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Derivation of Plastic Framing Requirements for Polar Ships

C. G. Daley[1]

[1]Faculty of Engineering and Applied Science, Memorial University, St. John's, Canada, A1B 3X5
email: cdaley@engr.mun.ca

ABSTRACT

A new IACS (International Association of Classification) standard for Polar Ship design, in the form of a Unified Requirement is being developed by an international committee with representatives from many classification societies and with the active participation of many polar nations. The framing structural requirements have been developed by a combination of analysis of existing rules and ships, finite element analysis and analytical solutions of plastic collapse mechanisms. This paper describes the derivation of the 3-hinge and asymmetrical shear plastic collapse mechanisms using work-energy principles. Energy methods are robust and well suited for developing design standards. The results are shown to compare well with non-linear finite element analyses of frame strength.

Key words: plastic design, ship structures, framing, ice class, Polar ships, IACS, shear and bending interaction.

NOTATION

a	length of shear panel
A1	shear factor in rule modulus equation for 3-hinge mechanism
A2	shear factor in rule modulus equation for shear mechanism
Af	area of the flange
Am	x-intercept of shear interaction equation
An	normalized web area
Ao	minimum web area
Aw	area of the web
b	height of the ice load patch
c	distance to edge of patch load
EWD	external work done
fz	function of *kz*
hw	height if the web
IWD	internal work done

j	number of fixed supports
kw	area ratio
kz	ratio of zp to Zp
L	length of frame
Mp	plastic moment for frame
mp	sum of plastic moments in plate and flange
Mpr	reduced plastic moment
N	shear force
P	pressure
$P1h$	pressure causing collapse for case of 0 fixed supports
$P2h$	pressure causing collapse for case of 1 fixed supports
$P3h$	pressure causing collapse for case of 2 fixed supports
Ps	pressure causing collapse for end load case
S	frame spacing
tf	thickness of the flange
tp	thickness of the shell plating
w	length of the ice load patch
wf	width of the flange
Zf	contribution of the flange to the plastic section modulus
Zm	y-intercept of shear interaction equation
Zn	normalized plastic section modulus
Zo	minimum plastic section modulus
Zp	plastic section modulus
zp	sum of plastic section moduli of plate and flange
zp_{flange}	plastic section modulus of the flange
$Zpmax$	maximum useful value of plastic section modulus
$Zpns$	a non-dimensional modulus
zp_{plate}	plastic section modulus of the plate
Zpr	reduced plastic section modulus
Zw	contribution of the web to the plastic section modulus
δ	deflection of the frame
σ_y	yield stress
τ	shear stress
τ_y	yield stress in shear

1. INTRODUCTION

Design is the process of specifying capability to satisfy anticipated demands. When designing ships for operation in ice, the inherent possibility of overloads must be considered. To mitigate against the consequences of overload conditions, there has been a move towards plastic design. At the design load level the structure is intended to exhibit some plastic behavior, yet maintaining substantial reserve against actual collapse or rupture. The derivations of strength shown below make use of plastic limit analysis. The load is determined by postulating the formation of a plastic mechanism, and equating internal and external work.

The method assumes rigid-plastic material behavior and ignores large strain and large deflection effects (such as strain hardening and secondary membrane stresses). This approach was developed as part of the development of the new Unified Requirement (UR) for Polar Ships, developed by the International Association of Classification Societies (IACS) [IACS, 2001]. The design ice loads for the UR are described in [Daley, 2000] (see Figure 1). A general discussion of the structural requirements in the UR is described in [Daley and Kendrick, 2000] and in [Daley, Kendrick and Appolonov, 2001]

The UR requires that frames be checked against several failure mechanisms. This paper describes and derives the nominal plastic strength for the case of a central patch load (see Figure 2), and an end (or asymmetrical) load patch. A related paper [Daley, 200x] discusses the application of these equations in a design/optimization exercise.

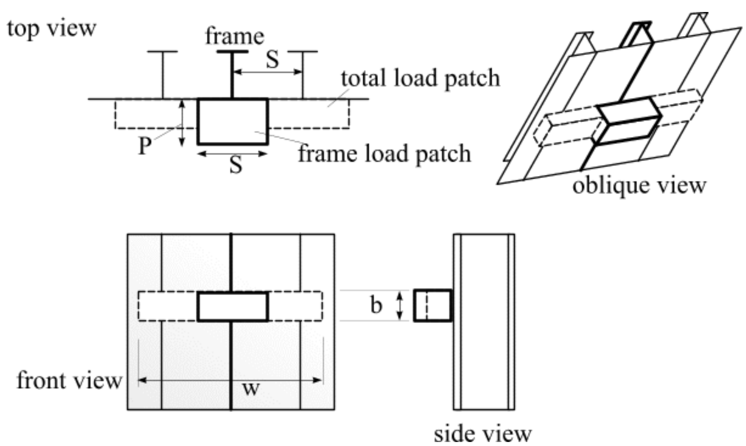


Figure 1. Design Ice Load Patch

2. CENTRAL LOAD CASE

The main case involves frames that are built-in on both ends (capable of transmitting a plastic moment to the surrounding structure, see Figure 2).

