# ENGI 4421 Probability and Statistics

Faculty of Engineering and Applied Science

# Some Practice Questions for Test 2

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1. The probability mass function for $X$ = "the number of major defects in a randomly selected appliance of a certain type" is

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $p(x)$ | .08 | .15 | .45 | .27 | .05 |

Compute

(a) $\mathrm{E}[X]$

(b) $\mathrm{V}[X]$ directly from the definition

(c) the standard deviation of $X$.

(d) $\mathrm{V}[X]$ using the shortcut formula

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2. The discrete random quantity $X$ has the following probability mass function:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p(x)$ | .1 | .2 | .3 | .4 |

(a) Verify that $p(x)$ is a valid p.m.f.

(b) Sketch the cumulative distribution function $F(x)$.

(c) Find the population mean of $X$.

(d) Find the population variance of $X$.

Now suppose that a random sample of size 100 is drawn from this population.

(e) Find the probability that the sample mean is less than 2.9 .

(f) Find the probability that the sample mean is less than 2.0 .

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3. [Modification of Devore, 3rd edition, ex. 3.2, q. 22, p. 93]

A small town situated on a main highway has two gas stations, A and B. Station A sells regular, extra, and premium gas for 139.9, 141.1, and 144.4 cents per litre, respectively, while B sells no extra, but sells regular and premium for 138.9 and 144.4 cents per litre, repectively. Of the cars that stop at station A, 50% buy regular, 20% buy extra, and 30% buy premium. Of the cars stopping at B, 60% buy regular and 40% buy premium. Suppose that A gets 60% of the cars that stop for gas in this town, while 40% go to B. Let $R$ = the price per litre paid by the next car that stops for gas in this town.

(a) Compute the p.m.f. of $R$. Then draw a probability bar chart of the p.m.f.
(b) Compute and graph the c.d.f. of $R$.
(c) If a randomly chosen customer has purchased regular gas, then what is the probability that the gas was purchased at station A?

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4. A box of silicon wafers will be withdrawn from a production line for more intensive testing, if and only if more than one defective wafer is found in a random sample of 20 wafers, drawn with replacement from the box.
Let $p$ be the true proportion of defective wafers in the box.
Let $X$ be the number of defective wafers in the random sample.

(a) Show that the probability distribution for $X$ is exactly binomial.
(b) Find, in terms of $p$, the probability that the box will be withdrawn from the production line for more intensive testing.
(c) Evaluate the probability in part (b) in the case when $p = .05$.

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5. The joint probability mass function $p(x, y)$ for a pair of discrete random quantities $X$, $Y$ is defined by the following table.

| ***p*** | | *y* | | |
|---|---|---|---|---|
| | | –1 | 0 | 1 |
| | –1 | .10 | .05 | .10 |
| *x* | 0 | .15 | .20 | .15 |
| | 1 | .10 | .05 | .10 |

(a) Find the correlation coefficient $\rho$ for $X$ and $Y$.
(b) Are $X$ and $Y$ stochastically independent?

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6. The probability density of a random quantity $X$ is given by:

$$f(x) = \begin{cases} kx\left(4-x^2\right) & (0<x<2) \\ 0 & (\text{elsewhere}) \end{cases}$$

(a) Find the value of $k$.
(b) Find the corresponding cumulative distribution function $F(x)$
(c) What are the ***odds*** that a random quantity having this distribution will take on a value greater than 1?

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7. Suppose that the duration (in years) of a construction job can be modelled as a continuous random quantity $t$ whose cumulative distribution function (c.d.f.) is given by

$$F(t) = \begin{cases} 0 & (t<0) \\ 2t-t^2 & (0 \le t \le 1) \\ 1 & (t>1) \end{cases}$$

(a) Find the probability that the construction job lasts longer than half a year.
(b) Find the median duration $\tilde{\mu}$, correct to two decimal places.
(c) Determine the corresponding probability density function $f(t)$.

