MEMORIAL UNIVERSITY Faculty of Engineering and Applied Science

Engineering 8674 - Design for Ocean and Ice Environments
Engineering 9096 - Sea Ice Engineering

MID-TERM EXAMINATION

Date: Fri., March 2, 2010 **Professor: Dr. C. Daley**
Time: 1:00 - 1:50 pm

Answer all questions. Total 50 marks. Each question is worth marks indicated **[x].**
Watch your time.
Short, clear answers are best, right on this exam paper. If you are having a problem (ie a road block) assume something, write down the assumption, and continue. Use the back of the sheets as needed.

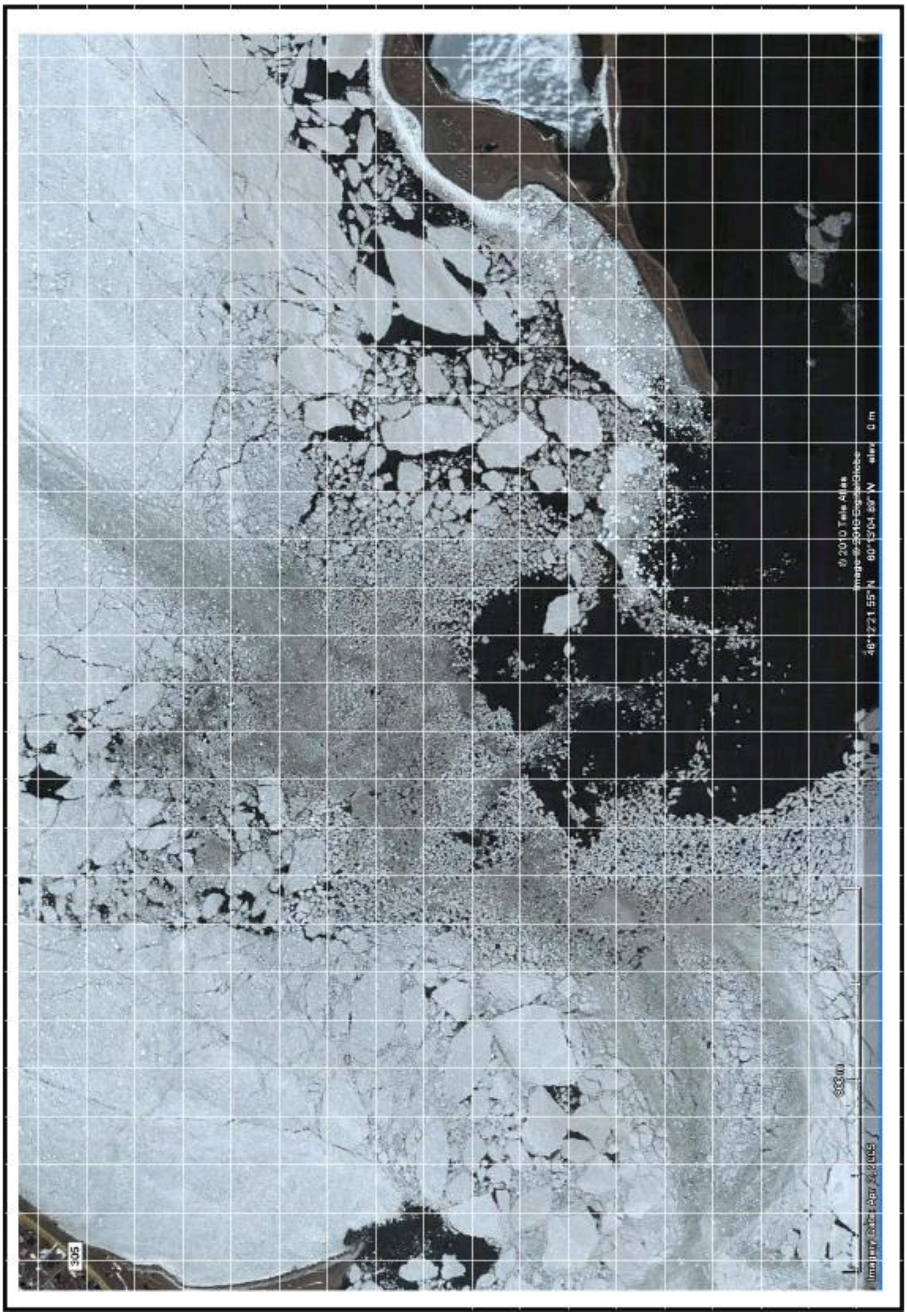
Good luck.