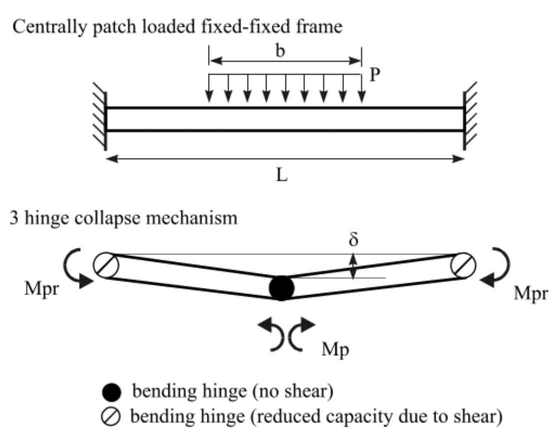


Figure 2. Centrally loaded fixed-fixed frame, with assumed plastic mechanism.

2.1 Energy Balance

We start by balancing internal and external work (see Figure 2). The external work depends on the external load and the deflection of the frame under the load, as shown on the left hand side of equation (1). The internal work includes the component from the central hinge, unaffected by shear, and the two edge hinges, which have reduced capacity due to shear. The internal work is shown on the right hand side of equation (1). Figure 3 shows other end conditions, with *j* being the number of fixed ends.

$$(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) = Mp \cdot \frac{4}{L} + Mpr \cdot \frac{4}{L} \tag{1}$$

where:

plastic moment: $Mp = Zp \cdot \sigma_y$ (2)

reduced plastic moment: $Mpr = Zpr \cdot \sigma_y$ (3)

On substitution of (2) and (3) into (1) we have:

$$(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) = 4 \cdot \frac{\sigma_y}{L} \cdot (Zp + Zpr) \tag{4}$$

The next step is to determine the values of Zp and Zpr.

2.2 Plastic Section Modulus

The plastic section modulus is used to calculate the moment capacity of a section when a plastic hinge has formed. Assuming perfectly plastic stresses, the balance of forces requires that the plastic neutral axis form at the half-area axis. The plastic section modulus is then the first moment of area about the plastic neutral axis. While this concept should work or all cross sections, it works best for ‘I’ sections, and less well for sections typical of frames attached to plating in ships. Often in ships, the frame area (web + flange) is less than the area of the shell plating. This results in the half-area axis, which is the plastic neutral axis, being inside the shell plating. Figure 3 illustrates the issue. In Figure 3(a), the state of strain and stress prior to full plasticity is shown. Because the distance to the flange is so much greater than the distance to the outer plate, the strain in the flange must be much larger than the plate. The neutral axis must be in the inner half of the plate and will normally be quite close to the inner side of the plate. For the inner fiber of the plate to reach full plasticity, the flange must have very large strains. This is both unrealistic and complicates the calculation of the plastic section modulus.

A more physically realistic and far simpler model of plastic section capacity for ship framing is to assume that the neutral axis forms at the intersection of the web and the shell (see Figure 3(b)). This assumption also implies that the stress in the plate is, on average, less than or equal to the yield stress. The force in the plate just balances the forces in the web and flange.

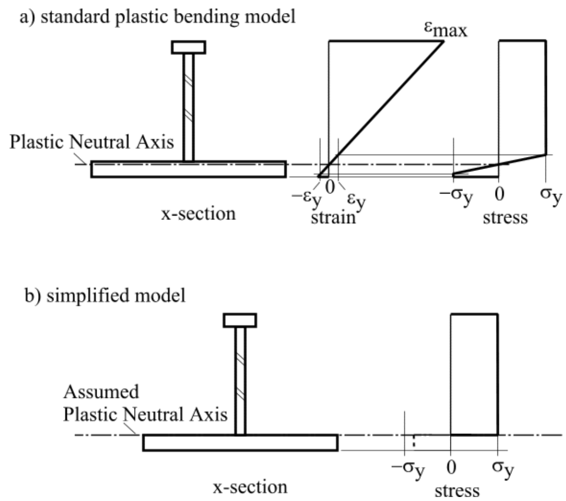


Figure 3. Simplified plastic modulus concept.

