ENGI 4421 Probability and Statistics

Faculty of Engineering and Applied Science

Some Practice Questions for Test 1

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1. Note that this question attempts to cover various aspects of descriptive statistics. In the actual test, there will be time to test only a small portion of what is presented here. Only the summary statistics and/or a boxplot or histogram, (not the full data set shown here), will be presented in a question on the test.

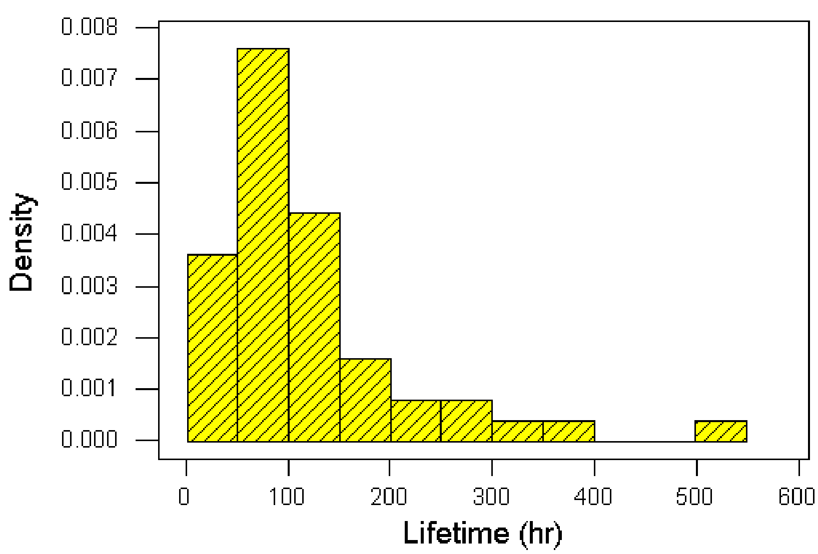
The actual time, in hours, to failure of a prototype mechanical component in a turbine, is measured on fifty occasions in an experiment. The raw results are displayed here, sorted into increasing order:

11	14	20	23	31	36	39	44	47	50
59	61	65	67	68	71	74	76	78	79
81	84	85	89	91	93	96	99	101	104
105	105	112	118	123	136	139	141	148	158
161	168	184	206	248	263	289	322	388	513

The summary statistics include $n = 50$, $\sum x = 5963$, $\sum x^2 = 1176795$

(a) From the summary statistics, calculate the sample mean and sample standard deviation for these data.

(b) From the histogram below, identify the modal class.



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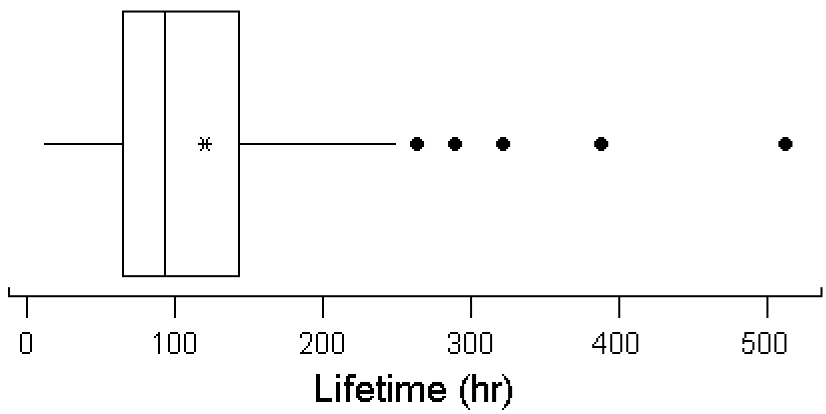
1 (c) What evidence is there, from the histogram, for skewness?

(d) Use the histogram only, to estimate the number of components in the sample, whose lifetimes are less than 50 hours.

(e) Find the median value from the original data.

(f) The asterisk in the boxplot below denotes the location of the sample mean. Describe the evidence for skewness that you can see in the boxplot.



(g) List all the outliers.

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2. A box contains 11 different [distinguishable] gear wheels. In how many ways can 3 gear wheels be drawn from the box, if they are drawn

(a) with replacement (order of selection matters)?

(b) without replacement (order of selection matters)?

(c) without replacement (order of selection is irrelevant)?

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3. Each of 12 motors of a certain type has been returned to a distributor because of the presence of a low-frequency vibration when the motor is running at its rated speed. Suppose that four of these 12 have defective bearings and the other eight have less serious problems. If they are examined in random order, let X = the number among the first six examined that have defective bearings. Compute

(a) $\mathrm{P}[X = 1]$

(b) $\mathrm{P}[X \geq 4]$

(c) $\mathrm{P}[1 \leq X \leq 3]$

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4 (a) An engineer states that the odds of a prototype microchip surviving a current of 3 μA for 2 hours is "4 to 1 against". What is the engineer's probability for this event?

(b) An engineer states that the probability of another prototype microchip surviving a current of 2 μA for 1 hour is .35 . What is the engineer's odds for this event?

