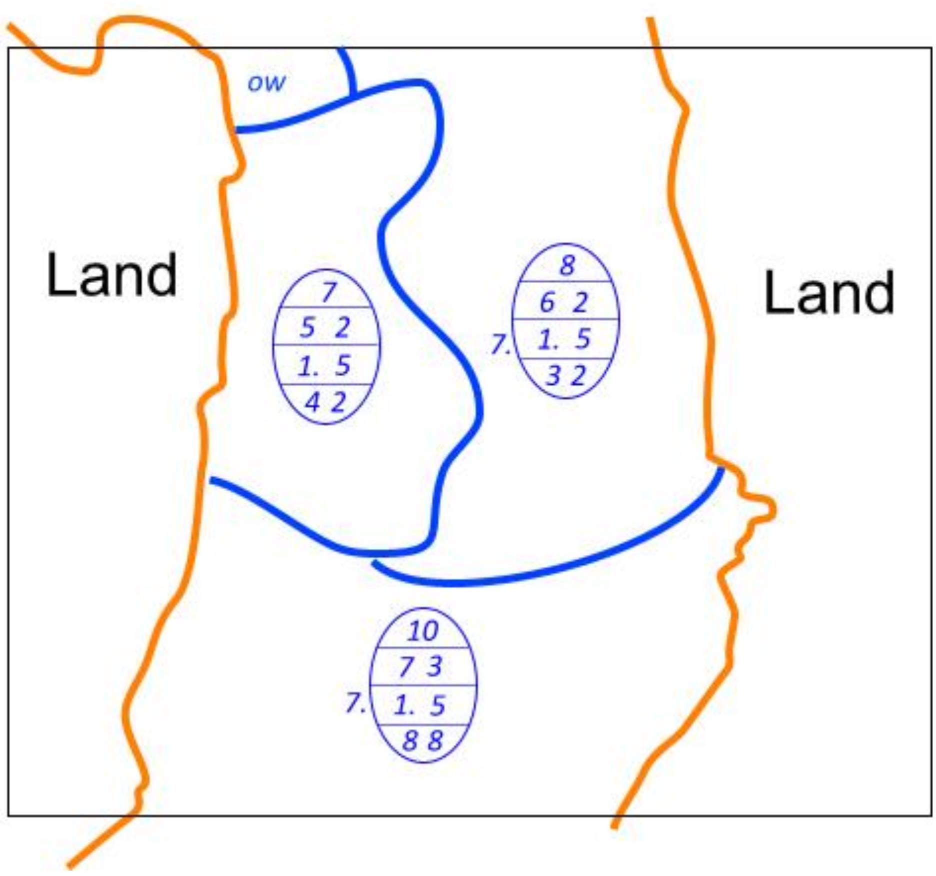
FOGO ISLAND



Closeup of rectangle

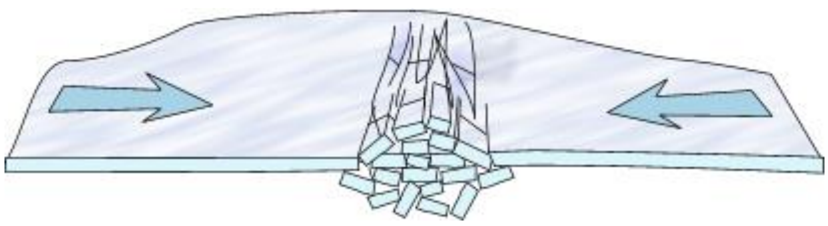


Ice Chart:



3. Natural processes [5]

Make a sketch of a pressure ridge, so that both top and cross section are evident.



4. Ice Mechanics [5]

Explain succinctly how/why the bending response of an ice sheet can be described with an ordinary differential equation, rather than needing to use partial differential equations.

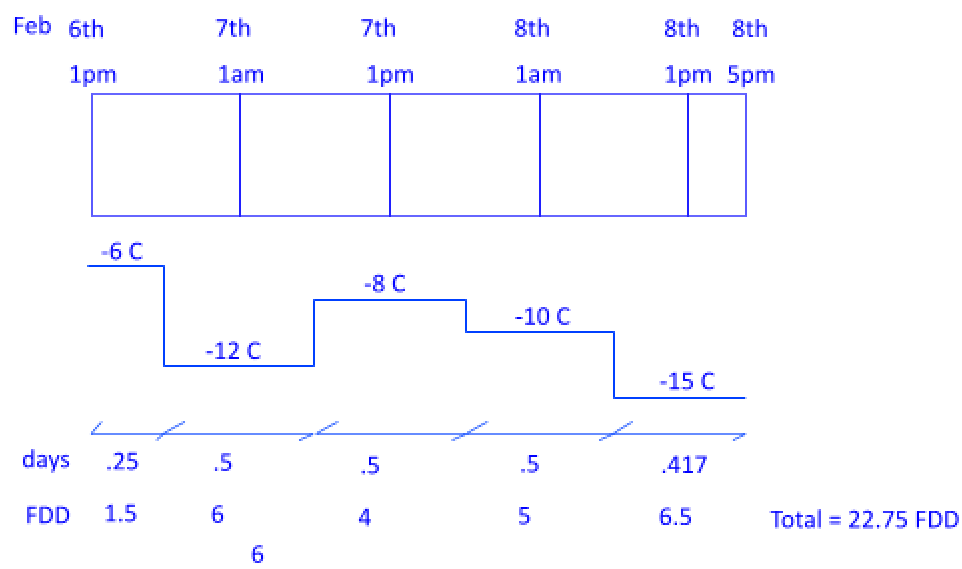
While ice is a volume (ie a 3D object),it is not usually necessary to model it with x,y and z independent coordinates. With near constant thickness it can be described as a plate with just x and y coordinates. However, even 2 independent variables require the use of partial differential equations. One more coordinate can be eliminated in certain situations. **If the load is a point load or is axially symmetric, then the problem (all responses) is axially symmetric and can be described with just one independent variable 'r'. With one independent variable the problem can be described with an ordinary differential equation.**

