1. Ordinary Differential Equations

Equations involving only one independent variable and one or more dependent variables, together with their derivatives with respect to the independent variable, are ordinary differential equations (**ODE**s). Similar equations involving derivatives and more than one independent variable are partial differential equations (to be studied in a later term).

Contents:

Appendix [not examinable]

The focus of the first tutorial is partial fractions. That material will be posted on the Brightspace (D2L) site for this course after the tutorial has taken place.

1.1 Integration by Parts

Two techniques from MATH 1001 need review, before we look at first order ordinary differential equations: partial fractions (in the first tutorial) and integration by parts.

Let $u(x)$ and $v(x)$ be functions of x. Then, by the product rule of differentiation,

$$\frac{d}{dx}(uv) = \frac{du}{dx}\cdot v + u\cdot\frac{dv}{dx}$$

Integrating with respect to x:

$$[uv] = \int u'v\,dx + \int uv'\,dx$$

This leads to the formula for integration by parts:

$$\boxed{\int uv'\,dx = [uv] - \int u'v\,dx}$$

Example 1.1.1

Find $I = \int x^3 e^{-x^2}\,dx$.

Example 1.1.2 [Repeated use of integration by parts]

Find $I = \int x^2 \cos x \, dx$.

Example 1.1.2 (continued)
Shortcut (a tabular form for repeated integrations by parts):

$I = \int x^2 \cos x \, dx$:

Example 1.1.3

Find $I = \int x^4 e^x \, dx$.

Example 1.1.4 (recursive use of integration by parts)

Find $I = \int e^{ax} \sin bx \, dx$, (where a, b are constants).

Example 1.1.4 (continued)

Example 1.1.5

Find $I_n(x) = \int x^n \ln x \, dx$, where n is any number except –1.

The derivative of $\ln x$ is known, but not the antiderivative (unless integration by parts is used!). This is therefore a rare instance where a polynomial factor is ***not*** in the left [differentiation] column.