Name: Sample Solutions Student #: ____________________

Engineering 4892 — Data Structures
Quiz 2

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Instructions: Answer all questions. Write your answers on this paper. This is a closed book test, no textbooks, notes, calculators or other aides are permitted.
Total points: 50

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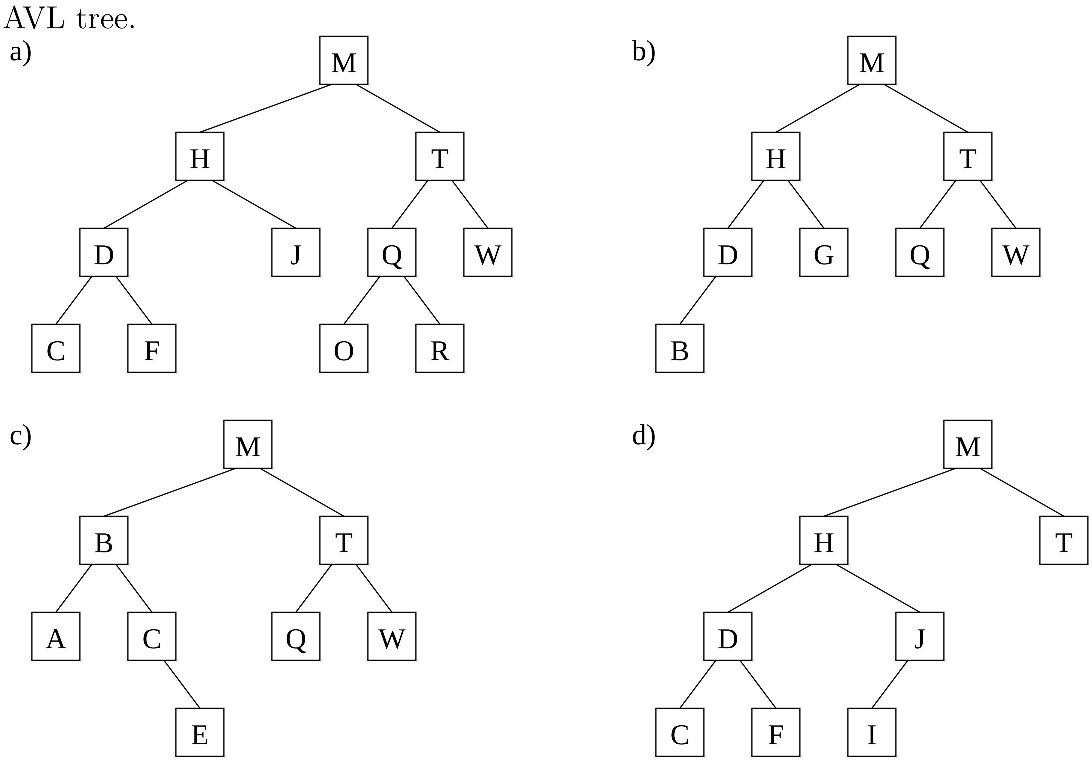
1. [8 points] For each of the following trees, fill in "true" or "false" in the appropriate cells of the table below to indicate if the tree is full, complete, a binary search tree and an AVL tree.



	Full	Complete	Binary Search	AVL
(a)	*True*	*False*	*True*	*True*
(b)	*False*	*True*	*False*	*False*
(c)	*False*	*False*	*True*	*True*
(d)	*False*	*False*	*True*	*False*

2. [14 points] Consider the following partial declaration of a doubly-linked **sorted** list class template. (The questions are on the following page.)

```
template <class T>
class SortedList
{
  public:
  // A list of type T.
  // Modeled by:
  //   L sequence of T
  //
  // INV: L is sorted in non-decreasing order.

  // Local types
    enum Status { Ok, NewFail, NoSuchElement };

  // Constructors
    SortedList();
    // Post: L = _

    SortedList(const SortedList<T>& r);
    // Post: L' = r.L

  // Destructor
    ~SortedList();

  // Accessors
    // ...
    Status getStatus() const { return err; }
    // Post: Result = status of last access

    int find(const T& e) const;
    // Pre: L is sorted in non-decreasing order.
    // Post: if e is in L then result = the index of the first occurance of e
    //                         and err' = Ok
    //       if e is not in L then err' = NoSuchElement

  // Mutators
    // ...

  private:
    // Representation:
    // head != 0 -> L = { head->data, head->next->data, ...  } /\
    // head = 0 -> L = _

    class Node {
      public:
        T data;
        Node* next;
        Node* prev;
        Node(T d = 0, Node *n = 0, Node* p = 0)
          : data(d), next(n), prev(p) { }
          // Note this assumes that T has a copy constructor.
    };
    Node* head;
    Node* tail;

    mutable Status err;   // Status of the last call.
};
```