The section modulus (assuming PNA is at the web/plate connection) is:

$$Zp = Zf + Zw \tag{5}$$

where:

$$Zf = Af \cdot \left(\frac{tf}{2} + hw + \frac{tp}{2} \right) \tag{6}$$

$$Zw = Aw \cdot \left(\frac{hw}{2} + \frac{tp}{2} \right) \tag{7}$$

This results in:

$$Zp = Af \cdot \left(\frac{tf}{2} + hw + \frac{tp}{2} \right) + Aw \cdot \left(\frac{hw}{2} + \frac{tp}{2} \right) \tag{8}$$

2.3 Reduced Plastic Section Modulus

Assume that the shear is carried by the web. As the shear load increases, the ability of the web to contribute to the bending moment is reduced. This loss of moment capacity is formulated as a loss of section modulus. The shear is assumed to affect only the web's ability to contribute to bending. The reduced capacity is therefore given by:

$$Zpr = Zf + Zw \cdot \sqrt{1 - \left(\frac{\tau}{\tau_y}\right)^2} \tag{9}$$

where

Shear stress in web: $$\tau = \frac{P \cdot b \cdot S}{2 \cdot Aw} \tag{10}$$

Yield shear stress: $$\tau_Y = \frac{\sigma_y}{\sqrt{3}} \tag{11}$$

The minimum allowable web area *Ao* corresponds to shear yield at both supports under the design load.

$$\tau_y = \frac{P \cdot b \cdot S}{2 \cdot Ao} \tag{12}$$

Solving for Ao:

$$Ao = \frac{1}{2} P \cdot b \cdot S \cdot \frac{\sqrt{3}}{\sigma_y} \tag{13}$$

which allows equation (9) to be stated as:

$$Zpr = Zp \cdot \left[1 - kw \cdot \left[1 - \sqrt{1 - \left(\frac{Ao}{Aw}\right)^2}\right]\right] \tag{14}$$

where *kw* is the ration of the web modulus (equation (7)) to the full plastic modulus:

$$kw = \frac{Zw}{Zp} \tag{15}$$

an approximate (within a few %) value for *kw* is :

$$kw = \frac{1}{1 + 2 \cdot \frac{Af}{Aw}} \tag{16}$$

2.4 Interaction Equations for Central Load

Moment and shear capacity interact, as shown by Equation 14. Figure 4 illustrates this interaction. As the shear force increases, the shear stress lowers the contribution of the web. When the web is fully plastic in shear, the moment capacity is minimized. For sections with flanges, the minimum moment (minimum modulus) becomes *Zf*. For flat bar sections the moment is zero. This approach gives recognition, in a simple practicable way, to the contribution of the flanges after the web has fully yielded. Normally, however, frames are designed to work in an intermediate range, with significant moment and shear capacity.

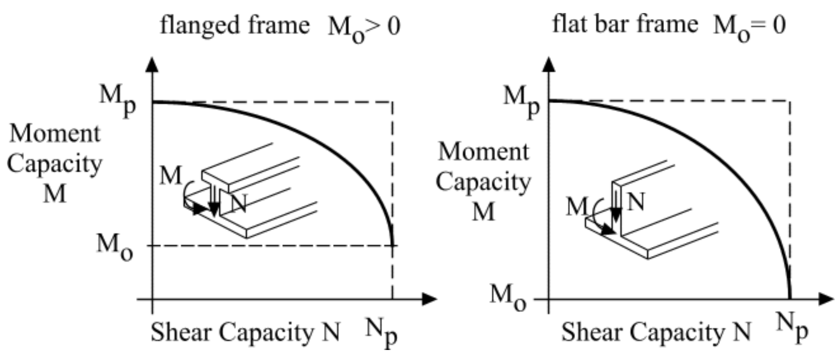


Figure 4. Interaction plot for moment and shear.

We denote the minimum modulus *Zo*, as that required if web is fully effective in bending (found by solving equation (4) with *Zp=Zpr*):

$$Zo = \frac{P \cdot b \cdot S}{8 \cdot \sigma_y} \cdot \left(1 - \frac{b}{2 \cdot L}\right) \cdot L \qquad (17)$$

Setting the normalized modulus to:

$$Zn = \frac{Zp}{Zo} \qquad (18)$$

and the normalized shear area to;

$$An = \frac{Aw}{Ao} \qquad (19)$$

and using equations (4), (8), (14), (18), and (19) the interaction equation between modulus and shear area requirements can be written in dimensionless form as:

$$Zn = \frac{2}{\left[2 + kw \cdot \left[\sqrt{1 - \left(\frac{1}{An}\right)^2} - 1\right]\right]} \tag{20}$$

or in dimensional form:

$$Zp = \frac{Zo \cdot 2}{\left[2 + kw \cdot \left[\sqrt{1 - \left(\frac{Ao}{Aw}\right)^2} - 1\right]\right]} \tag{21}$$

Using Equation (21) we plot the interaction equation for various *Af/Aw* ratios (see Figure 5). The acceptable region is above the curves. The inclusion of the interaction effects shown in Figure 6. is a significant development in the IACS Polar Rules. The requirement better represents the behavior of frames, but requires more effort to determine actual scantlings.

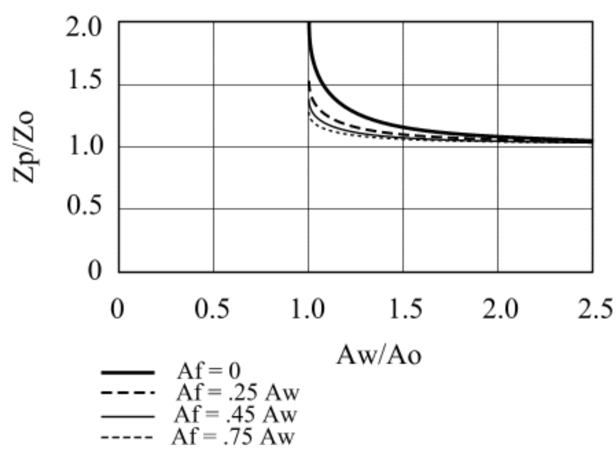


Figure 5. Interaction plot for modulus and web area for 3-hinge collapse.