NAME: ______________________________

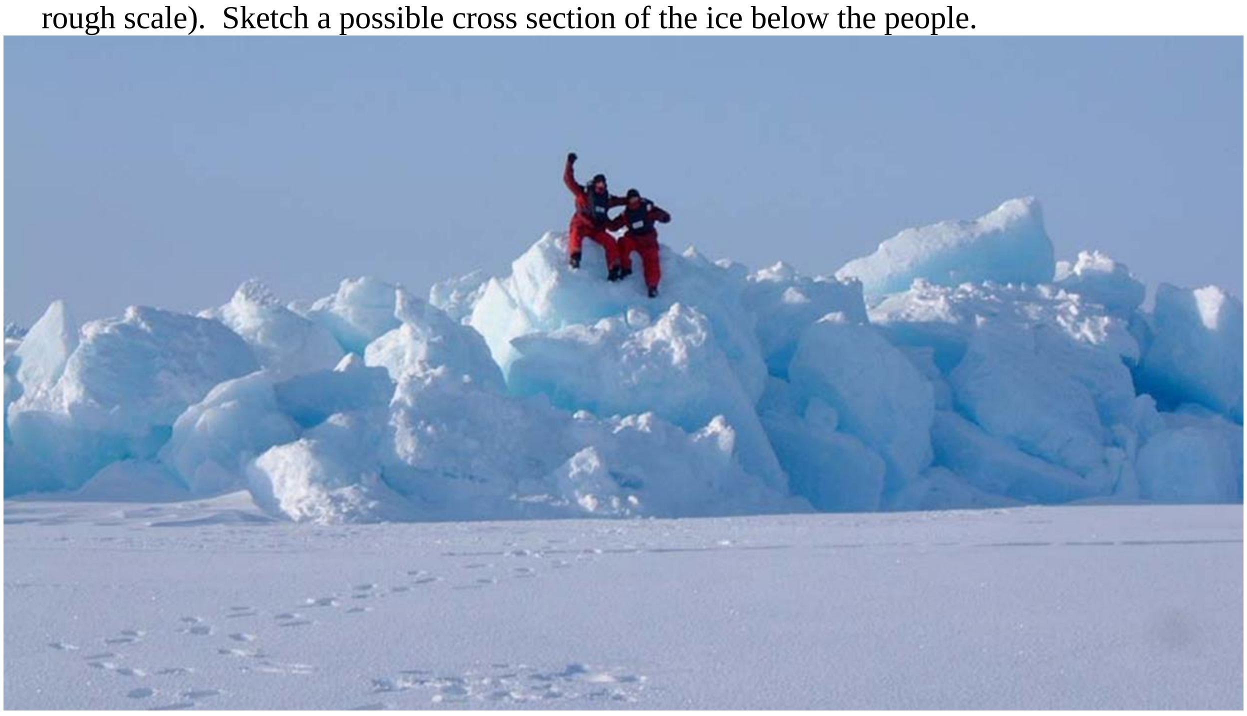
STUDENT NUMBER: ______________________________

1. The image below is of the Sydney NS ferry harbor in winter. Divide the water into regions and give an egg code for each (about 4 egg codes).

