1) The joint probability mass function $p(x, y)$ for random quantities $X, Y$ is defined by the table:

| | $p(x, y)$ | $Y$: –1 | 0 | 1 | |
|---|---|---|---|---|---|
| $X$ | –1 | .06 | .09 | .15 | |
| | 0 | .10 | .15 | .25 | |
| | 1 | .04 | .06 | .10 | |
| | | | | | |

(a) Verify that $p(x, y)$ is a valid probability mass function. [2]

(b) Find the correlation coefficient $\rho_{X, Y}$. [7]

(c) Are the random quantities $X, Y$ independent? Why or why not? [4]

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2) Lamps from a certain factory are known to have lifetimes $T$ that are independent random quantities following an exponential distribution with a mean lifetime of 10,000 hours.

(a) Show that the probability $p$ that a randomly chosen lamp has a lifetime exceeding 23,026 hours is 0.100 00, correct to five decimal places. [4]

(b) A random sample of ten such lamps is tested. Let $X$ be the number of lamps in this sample that have lifetimes exceeding 23,026 hours. Does $X$ follow a binomial distribution exactly, approximately or not at all? Justify your answer. [2]

(c) Assume that $p = 0.1$ exactly. Write down the value of $\mathrm{E}[X]$. [2]

(d) Find $\mathrm{P}[X < 2]$. [3]

(e) Another random sample of 100 lamps is tested. Estimate the probability that the sample mean lifetime $\overline{T}$ will be less than 9,000 hours. [4]

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3) Two percent of all items from a production line are known to be defective. [10]
A quality control process rejects a defective item 99% of the time and it rejects a good (non-defective) item 5% of the time.

Given that the quality control process has just rejected an item, find the odds that the item is, indeed, defective.

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4) A cumulative distribution function $F(x)$ of a continuous variable $x$ is defined by

$$F(x) = \begin{cases} 0 & (x < 0) \\ 21x^5 - 35x^6 + 15x^7 & (0 \le x \le 1) \\ 1 & (x > 1) \end{cases}$$

(a) Evaluate $\mathrm{P}\left[X > \frac{1}{2}\right]$ exactly. Leave your answer as a fraction. [6]

(b) Find the probability density function (p.d.f.) for this c.d.f. in its simplest form; [that is, factor $f(x)$ as much as possible.] [6]

*BONUS QUESTION*:

(c) Find the population mean $\mu$ as a fraction reduced to its lowest terms. [+3]

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