2.5 Capacity Equation for 3-Hinge Collapse

While equation (21) is useful for determining the required modulus, it is less useful for checking the compliance of a specified frame. To check a frame, comparison of the design load with the capacity of the frame is simpler. The capacity for 3 hinge collapse (centered load), is found by solving for the pressure. Combining equations (4), (13), (14), (21) gives :

$$(P \cdot b \cdot S)\left(1-\frac{b}{2 \cdot L}\right) = 4 \cdot \frac{\sigma_y}{L} \cdot Zp \cdot \left[2 + kw \cdot \left[\sqrt{1-\left(\frac{\frac{1}{2} \cdot P \cdot b \cdot S \cdot \frac{\sqrt{3}}{\sigma_y}}{Aw}\right)^2} - 1\right]\right] \quad (22)$$

solving for *P*, we have the pressure to cause 3 hinge collapse:

$$P_{3h} = \frac{(2-kw) + kw \cdot \sqrt{1-48 \cdot Zpns \cdot (1-kw)}}{12 \cdot Zpns \cdot kw^2 + 1} \cdot \frac{Zp \cdot \sigma_y \cdot 4}{\left[S \cdot b \cdot L \cdot \left(1-\frac{b}{2 \cdot L}\right)\right]} \quad (23)$$

where the term *Zpns* is:

$$Zpns = \left[\frac{Zp}{Aw \cdot L \cdot \left(1-\frac{b}{2 \cdot L}\right)}\right]^2 \quad (24)$$

for the term under the root sign in equation (23) to stay positive, *Zp* must be less than *Zpmax*, where;

$$Zpmax = \sqrt{\frac{1}{48 \cdot (1-kw)}} \cdot Aw \cdot L \cdot \left(1-\frac{b}{2 \cdot L}\right) \quad (25)$$

note: in cases in which Zp > Zpmax, the frame will first fail by shear at both supports (central load). In this case the capacity is nominally limited by:

$$Plim = 2 \cdot \frac{Aw \cdot \sigma_y}{\sqrt{3} \cdot S \cdot b} \quad (26)$$

3. END LOAD CASE

The main case involves frames that are built-in on both ends (capable of transmitting a plastic moment to the surrounding structure, see Figure 6). In the case of the far end pinned the solution results in a value of 'a' greater than L/2, and is thus illogical. Practically, this means that we only check this mechanism for fixed-fixed boundary conditions, and that pinned connections are not to be allowed in the ice-strengthened areas.

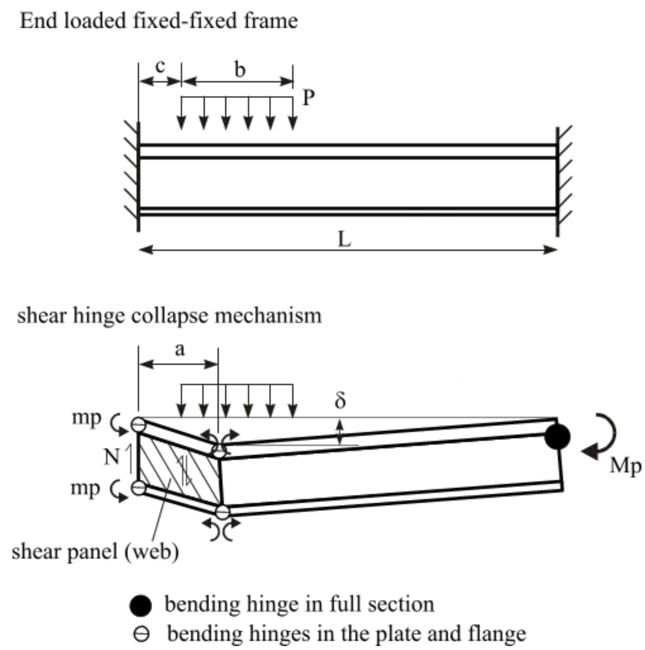


Figure 6. End loaded fixed-fixed frame, with assumed plastic mechanism.