[10]

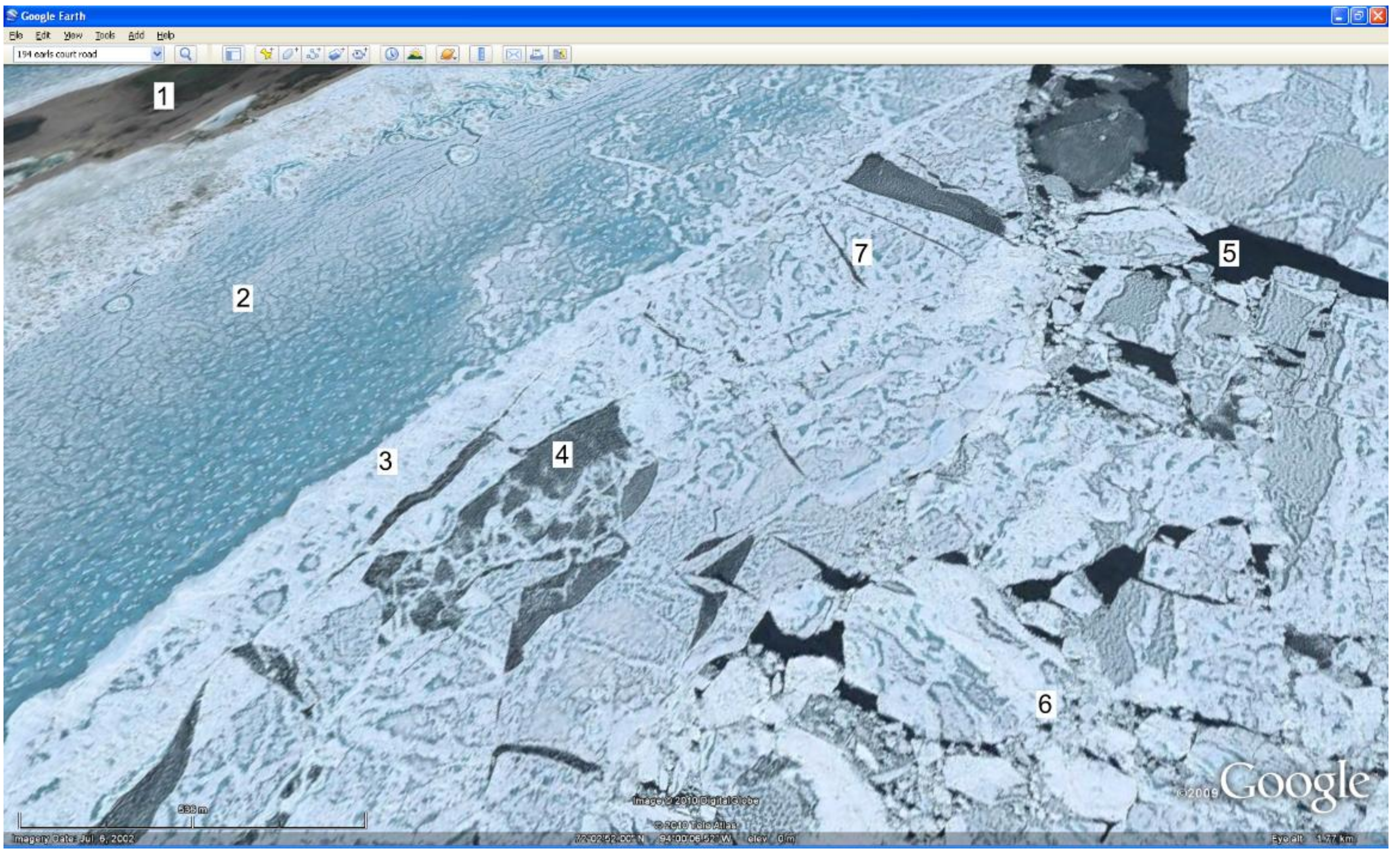


2. The photograph below was taken near the North Pole in 2004. Describe the ice feature in the photo. Estimate both the height (in air) and depth (in water) of the feature (use the people for rough scale). Sketch a possible cross section of the ice below the people. **[10]**



3. Natural ice [10]

Examine this image closely. It's an image of ice taken by Satellite. Describe the various features you see, with reference to the <u>type, condition or other aspect of the ice</u>. Seven numbers are on the image. You can use the numbers to help organize your description. (ie Tell me what you see, and what it means!)