a) [10 points] Give an implementation of the `find` method such that it satisfies the given specification. Your function should stop searching as soon as a match is found or there is no possibility of finding one.

```
template <class T>
int find(const T& e) const
// Pre: L is sorted in non-decreasing order.
// Post: if e is in L then result = the index (zero based) of the first
//                                   occurance of e
//                          and err' = Ok
//       if e is not in L then err' = NoSuchElement
{
  int index = 0;
  Node* cur = head;
  while (cur != 0 && cur->data < e) {
    cur = cur->next;
    index++;
  }
  if (cur != 0 && cur->data == e) {
    err = Ok;
  } else {
    err = NoSuchElement;
  }
  return index;
}
```

b) [2 points] What is the time complexity (in big-Oh notation) of your `find` method? (Be sure to specify what n represents.)

$O(n)$ where n is the length of the list.

c) [2 points] The `insert` method must ensure that the class invariant is kept true (i.e., that the list is sorted in non-decreasing order) by inserting elements in the correct position of the list. What is the time complexity (in big-Oh notation) of this algorithm? (Again be sure to specify what n represents.)

$O(n)$ where n is the length of the list.

3. [10 points] The number of 1's in a binary representation of a natural number, N, is equal to the number of 1's in the binary representation of $N/2$ plus 0 if N is even, or 1 if N is odd. For example, the binary representation of 6 is 110, which contains two 1's, as does the binary representations of 3 (11), which is 1+ the number of 1's in the binary representation of 1. Consider a recursive function `int numOnes(int n)` that computes this number.

a) [1 points] What is the base case for the recursion?

$n = 0$

b) [1 points] What is the variant expression for the recursion?

n

c) [8 points] Give an implementation of the function.

```
int
numOnes(int n)
{
  int result = 0;
  if (n > 0) {
    result = numOnes(n/2) + n%2;
  }
  return result;
}
```

4. [18 points] Consider the following partial implementation of a binary tree class. (Note: **Not** a binary *search* tree.)

```
template <class T>
class BinaryNode // The node of the binary tree
{
  public:
    T key;
    BinaryNode* left;
    BinaryNode* right;
    BinaryNode(BinaryNode *l = 0, BinaryNode* r = 0)
      : left(l), right(r) { }
    BinaryNode(const T& k, BinaryNode *l = 0, BinaryNode* r = 0)
      : key(k), left(l), right(r) { }
};

template <class T>
class BinaryTree
{
  public:
    enum Status { Ok, NewFail, NoSuchElement, DuplicateElement };
  // Constructors
    BinaryTree();
    // Post: empty tree constructed

    BinaryTree(const BinaryTree<T>& r);
    // Post: tree is a copy of r

  // Destructor
    virtual ~BinaryTree();

  // Mutators
    Status insert(const T& x);
    // Post: x is inserted in the tree keeping it optimally filled

    // ...

  protected:
    BinaryNode<T>* root;
    mutable Status err;   // Status of the last call.

  private:
    Status rInsert(BinaryNode<T>*& r, const T& x);
    int rMinSpace(BinaryNode<T>* r) const;
    // ...
};

/******************************************************************
 * insert -- insert an element in the tree, keeping it filled
 *
 * Post:
 ******************************************************************/
template <class T>
BinaryTree<T>::Status
BinaryTree<T>::insert(const T& x)
{
  return rInsert(root, x);
}
```

a) [8 points] An alternative technique for deciding where to insert new nodes is to look for the **shortest** path from the root to an empty sub-tree (i.e., a 'hole' where a node can be inserted) and insert there. Give a *recursive* implementation for `rMinSpace` such that it returns the number of nodes in the shortest path to an empty sub-tree of `r`. (You may use the STL template function `T min(T l, T r)`, which returns the minimum of its arguments.)

```
template <class T>
int
BinaryTree<T>::rMinSpace(BinaryNode<T>* r) const
{
  int result = 0;
  if (r != 0) {
    result = 1 + std::min(rMinSpace(r->left), rMinSpace(r->right));
  }
  return result;
}
```

b) [2 points] What is the time complexity (in big-Oh notation) of your `rMinSpace` function, above? Be sure to state what n represents.

$O(n)$ where n is the number of nodes in the tree.

c) [8 points] Give a *recursive* implementation of `rInsert` that inserts in the sub-tree containing the closest 'hole', or the left sub-tree if the distance to the closest hole is the same in both sub-trees.

```
template <class T>
BinaryTree<T>::Status
BinaryTree<T>::rInsert(BinaryNode<T>*& r, const T& x)
{
  if (r == 0) {
    r = new(std::nothrow) BinaryNode<T>(x);
    if (r != 0) {
      err = Ok;
    } else {
      err = NewFail;
    }
  } else if (rMinSpace(r->left) <= rMinSpace(r->right)) {
    err = rInsert(r->left, x);
  } else {
    err = rInsert(r->right, x);
  }
  return err;
}
```