3.1 Energy Balance

The external work done is found by integrating the external load over the deformation (for δ =1). The general equation (see Figure 7) for the external work (*EWD*) is;

$$EWD = P \cdot S \, \frac{(a-c)\left(\frac{1}{2}(c+a)\right)}{a} + \frac{(b+c-a)\left(L-\frac{1}{2}(a+b+c)\right)}{L-a} \tag{27}$$

We can simplify this by finding the location (value of *c*) which maximizes the work done. This is done by using;

$$\frac{\partial}{\partial c} EWD = 0 \tag{28}$$

When solving the above for *c* we get;

$$c = a \cdot \left(1 - \frac{b}{L}\right) \tag{29}$$

Substituting (29) into (27) , we get;

$$EWD = P \cdot b \cdot S \cdot \left(1 - \frac{b}{L}\right) \tag{30}$$

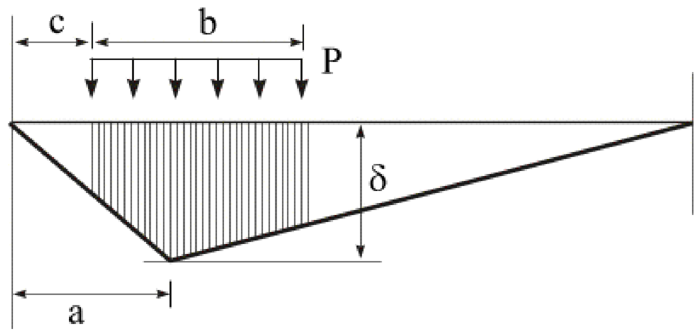


Figure 7. Shape of asymmetrical collapse.

The internal work (*IWD*) per unit deflection includes the plastic work done by the shear panel, the 4 small plastic hinges in the flanges and the large plastic hinge at the far end. The equation is;

$$IWD = N + \frac{Mp}{L-a} + mp \cdot \left(\frac{2}{a} - \frac{1}{L-a}\right) \tag{31}$$

where:

shear force in web: $N = Aw \cdot \frac{\sigma_y}{\sqrt{3}}$ (32)

plastic moment in full frame: $Mp = Zp \cdot \sigma_y$ (33)

sum of local plastic moments in plate and flange: $mp = zp \cdot \sigma_y$ (34)

sum of local plastic section mod. $zp = zp_{plate} + zp_{flange}$ (35)

local plastic section modulus of shell plate $zp_{plate} = S \cdot \frac{tp^2}{4}$ (36)

local plastic section modulus of flange $$zp_{flange} = wf \cdot \frac{tf^2}{4}$$ (37)

section modulus of frame (assumes NA at plate/web join) $$Zp = Af \cdot \left(\frac{tf}{2} + hw + \frac{tp}{2}\right) + Aw \cdot \left(\frac{hw}{2} + \frac{tp}{2}\right)$$ (38)

Equating EWD with IWD gives an energy balance equation of :

$$(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) = \sigma_y \cdot \left[\frac{Aw}{\sqrt{3}} + Zp \cdot \left[\frac{1}{L-a} + kz \cdot \left(\frac{2}{a} - \frac{1}{L-a}\right)\right]\right] \quad (39)$$

where:

ratio of local to total moduli $$kz = \frac{zp}{Zp}$$ (40)

The value of a will be that which minimizes the internal work. This is found by taking the derivative of IWD with respect to '*a*' and setting it to zero. This gives;

$$\frac{d}{da}\left[\frac{1}{L-a} + kz \cdot \left(\frac{2}{a} - \frac{1}{L-a}\right)\right] = 0 \quad (41)$$

Solving (41) for a/L, we get;

$$\frac{a}{L} = \frac{1}{2 \cdot (kz-1)} \cdot \left[4 \cdot kz - 2 \cdot \sqrt{2 \cdot kz^2 + 2 \cdot kz}\right]$$ exact (42)

This is the exact solution. An approximate solution is;

$$\frac{a}{L} = .64 \cdot kz^{.3333}$$ approximate (43)

When we plot the two equations (Figure 8), we see that equation (43) is a very good approximation to (42). Note that *a/L* is only a function of *kz*, so this comparison will hold for all cases.

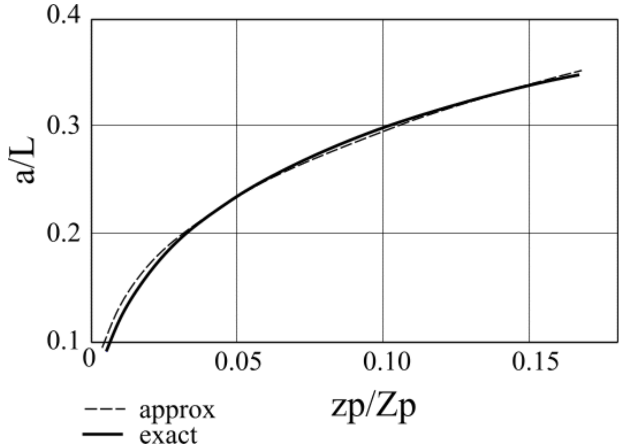


Figure 8. Comparison of exact and approximate values of a/L

Now, we can write the energy equation as;

$$(P\cdot b\cdot S)\left(1-\frac{b}{2\cdot L}\right)=\sigma_y\cdot\left[\frac{Aw}{\sqrt{3}}+\frac{Zp}{L}\cdot fz\right] \qquad (44)$$

where *fz* depends on *a/L* and *kz*. *fz* can be expressed as exactly (substituting (42) into (39) and solving exactly), near-exact (substituting (43) into (39) and solving exactly) or approximately (fitting the exact solution to a simpler equation)). The exact solution for *fz* is:

$$fz=\frac{-(kz-1)^2\cdot\sqrt{2}\cdot\sqrt{kz^2+kz}}{\left(\sqrt{2}\sqrt{kz^2+kz}-2\cdot kz\right)\cdot\left(\sqrt{2}\sqrt{kz^2+kz}-kz-1\right)} \qquad (45)$$