4. Ice Growth [10]

The air temperature is minus 20°C at night with a light wind blowing. The ice on a lake is 10cm thick. There is a 3cm snow cover. The snow has a thermal conductivity of 0.2 W/m/°C. How much will the ice grow in one hour?

5. Ice Loads [10]

Think of a fixed offshore structure, something like the Hibernia GBS, but sitting in the Beaufort Sea. Discuss how Croasdale's limit load concepts can be used to help determine the design load.

Formulae

Table 1: Physical Properties of Fresh Water Ice 1h at 0°C

Density	917 kg /m^3
Melting point	0°C
Specific heat (heat capacity)	2.01 kJ / kg /°C
Latent heat of fusion (or melt)	334 kJ / kg
Thermal conductivity	2.2 W / m / °C
Linear expansion coefficient	55·1O^{-6} / °C
Vapor pressure	610.7 Pa
Refractive index	1.31
Acoustic velocity	
longitudinal wave	1928 m/s
transverse wave	1951 m/s

$$Q = \frac{k_i \cdot \Delta T}{h_i} \quad (1)$$

Where Q is heat flux in W/m^2, k_i is thermal conductivity in W / m / °C, h_i is the ice thickness in m and ΔT is the temperature difference across the ice sheet.

$$\frac{\Delta h_i}{\Delta t} = \frac{Q}{L \cdot \rho_i} = \frac{k_i \cdot \Delta T}{h_i \cdot L \cdot \rho_i} \quad (2)$$

Where t is time, and ρ_i is density of the ice.
the standard growth equation:

$$h_i = \sqrt{\frac{2 \cdot k_i}{L \cdot \rho_i} FDS}$$

Converted to freezing degree days:

$$h_i = .037\sqrt{FDD} \quad \text{[m]}$$

A more practical estimate would be found with the following equation (Cammaert and Muggeridge, 1988);

$$h_i = .025\sqrt{FDD} \quad \text{[m]}$$

$$\nu_b = .001 \cdot S\left(.53 - \frac{49.2}{T}\right)$$

where T is temperature (°C), S is salinity (ppt) and ν_b is the brine volume ratio.

Strength has been empirically related to brine volume as follows: Peyton (1966) proposed Cook Inlet (south of Alaska):

$$\sigma = 1.65\left(1 - \sqrt{\frac{\nu_b}{275}}\right)$$

Frederking and Timco (1980) report Southern Beaufort Sea:

$$\sigma = 24\left(1 - \sqrt{\frac{\nu_b}{33}}\right)$$

Peyton (1966) suggested the following mean (regression) values for tensile strength:

$$\sigma_t = 0.82\left(1 - \sqrt{\frac{\nu_b}{142}}\right) \quad \text{case a) across grain}$$

$$\sigma_t = 1.54\left(1 - \sqrt{\frac{\nu_b}{311}}\right) \quad \text{case b) along grain (18)}$$

The four point bending test is a lab test conducted with a machined (or at least cut) sample of ice. The strength value from such a test is;

$$\sigma_f = \frac{3 \cdot P \cdot a}{w \cdot h^2} \quad (21)$$

The cantilever beam test. In the simplest case the flexural strength is found from the formula;

$$\sigma_f = \frac{3 \cdot P \cdot L}{w \cdot h^2} \quad (22)$$

1.1 Beam on Elastic Foundation

$$EI\frac{d^4 v}{dx^4} + kv = 0 \quad (23)$$

Where v is deflection, EI is the beam stiffness and k is the foundation modulus. For water density ρ and beam of width w, $k = \rho g w$. For later convenience we define a term called the characteristic length λ;

$$\lambda = \left(\frac{4EI}{k}\right)^{1/4} \quad (24)$$

To find the influence of the water foundation, the solution of the differential equation (eqn 23) is needed. For the case of a semi-infinite beam, with an end load P, the solution of the system is;