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8. Space craft are to be landed at some given point "O" on the moon's surface, and we wish to observe the position of an actual landing point relative to the aiming point O. Assume that the landing errors are small enough so that it is reasonable to measure them in the tangent plane to the moon's surface which touches O. Let us further establish a local rectangular coordinate system in that tangent plane which is centered on O and which is oriented so that the $x$ axis points east and the $y$ axis points north.

Suppose that the probability that the space craft lands more than 10 km from the aiming point O is zero and, further, that the probability that the spacecraft lands in some particular region (no point of which is more than 10 km from O) is directly proportional to the area of that region.

Find the probability that the space craft will land
(a) less than 5 km from O
(b) more than 1 km yet less than 3 km from O
(c) north of the aiming point
(d) north-east of the aiming point
(e) south of the aiming point and more than 5 km from O
(f) exactly due south of O.

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9. Let $X$ have the Pareto probability density function

$$f(x) = \begin{cases} \dfrac{k\,t^k}{x^{k+1}} & (x \ge t) \\ 0 & (x < t) \end{cases}$$

where $k$ and $t$ are both positive parameters.

(a) Find necessary conditions on $k$ to ensure that $\mathrm{E}\left[X^n\right]$ is finite; ($n$ = a positive integer).

(b) Hence find the range of values of $k$ for which the variance $\mathrm{V}[X]$ is finite.

(c) Find an expression (in a simplified form) for the variance when it is finite.

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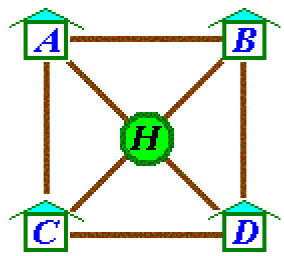
10. Adam Ant can travel only along the twigs connecting his home ***H*** with his aunts' homes *A*, *B*, *C*, *D*, as shown in the diagram.




One fine day, Adam decides to visit his aunt at house *A*. Then it is time to return home. Being a confused young ant, his choice of route is random (equal probability for each twig). Let $X$ = the number of aunts he visits before he next returns to his home. Thus $X = 1$ represents the event "Adam Ant returns to his home immediately after his initial visit to Aunt *A*" and $\mathrm{P}[X = 1] = 1/3$ .

(a) Find the probability mass function for the random quantity $X$.

Challenging questions:

(b) Find $\mathrm{E}[X]$.

(c) Show that the probability that Adam Ant never visits aunt $C$ is 4/7.

(d) What are the odds that Adam Ant visits both aunt $B$ and aunt $C$ at least once?

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11. A certain atmospheric sensing device has an advertised service life of one year, but it is vulnerable to severe icing. If it survives the full year, then the profit to the owner is 50 (in some units of currency). If an ice storm severe enough to be fatal to the device occurs during its service life, then the profit is reduced by 90 units, unless the owner had invested in protection (costing 15 units), in which case the profit is reduced by 20 units.

An analysis method is available (at a cost of 5 units) which will suggest whether or not such icing will occur during the service life of the device. The analysis is not perfect: in only 95% of all cases where icing fatal to the device occurs, the analysis correctly suggests that such icing will occur. The analysis also suggests fatal icing (incorrectly) in 20% of all cases when no such icing happens.

It is also known from past experience that the probability of icing fatal to the device during the year is 20%.

(a) Given that the analysis suggests that fatal icing will occur, find the probability that such icing happens.
(b) Given that the analysis suggests that fatal icing will not occur, find the probability that such icing does happen.
(c) Construct a decision tree for this problem and determine the optimum strategy for the owner.

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12. A box contains 12 good items and 3 defective items from a factory's production line. A manager selects four items at random from the box, without replacement. Let $X$ = (the number of defective items in the random sample).
(a) Show that the probability distribution of $X$ is **not** binomial.
(b) Find the probability mass function $p(x)$.
(c) Find $P[X = 4]$ (Note: this part can be attempted without part (b)).
(d) Find $P[X < 3]$ .
(e) If the random sample were drawn **with** replacement, would the probability mass function be binomial?
(f) If the random sample were drawn **with** replacement, what would the value of $P[X = 4]$ be?

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