OPTION – Do Question 5 <u>or</u> 6. **[15]**

5. Use the following data to estimate the ice thickness on Feb 8 at 5pm;

Date	Feb 6	Feb 7	Feb 7	Feb 8	Feb 8	Feb 8
Time	1pm	1am	1pm	1am	1pm	5pm
Air temp	-6 C	-12 C	-8 C	-10 C	-15 C	
Ice thk.	1cm					?

we assume that the 1pm and 1 am tempertures are valid for the 6 hrs befor and after:



First we find the FDD needed to freeze the 1cm of ice that we started with:

$$h_{start} = 0.01 \text{ m}$$

$$h_{start} = .025\sqrt{FDD_{start}}$$

$$.01 = .025\sqrt{FDD_{start}}$$

$$FDD_{start} = \left(\frac{.01}{.025}\right)^2 \qquad FDD_{start} = 0.16 \text{ FDD}$$

Now we add the additional FDD and find the final thickness:

$$FDD_{end} = .16 + 22.75 = 22.91 \text{ FDD}$$

$$h_{end} = .025\sqrt{22.91}$$

$$h_{end} = 0.1196 \text{ m} = 12\text{cm} \quad \text{<= ANS}$$

the final ice thickness is 12 cm

6. You are standing on level fast ice, 2m thick, and you are 1 km from shore. The ice extends 50km further offshore and the wind is blowing towards shore. The shore is a vertical flat rock cliff. You have an accurate measurement of your position, and you determine that you are moving towards shore at 1mm/min. Estimate the ice pressure on the rock face at the shore.

the speed is: $.001 \text{ m}/60 \text{ s} = .0000167 \text{ m/s} = 1.67 \times 10^{-5}$

the strain rate is: $.0000167 \text{ m/s} / 1000\text{m} = 1.67 \times 10^{-8}$ strain/sec

the creep stress from figure 1 is approx 0.3 MPa <= ANS

Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m^3
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·1O^{-6} / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i} \quad (1)$$

Where Q is heat flux in W/m^2, k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i} \quad (2)$$

Where t is time, and ρ_i is density of the ice.
the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$

Converted to freezing degree days:

$$h_i = .037\sqrt{FDD} \quad [\text{m}]$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025\sqrt{FDD} \quad [\text{m}]$$

$$\nu_b = .001 \cdot S\left(.53 - \frac{49.2}{T}\right)$$

where T is temperature (°C), S is salinity (ppt) and ν_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65\left(1 - \sqrt{\frac{\nu_b}{275}}\right)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24\left(1 - \sqrt{\frac{\nu_b}{33}}\right)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82\left(1 - \sqrt{\frac{\nu_b}{142}}\right) \quad \text{case a) across grain}$$

$$\sigma_t = 1.54\left(1 - \sqrt{\frac{\nu_b}{311}}\right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2} \quad (21)$$

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2} \quad (22)$$

1.1 Beam on Elastic Foundation

$$EI\frac{d^4 v}{dx^4} + kv = 0 \quad (23)$$

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, $k = \rho g w$. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k}\right)^{1/4} \quad (24)$$

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection: $v = \frac{2P}{\lambda k} \cdot e^{-x/\lambda} \cdot \cos x/\lambda$

Slope: $\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$

Moment: $M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$

Shear: $Q = -P \cdot e^{-\beta x} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4}\lambda$$

$$\lambda = \left(\frac{E}{3\rho g}\right)^{1/4} h^{3/4} \quad (29)$$

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4} \quad (30)$$

$$\ell = \sqrt[4]{D/k} \quad (55)$$

$$\omega = w/\ell \quad (6) \quad (56)$$

$$x = r/\ell \quad (57)$$

$$p' = p/k \quad (58)$$

For a concentrated load the (7) solution is;

$$w = \frac{p\ell^2}{2\pi D} kei(r/\ell)$$

$$w_{\max} = \frac{p\ell^2}{8D} \quad (8) \quad (61) \quad (13)$$

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{\max}}} \quad (62)$$

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log\left[\frac{E \cdot h^3}{k \cdot c^4}\right]} \sigma_{flex} \cdot h^2 \quad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2$$