Deflection: $v = \frac{2P}{\lambda k} \cdot e^{-x/\lambda} \cdot \cos x/\lambda$

Slope: $\theta = -\frac{2P}{\lambda^2 k} \cdot e^{-x/\lambda} \cdot (\cos(x/\lambda) + \sin(x/\lambda))$

Moment: $M = \lambda P \cdot e^{-x/\lambda} \cdot \sin(x/\lambda)$

Shear: $Q = -P \cdot e^{-\beta x} \cdot (\cos(x/\lambda) - \sin(x/\lambda))$

The maximum bending moment occurs at a distance x_c from the free end:

$$x_c = \frac{\pi}{4}\lambda$$

$$\lambda = \left(\frac{E}{3\rho g}\right)^{1/4} h^{3/4} \quad (29)$$

Which for typical ice properties is;

$$\lambda = C \cdot h^{3/4} \quad (30)$$

$$\ell = \sqrt[4]{D/k} \quad (55)$$

$$\omega = w/\ell \quad (6) \quad (56)$$

$$x = r/\ell \quad (57)$$

$$p' = p/k \quad (58)$$

For a concentrated load the (7) solution is;

$$w = \frac{p\ell^2}{2\pi D} kei(r/\ell)$$

$$w_{max} = \frac{p\ell^2}{8D} \quad (8) \quad (61) \quad (13)$$

$$\ell = \sqrt{\frac{p}{8 \cdot k \cdot w_{max}}} \quad (62)$$

This can be re-expressed as a bearing capacity that as;

$$P = \frac{4}{\log\left[\frac{E \cdot h^3}{k \cdot c^4}\right]} \sigma_{flex} \cdot h^2 \quad (67)$$

$$\cong 0.5 \cdot \sigma_{flex} \cdot h^2 \quad (15)$$

Korzhavin

$$p_{ice} = I \cdot m \cdot k \cdot \sigma_c \quad (70)$$

Where

I - indentation (confinement) factor, say 2.5 – 3.0 for narrow indentors (16)
m - shape factor (above)
k - contact factor (starts at 1 for full contact and drops to .6 with loss of contact for spalling) (17)
σ_c – uniaxial crushing strength

Seaki's force equation can be converted to show that effective pressure decreases with contact width;

$$\bar{p} = \frac{F}{b \cdot h} = 3.9 \cdot \sigma_c \cdot b^{-.39} \quad (71)$$

kinetic energy (KE):

$KE = \frac{1}{2} m \cdot v^2$ (for head-on collisions)(72)

$KE = \frac{1}{2} m_e \cdot v_n^2$ (for oblique collisions)(73)

m_e is the 'effective' mass

v_n is the 'normal' velocity

crushing energy (IE):

$$IE = \int_0^x F(x)dx \quad (74)$$

$$F(x) = p(A(x)) \cdot A(x) \quad (75)$$

$$V = N_c(\cos\theta - \mu\sin\theta) \quad (76)$$

$$H = N_c(\sin\theta + \mu\cos\theta) \quad (77)$$

$$\sigma_{flex} = \frac{V \cdot L}{\frac{1}{6}t^2} \quad (79)$$

The tensile stress in the ice is;

$$\sigma_t = \sigma_{flex} - \frac{H}{t} \qquad (80)$$

Combining the above and letting $L = 10t$ gives;

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (81)$$

Therefore, the vertical, horizontal and normal forces (per unit width), become;

$$V = \frac{\sigma_t \cdot t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (82)$$

$$H = \frac{\sigma_t \cdot t}{\left(60\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} - 1\right)} \qquad (83)$$

$$N_c = \frac{\sigma_t \cdot t}{(60 \cdot (\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta))}$$

