ENGI 4421 Probability and Statistics
Faculty of Engineering and Applied Science

# Problem Set 9

## Hypothesis Tests

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1. A circuit breaker is rated to cut the power supply when the current passing through it reaches 20 A. If the circuit breaker trips at a current below the rated value of 20 A, then customers will complain that it trips too often without good cause. If the circuit breaker trips only at a current above the rated value of 20 A, then the manufacturer risks liability for the extensive damage that electrical overloads can cause.

   A random sample of 100 newly-produced circuit breakers is tested to check whether or not the mean trip current is at the rated value of 20 A. The sample mean trip current is 19.7 A and the sample standard deviation is 0.62 A.

   (a) State the appropriate null and alternative hypotheses.
   (b) Is there sufficient evidence to assert that the true mean trip current is not at the rated value?

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2. A production process gives components whose strengths are normally distributed with a mean of 400 N and a standard deviation of 11.5 N. A modification is made to the process which cannot reduce but may increase the mean strength. It may also change the variance. The strengths of nine randomly selected components from the modified process, in Newtons, are:

   396 402 409 409 414 398 394 436 418

   Test, at a five per cent level of significance, the hypothesis that the mean strength has not increased.

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3. A transport firm is very suspicious of the tyre company's claim that the average lifetime of its tyres is at least 45,000 kilometres. The transport company decides to check this claim by fitting forty of these tyres to its trucks, the tyres being a random sample.

   A mean lifetime of 44,164 km with a sample standard deviation of 2,106 km is observed. What may the transport firm conclude about the claim at a level of significance of one per cent?

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4. A garage wants to know if a more expensive type of radial tyre has a tread life significantly more than 10,000 km beyond the tread life of a cheaper bias-ply tyre. Only if this is the case will the garage invest in the more expensive type of tyre. A random sample of forty tyres of each type is tested and the tread lives are measured. The radial tyres have a mean tread life of 36,500 km with a standard deviation of 2,200 km, while the bias ply tyres have a mean tread life of 23,800 km with a standard deviation of 1,500 km.

   Based on these data, should the garage invest in the radial tyres?

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5. A designer claims that a new type of hull increases the average sustained speed of a speedboat by more than 2 km/h over the average sustained speed of the existing hull design. Random samples of speedboats of the two designs are tested and their sustained speeds (in km/h) are measured on a test course under identical water conditions:

   **New design ($x_A$):**
   42 36 38 37 39 36

   **Old design ($x_B$):**
   30 37 33 31 34

   Conduct an appropriate hypothesis test at a level of significance of 5%. Is there sufficient evidence to accept the designer's claim? What assumptions have you made?

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6. Random samples are drawn from two independent populations, producing the following summary statistics:

$$X: \quad n_X = 50 \qquad \bar{x} = 643 \qquad s_X = 26$$
$$Y: \quad n_Y = 90 \qquad \bar{y} = 651 \qquad s_Y = 32$$

   Are these data consistent with the hypothesis that the two population means are equal?

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7. [Bonus question, to provide practice in the supplementary topic of type II error probabilities.]
Find the probability of committing a type II error when the true population mean is $\mu = 104$ and an upper-tail hypothesis test is conducted at a level of significance of five per cent with a random sample of size 25 on the null hypothesis that $\mu = 100$. It is known that the population variance is 100.
Repeat your calculation in the case when the level of significance of the hypothesis test is one per cent.

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8. [Bonus question, to provide practice in the supplementary topic of type II error probabilities. This question is a modification of Devore, 7th ed., Ch. 8.3, p. 310, q. 39.]

A university library ordinarily has a complete shelf inventory done once every year. Because of new shelving rules instituted the previous year, the head librarian believes it may be possible to save money by postponing the inventory. The librarian decides to select 800 books at random from the library's collection and to have them searched in a preliminary manner. If the evidence indicates strongly that the true proportion of misshelved or unlocatable books is less than .02, then the inventory will be postponed.

(a) Among the 800 books searched, 12 were misshelved or unlocatable. Test the relevant hypotheses (at a level of significance of .05) and advise the librarian what to do.
(b) If the true proportion of misshelved and lost books is actually .01, what is the probability that the inventory will be [unnecessarily] taken?
(c) If the true proportion is .05, what is the probability that the inventory will be postponed?
(d) What types of errors are the events described in parts (b) and (c) above?

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