ENGI 4421 Probability and Statistics
Faculty of Engineering and Applied Science

Problem Set 3

Probability; Counting Techniques

In questions 1-4, evaluate manually and show your working:

1. $^{10}C_3$

---

2. $^{10}P_3$

---

3. $^{11}C_2$

---

4. $^{200}C_{198}$

---

5. A truck is carrying six steel pipes and three copper pipes.
A random sample of four pipes is taken [without replacement] from the truck.
Find (and show your working):
   (a) the number of ways there can be **exactly** two copper pipes in the sample.
   (b) the number of ways there can be **at least** two copper pipes in the sample.

---

6. Just as the binomial coefficient

$$^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

counts the number of distinct partitions of n items into two piles, one of r items and the other of the remaining $(n - r)$ items, so the multinomial coefficient

$$\binom{n}{r_1\ r_2\ r_3 \cdots r_k} = \frac{n!}{r_1!\,r_2!\,r_3!\cdots r_k!}$$

counts the number of distinct partitions of n items into k piles, one of r_1 items, another of r_2 items, another of r_3 items and so on, (where, in this case, $r_1 + r_2 + r_3 + \ldots + r_k = n$). [The binomial coefficient is then a special case of the multinomial coefficient, with $k = 2$, $r_1 = r$ and $r_2 = n - r$.]

Use the appropriate multinomial coefficient to find the number of distinct ways in which a manager can divide a work crew of twelve employees into groups of three, four and five people.

---

7. Find the number of distinct ways of rearranging the letters in the word "STATISTICS" among themselves.

---

8. A bag contains four tiles, each marked with a letter, one each of A B C D .
   (a) If the four letters A B C D are drawn at random **without** replacement, what is the probability that the letter B will be drawn immediately after the letter A ?
   (b) If the four letters A B C D are drawn at random **with** replacement, what is the probability that the letter B will be drawn immediately after the letter A ?

---

9. In a competition to award two identical contracts, five firms are tied with the lowest acceptable tender. It is decided to award the contracts to exactly two of the five firms by a random process, so that each pair of firms has an equal chance of being chosen.
   The five firms have 4, 11, 12, 15 and 19 complete years' experience respectively.

   [Note that two firms with 5 years 11 months' experience would each have only five complete years and a total between them of only ten complete years, not eleven or twelve. The quantity "years" in this question is therefore a discrete quantity, taking on non-negative integer values only.]

   Find the probability that the two chosen firms have a total of
   (a) at least 25 complete years' experience.
   (b) more than 30 complete years' experience.

[*BONUS QUESTION*]

10. Three identical boxes each contain two marbles. One box contains two red marbles, another box contains two blue marbles and the remaining box contains one of each. All three boxes are closed and are labelled incorrectly.
    You are permitted to select any box and to withdraw one marble (without replacement) from that box. You may repeat this process of selecting any box and withdrawing a marble without replacement as many times as necessary, until you are able to deduce the true contents of all three boxes.
    What is the *least number of marbles* that you must withdraw in order to be sure of identifying the contents of all three boxes correctly?
    Justify your answer *with full working.*

[*BONUS QUESTION*]

11. A standard deck of 52 playing cards contains four suits, each of thirteen ranks (ace, king, queen, jack and 10 down to 2). When five cards are dealt from this deck, find the odds that four of these five cards are all of the same rank (a hand known as "four of a kind" or "quads").

☜ Back to the index of questions On to the solutions to this problem set ☞