A purely vertical load causes failure at a load of;

$$V_{vert} = \frac{\sigma_t \cdot t}{60} \qquad (85)$$

The ratio of the N_c force to V_{vert} is;

$$\frac{N_c}{V_{vert}} = \frac{1}{((\cos\theta - \mu\sin\theta) - (\sin\theta + \mu\cos\theta)/60)}$$

The net forces applied to the ice are;

$$V = N_c(\cos\theta - \mu\sin\theta) - W\sin\theta \cdot (\sin\theta + \mu\cos\theta)$$

$$H = N_c(\sin\theta + \mu\cos\theta) + W\cos\theta \cdot (\sin\theta + \mu\cos\theta)$$

Again using equations (79) and (80);

$$\sigma_t = \frac{V}{t}\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \qquad (89)$$

This can be rearranged to give the required vertical force to cause flexural (tensile) failure in the ice:

$$V = \frac{\sigma_t t}{\left(60 - \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} \qquad (90)$$

With (87), the required normal force Nc is;

$$N_c = \frac{V + W\sin\theta \cdot (\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad (91)$$

$$P = C \cdot A^{-0.5}$$

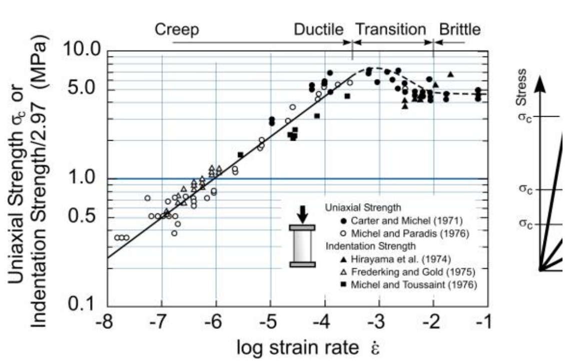


Figure 1. Influence of strain rate on uniaxial strength.

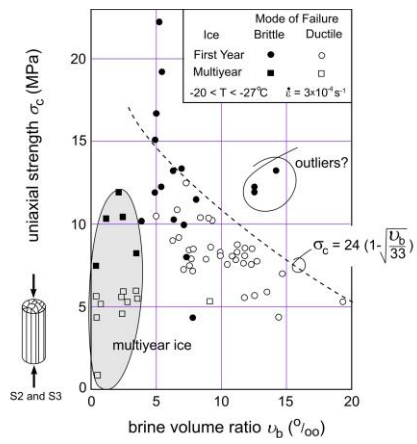


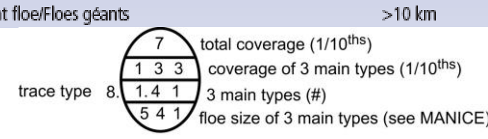
Figure 2. Compressive strength of Beaufort Sea ice.

(84)

Color	Ice Type		Thickness		Egg Code #	
	Ice Free					
	< 1/10 unspecified ice					
	New Ice		< 10 cm		1,2	
	Grey Ice		10 - 15 cm		4	3
	Grey-White Ice		15 - 30 cm		5	
	Thin FY	First-year Ice (FY)	30 - 70 cm	>30 cm	7 8/9	6
	Medium FY		70 - 120 cm		1.	
	Thick FY		> 120 cm		4.	
	Second-year Ice	Old Ice			8.	7.
	Multi-year Ice				9.	

Floe Size/Grandeur des floes ($F_aF_bF_c$)

Description/Élément	Width/Extension	Code
Pancake ice/Glace en crêpes		0
Small ice cake, brash ice/Petit glaçons, sarrasins	<2 m	1
Ice cake/Glaçons	2-20 m	2
Small floe/Petits floes	20-100 m	3
Medium floe/Floes moyens	100-500 m	4
Big floe/Grands floes	500-2000 m	5
Vast floe/Floes immenses	2-10 km	6
Giant floe/Floes géants	>10 km	7



In this example, there is 7/10ths ice, made up of 1/10th medium FY, 3/10ths grey ice and 3/10ths new ice. There is also a trace of second year ice.