(15)

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \quad (70)$$

Where

I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors (16)

m - shape factor (above)

k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling) (17)

σ_c – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\overline{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \quad (71)$$

kinetic energy (KE):

$KE = \frac{1}{2} m \cdot v^2$ (for head-on collisions)(72)

$KE = \frac{1}{2} m_e \cdot v_n^2$ (for oblique collisions)(73)

m_e is the 'effective' mass

v_n is the 'normal' velocity

crushing energy (IE):

$$IE = \int_0^x F(x)dx \quad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \quad (75)$$

$$V = N_c(\cos\theta - \mu\sin\theta) \quad (76)$$

$$H = N_c(\sin\theta + \mu\cos\theta) \quad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6}t^2} \quad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \qquad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t}(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}) \qquad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta})} \qquad (82)$$

$$H = \frac{\sigma_t \cdot t}{(60\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} - 1)} \qquad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{(60 \cdot (\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta))}$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \qquad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta)/60)}$$

The net forces applied to the ice are;

$$V = N_c(\cos\theta - \mu\sin\theta) - W\sin\theta \cdot (\sin\theta + \mu\cos\theta)$$

$$H = N_c(\sin\theta + \mu\cos\theta) + W\cos\theta \cdot (\sin\theta + \mu\cos\theta)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t}(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}) \qquad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice:

$$V = \frac{\sigma_t t}{(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta})} \qquad (90)$$

With (87), the required normal force Nc is;

$$N_c = \frac{V + W\sin\theta \cdot (\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad (91)$$

$$P = C \cdot A^{-0.5}$$

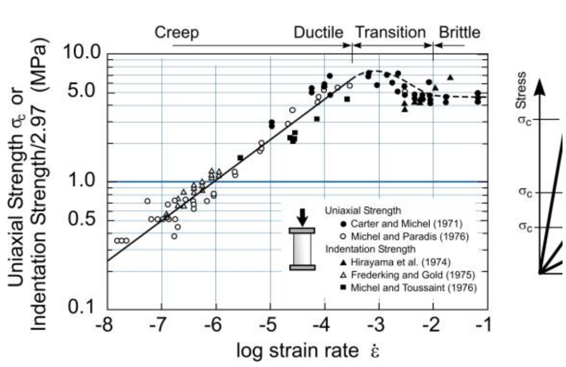


Figure 1. Influence of strain rate on uniaxial strength.

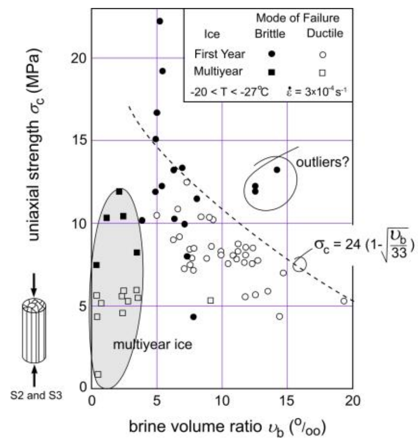


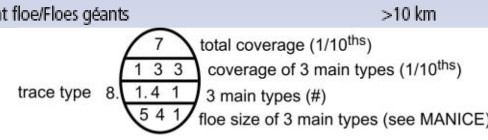
Figure 2. Compressive strength of Beaufort Sea ice.

(84)

Color	Ice Type		Thickness		Egg Code #	
	Ice Free					
	< 1/10 unspecified ice					
	New Ice		< 10 cm		1,2	
	Grey Ice		10 - 15 cm		4	3
	Grey-White Ice		15 - 30 cm		5	3
	Thin FY	First-year Ice (FY)	30 - 70 cm	>30 cm	7 8/9	6
	Medium FY	First-year Ice (FY)	70 - 120 cm	>30 cm	1.	6
	Thick FY	First-year Ice (FY)	> 120 cm	>30 cm	4.	6
	Second-year Ice	Old Ice			8.	7.
	Multi-year Ice	Old Ice			9.	7.

Floe Size/Grandeur des floes ($F_aF_bF_c$)

Description/Élément	Width/Extension	Code
Pancake ice/Glace en crêpes		0
Small ice cake, brash ice/Petit glaçons, sarrasins	<2 m	1
Ice cake/Glaçons	2-20 m	2
Small floe/Petits floes	20-100 m	3
Medium floe/Floes moyens	100-500 m	4
Big floe/Grands floes	500-2000 m	5
Vast floe/Floes immenses	2-10 km	6
Giant floe/Floes géants	>10 km	7



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.