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5. [Devore 3rd ed., ex. 2.4, q. 47, p. 67, modified]

A mathematics professor teaches both morning and afternoon sections of a course.
Let A = {the professor gives a bad morning lecture}
and B = {the professor gives a bad afternoon lecture}.
If $\mathrm{P}[A] = .3$, $\mathrm{P}[B] = .2$ and $\mathrm{P}[A \wedge B] = .1$, then calculate the following probabilities (a Venn diagram might help) **and** calculate the equivalent odds:

(a) $\mathrm{P}[B \mid A]$
(b) $\mathrm{P}[\sim B \mid A]$
(c) $\mathrm{P}[B \mid \sim A]$
(d) $\mathrm{P}[\sim B \mid \sim A]$
(e) If, at the conclusion of the afternoon class, the professor is heard to mutter "what a rotten lecture", then what is the probability that the morning lecture was also bad?

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6. Fair Odds vs. Bookmaker's Odds [Bonus material]

In a five horse race, you can place a bet of \$10 on event E_i (= horse i wins). If that event occurs, then you win the bookmaker's stake that is based on the quoted odds. The bookmaker quotes odds of

$$r_1 = \text{ 5 to 4 on,} \quad r_2 = \text{ 3 to 1 against,} \quad r_3 = \text{ 7 to 2 against,} \quad r_4 = r_5 = \text{ 17 to 1 against}$$

(a) Are these odds fair? *Justify your answer* by determining whether or not the probabilities associated with these odds are coherent.
(b) What are the bookmaker's least and greatest profits if only one bet is placed on each of all five horses?
(c) What are the bookmaker's least and greatest profits if the numbers of bets placed are 40, 18, 16, 4 and 4 on each of horses i = 1, 2, 3, 4, 5 respectively?
(d) What should the quoted odds be if this set of probabilities is re-scaled to be coherent?

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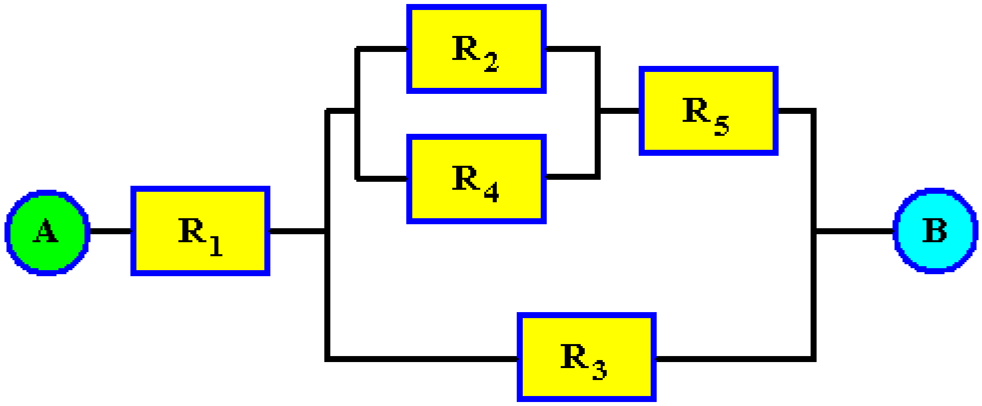
7. An electronic [or structural] system consists of five electronic [or structural] components arranged as follows:



Each component is operative or fails under load. The probability of failure for each individual component is .01. The entire assembly fails only if the path from A to B is broken. The sample space S consists of all possible arrangements of operative and inoperative components.

(a) Show that $n(S) = 32$.
Let E_1 = "the assembly is operative";
E_2 = " R_2 has failed but the assembly is operative";
E_3 = " R_3 has failed but the assembly is operative";
and F = "the assembly has failed".

(b) Are E_1 and E_2 mutually exclusive?

(c) Are E_2 and E_3 mutually exclusive?

(d) Write down all the sample points in the event F, (that is, list all arrangements of operative and inoperative components that lead to the failure of the entire assembly). Hence find $n(F)$.

(e) Are the sample points in the event F equally likely?

(f) What is the probability that the assembly fails?
[You may assume that component failures are independent of each other]

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8. There are seven candidates in an election for three officers on the executive committee of a club. In how many distinct ways can the voting members of the club fill the three vacancies if

(a) the three officer positions are identical (for example, they are all committee members without specific office);

(b) the three officer positions are distinct (for example, the person with the most votes becomes president and the runner-up becomes vice president);

9. A certain rare disease is known to occur in 1% of the population.
A diagnostic test exists for this disease, but the test is not perfect.
If a person has the disease, then the test will [correctly] detect the disease 98% of the time.
If a person does not have the disease, then the test will [incorrectly] claim a detection of the disease 10% of the time.
Given a positive test result [implying that the person has the disease], what are the odds that the test result is correct, [the person really does have the disease]?

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10. Adam Ant is determined to return to his home `I` from his Aunt's house `A` over the lattice of twigs shown below.

```
A ---- B ---- C
|      |      |
|      |      |
D ---- E ---- F
|      |      |
|      |      |
G ---- H ---- I
```

Being only a young ant, he can only move south (down the page) or east (right) at each junction. Where there is a choice, Adam is equally likely to choose each of the two twigs.

(a) Write down the complete list of outcomes.
(For example, one of the possible outcomes is the path `ABEFI`).

(b) Let X represent the number of junctions along the path at which Adam has a choice of twig. Write down the complete set of possible values of X .

(c) For each possible value of X, find the probability that that value of X occurs.
[Be careful: P[X = 2] is **not** 1/3 !]

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