The near-exact solution is (uses approx *a/L*);

$$fz=\frac{1}{1-.64\cdot kz^{.3333}}+kz\cdot\left(\frac{2}{.64\cdot kz^{.3333}}+\frac{1}{1-.64\cdot kz^{.3333}}\right) \qquad (46)$$

The approximate solution for *fz* is;

$$fz=1.1+5.75\cdot kz^{.7} \qquad (47)$$

Equations (45), (46), (47) are plotted in Figure 9. The plot shows that all three equations are equivalent (<1% error). Again, as *fz* is only a function of *kz*, this comparison will be the same for all cases.

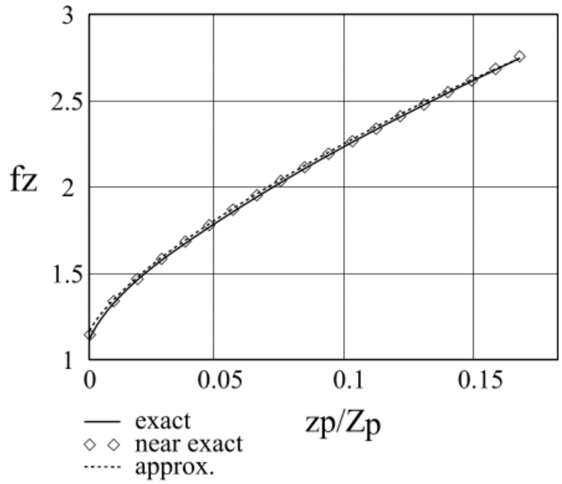


Figure 9. Comparison of formulae for fz.

The above derivations have enabled the development of a relatively simple energy equation, which accounts for the optimal load and hinge locations. Figure 9 shows that the simplifications have not diminished the accuracy of the solution. Thus, the energy balance equation takes a simpler form;

$$(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) = \sigma_y \cdot \left[\frac{Aw}{\sqrt{3}} + \frac{Zp}{L} \cdot (1.1 + 5.75 \cdot kz^{.7})\right] \qquad (48)$$

3.2 Interaction Equations for End Load Case

Equation (48) can be re-stated in a non-dimensional form, similar in form to the interaction equations for the 3-hinge case:

$$1 = \frac{Aw}{(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) \cdot \frac{\sqrt{3}}{\sigma_y}} + \frac{Zp}{(P \cdot b \cdot S)\left(1 - \frac{b}{2 \cdot L}\right) \cdot \frac{L}{\sigma_y}} \cdot (1.1 + 5.75 \cdot kz^{.7}) \qquad (49)$$

As in the case of 3 hinge, we define the minimum web area *Ao*: and Zo as before (equations (13), (17)). Using *Ao* and *Zo*, we can re-write the capacity equation as:

$$1 = \frac{Aw}{2 \cdot \left(1 - \frac{b}{2 \cdot L}\right) \cdot Ao} + \frac{Zp}{8 \cdot Zo} \cdot (1.1 + 5.75 \cdot kz^{.7}) \tag{50}$$

which is simplified to:

$$1 = \frac{An}{2 \cdot \left(1 - \frac{b}{2 \cdot L}\right)} + \frac{Zn}{8} \cdot (1.1 + 5.75 \cdot kz^{.7}) \tag{51}$$

where *Zn* and *An* are normalized values from Equations (19) and(20). By re-arranging (51), we get the interaction equation;

$$Zn = \frac{8}{(1.1 + 5.75 \cdot kz^{.7})} \cdot \left[1 - \frac{An}{2 \cdot \left(1 - \frac{b}{2 \cdot L}\right)}\right] \tag{52}$$

This interaction equation is a straight line. The *y*-intercept is defined as *Zm*, and the *x*-intercept is *Am*. The equation can be written as;

$$Zn = Zm \cdot \left(1 - \frac{An}{Am}\right) \tag{53}$$

where:

y-intercept (modulus axis) $$Zm = \frac{8}{(1.1 + 5.75 \cdot kz^{.7})} \tag{54}$$

x-intercept (shear area axis) $$Am = 2 \cdot \left(1 - \frac{b}{2 \cdot L}\right) \tag{55}$$

Note that the interaction diagram depends on *Am* (which depends only on the patch/span ratio b/L) and *Zm* (which depends only on the modulus ratio *kz=zp/Zp*). This allows us to add a *kz* axis to the modulus axis, and a *b/L* axis to the shear area axis.

The interaction equation (53), shows some interesting properties. The equation is linear, meaning that an increase in web area will result in a constant decrease in the section modulus requirement. Two extreme cases (neither actually possible) involve a shear area of *Am*, with zero modulus, or a section modulus of *Zm* with zero web area. In the first case the entire load would be carried by the web (the plastic shear panel, with no load transmitted to the far end). At the other extreme, with no shear capacity, all the load would be carried by the far end, as a plastic cantilever beam. For all realistic cases the load is carried in three ways; through the shear panel in the web to the near end, through the plate and flange to the near end, and through the full frame to the far end. A trade-off among the web, flanges and full modulus allow for various possible designs.

The interaction plot for shear is illustrated in Figure 10. Two cases are illustrated to show a range of possible interaction equations for the shear mechanism. Also shown are interaction equations for the 3-hinge collapse mechanism (for the centered load). The three hinge case will require Aw/Ao and Zp/Zp to both be greater than one. The two example 3-hinge interaction curves cover a range of possibilities (the upper is for a flat bar and other is a typical flanged frame). It is clear that there will be cases in which the shear mechanism will never govern. In the case where the load length covers most or all of the frame (b/L ~1), the shear curve will always lie below the 3-hinge curves. In the case of a very concentrated load, especially on a frame with small flanges, shear collapse will more likely govern.

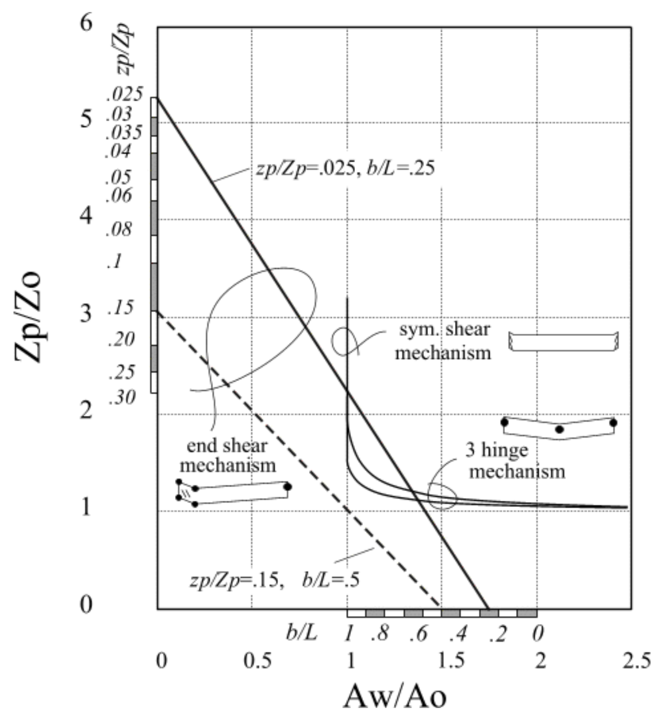


Figure 10. Interaction plot for asymmetrical shear collapse. The equation depends on the load patch length and on the ratio *zp/Zp*. The 3-hinge interaction is also shown.

3.3 Capacity Equation for Shear Collapse

The capacity equation is just a re-arrangement of (48);

$$Ps = \frac{\sigma_y}{b \cdot S\left(1 - \frac{b}{2 \cdot L}\right)} \cdot \left[\frac{Aw}{\sqrt{3}} + \frac{Zp}{L} \cdot (1.1 + 5.75 \cdot kz^{.7})\right] \qquad (56)$$

4. COMPARISON WITH PLASTIC FE ANALYSIS

Two frames were analyzed using non-linear finite element analysis. In both cases, a patch load was applied in the center. The frames are shown in Figure 11:

The FE load-deflection curves for the central load are plotted in Figure 12. Also shown are the values calculated from equation (23) (note: Equation (23) only gives a load, not a deflection.) The calculated values agree very well with the onset of large permanent deformations.

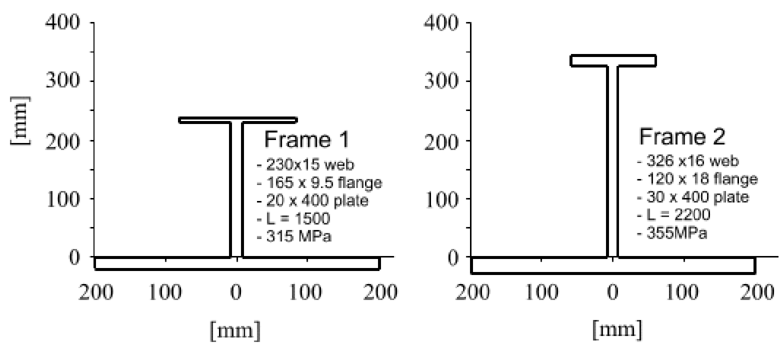


Figure 11. Frames used in validation exercise.

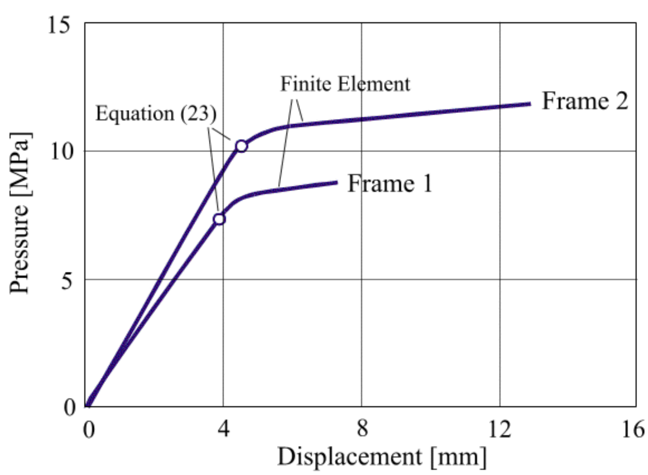
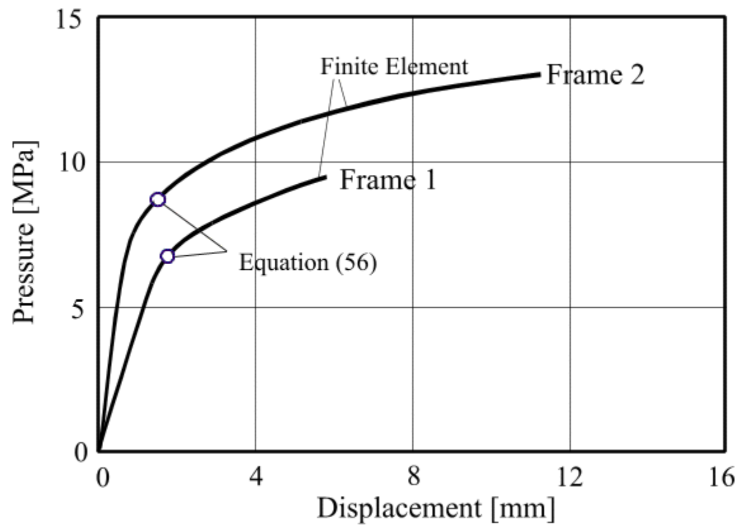


Figure 12. Comparison of equations with finite-element results for central load case.

The FE load-deflection curves for the end load case and the values calculated from equation (56) are shown below (see Figure 7). The calculated values agree very well with the onset of large permanent deformations.



note: the Frame 1 pressure was applied over 300 mm, while the Frame 2 pressure was applied over 710 mm.

Figure 13. Comparison of Equation (56) with finite-element results.

The finite element analyses show that frames no not actually collapse when plastic mechanisms form. This is due to the axial forces and large deformations, which are not considered in the energy methods. The equations produced by the energy methods are useful as design equations, representative of the onset of large deflections and large strains.

5. RULE EQUATIONS

The above derivations lead to the following rule equations. For checking the symmetrical load case, equation (13) gives the minimum shear area *Ao*. The required section modulus *Zp*, equivalent to equation (21), which with slight re-arrangement can be written as:

$$Zp = \frac{(P \cdot b \cdot S \cdot L)}{4 \cdot \sigma_y} \cdot \left(1 - \frac{b}{2 \cdot L}\right) \cdot A1 \qquad (57)$$

where:

$$A1 = \frac{1}{1 + \frac{j}{2} + \frac{j}{2} \cdot kw \cdot \left[\sqrt{1 - \left(\frac{Ao}{Aw} \right)^2} - 1 \right]} \tag{58}$$

with *kw* is defined by Equation (16), and *j* is the number of fixed supports (normally 2).

The check that adequate capacity exists to prevent shear collapse with an asymmetrical load results from re-arranging (52). The additional formula for *Zp*, to augment the central load case is;

$$Zp = \frac{P \cdot b \cdot S \cdot L}{4 \cdot \sigma_y} \cdot \left(1 - \frac{b}{2 \cdot L} \right) \cdot A2 \tag{59}$$

where:

$$A2 = \frac{\left[1 - \frac{1}{2 \cdot \frac{Ao}{Aw} \cdot \left(1 - \frac{b}{2 \cdot L} \right)} \right]}{(.275 + 1.438 \cdot kz^{.7})} \tag{60}$$

Equations (57) and (59) are identical except for the *A1* and *A2*.

6. CONCLUSION

The derivations of the rule equations for checking framing modulus and shear strength have been presented. The final equations for both central 3-hinge collapse and end load shear collapse both show an interaction between bending and shear strength. This allows designers to trade off shear area against section modulus. The interactions can be seen in the interaction plots presented. Optimal and effective design requires an understanding of these effects, in combinations with the structural stability (tripping and buckling) rules and the constraints of available sections geometry.

The new Unified Requirement makes a significant step forward in expressing the load and structural response in realistic terms. The loads reflect actual measured values, and the structural equations reflect both plastic behavior and interaction effects. This, the author believes, lends a greater level of safety and certainty to these rules.

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